Note on the ARCES Dispersion

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1 Context

As part of our radial-velocity (RV) follow-up of eclipsing-binaries (Gaulme et al., 2016), we measure if our spectra are properly calibrated in wavelength by checking whether telluric lines line up from epoch to epoch. We indeed find some residual shifts from spectrum to spectrum. My next question was: once we have measured a $\delta\lambda$, do we shift the whole spectrum of this amount, or do we shift it in a non-linear fashion? For example, in the paper Rawls et al. (2016), it is stated that measured shifts are applied in the velocity space, i.e., with a constant shift in terms of δV.

I have asked Russet McMillan at APO, who answered:
"I don't really know, but I thought it was a shift in pixel position actually. That's not the same as either lambda or RV. But this is based on the assumption that the shifts probably come from a motion of the grating, which hasn't really been proved. We just suspect the grating because the shifts got a little worse after the grating was changed out. But it's also possible that more than one thing is moving.

You should be able to answer this question by looking at the ThAr spectra: even if they don't have the full precision you need, they have lots of lines from blue to red. Compare two ThAr spectra at different times or temperatures, with a significant difference between them. Find out if those shifts match wavelength or RV or pixel position over the range of the spectrum, and then you will know what rule to apply to your telluric lines."

I will certainly take a look at ThAr lines, but at the same time I had already checked several groups of telluric lines, and it looked that a shift in RV seemed more appropriate (see Fig. 1). In this document, I show that shifts must indeed be applied in the velocity space.

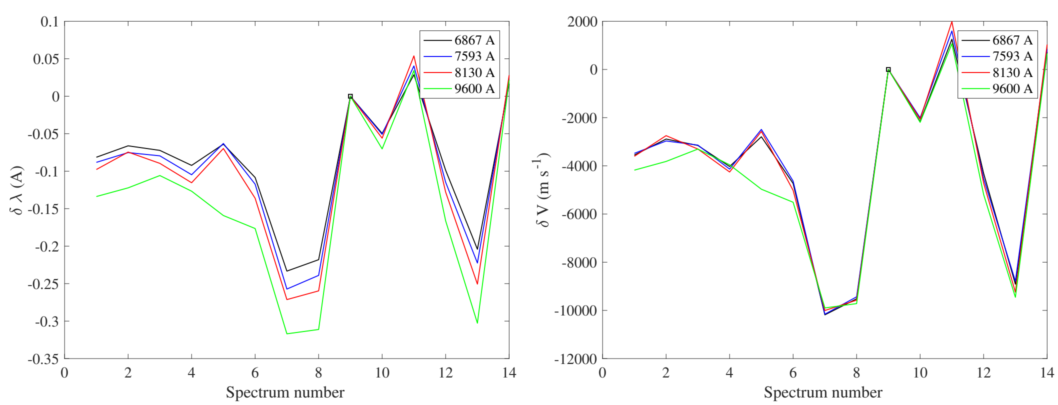

Figure 1: Relative positions of telluric lines as function of time (each spectrum was taken at different dates). Data were taken with ARCES on the KIC 4054905 *Kepler* target. The reference spectrum is indicated with a black square and was selected based on signal-to-noise (SNR) criteria. Shifts were measured on four groups of telluric lines, whose wavelengths are indicated in the legends of each panel (6867, 7593, 8130, and 9600 Å). Telluric lines at 9600 Å are noisier because of the detector's reduced sensitivity with respect to the mid visible domain. Shifts were measured from 1D spectra that were linearly sampled in wavelength, from a weighted cross-correlation (weighting based on the spectrum's derivative). The left panel displays the measured shift in wavelength (Å), while the right panel displays the same in RVs.

2 Spectrometer's dispersion law

On the Apache Point observatory's webpage, I found a table indicating the dispersion law of the échelle spectrometer (`http://www.apo.nmsu.edu/arc35m/Instruments/ARCES/`). More precisely, it displays the dispersion expressed in Å pixel^{-1} per order of the spectrometer. In Fig. 2, I plot the dispersion as function of the mean wavelength of each order. It appears to be almost strictly linear, with a dispersion ranging from about 0.045 Å pixel^{-1} at 3500 Å to 0.13 Å pixel^{-1} at 10,000 Å. I fitted the dispersion $D(\lambda)$ per pixel as function of wavelength:

$$D(\lambda) = A\lambda + B, \quad \text{where } A = 1.2719\ 10^{-5} \text{ pixel}^{-1}, \text{ and } B = -9.0208\ 10^{5} \text{ Å pixel}^{-1} \tag{1}$$

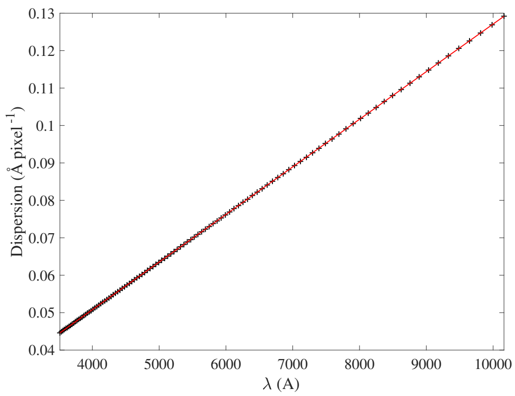

Figure 2: Dispersion of the ARCES échelle spectrometer in Å per pixel, as function of wavelength expressed in Å. Black crosses represent the values from the table, and the red curve is a linear fit of it.

To convert the impact of a pixel shift δx – uniform along the spectrum – into wavelength $\delta\lambda$, we must express the dispersion as function of pixels. The wavelength $\lambda(x)$ at a given pixel x expresses as follows:

$$\lambda(x) = \lambda_0 + \int_0^x D(\lambda(x))dx, \tag{2}$$

where $\lambda_0 \equiv \lambda(x = 0)$. We can derive both sides of the equation with respect to x, and obtain:

$$\frac{\mathrm{d}\lambda(x)}{\mathrm{d}x} = \frac{\mathrm{d}\lambda_0}{\mathrm{d}x} + D(\lambda(x)) \tag{3}$$

$$\text{i. e.,} \quad \frac{\mathrm{d}\lambda(x)}{\mathrm{d}x} = A\ \lambda(x) + B \tag{4}$$

The solution of this differential equation is

$$\lambda(x) = Ke^{Ax} - \frac{B}{A} \qquad \text{with} \qquad K = \left(\lambda_0 + \frac{B}{A}\right) e^{-A} \tag{5}$$

3 Shifts are in velocity space

It is now straightforward to demonstrate that a shift in pixels implies a uniform shift in velocity, and *not* in wavelength. By deriving Eq. 5, we get:

$$\frac{\mathrm{d}\lambda(x)}{\mathrm{d}x} = KAe^{Ax} \tag{6}$$

$$= AKe^{Ax} - B + B \tag{7}$$

$$= A\lambda(x) + B. \tag{8}$$

Therefore, for an infinitesimal shift δx along the x-axis, we have an infinitesimal change in wavelength $\delta\lambda$:

$$\delta\lambda = A\left(\lambda(x) + \frac{B}{A}\right)\ \delta x \tag{9}$$

By considering that $B/A = -7.0922$ Å, and λ belongs to the range $[3500, 10\,000]$ Å, it is safe to write:

$$\delta x \approx \frac{1}{A}\,\frac{\delta\lambda}{\lambda} \tag{10}$$

In other words:

$$\delta x \approx \frac{1}{A}\,\frac{\delta V}{c} \tag{11}$$

where c is the speed of light. A shift in x is thus proportional a shift in radial velocity.

4 References

Gaulme, P., McKeever, J., Jackiewicz, J., et al. 2016, ApJ, 832, 121

Rawls, M. L., Gaulme, P., McKeever, J., et al. 2016, ApJ, 818, 108