1. In a single quasar spectrum covering the redshift range $2.6 \leq z \leq 2.8$ you find many Ly$\alpha$ absorbers and obtain a list of H$\text{I}$ column densities [provided electronically on the web site as log $N(\text{H}I)$]. Compute the mean gas density, $\Omega_g$, and its uncertainty using both (a) the column density distribution method and (b) the mean column density method. Assume $H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Compare your results from the two methods.

Recall that

$$\Omega_g = \frac{H_0 \mu m_H}{c} \rho_c \langle N \rangle X(X),$$

where

$$\rho_c = \frac{3H_0^2}{8\pi G}.$$

(a) For the column density distribution method, use

$$f(N) = CN^{-\beta}$$

and assume $C = 1.4 \times 10^{10} [\Delta X^{-1} \text{ cm}^{-2}]$ with a 10% uncertainty, and $\beta = 1.5 \pm 0.05$.

(b) For the mean column density method, use

$$\langle N \rangle = \frac{1}{m} \sum_{i=1}^{m} N_i.$$ 

You will need to compute $\Delta X = X(z = 2.8) - X(z = 2.6)$ for your quasar; assume perfect sensitivity and an Einstein de–Sitter cosmology.

2. (a) Using Eq. 10 from Schaye (2001, ApJ, 559, 507), (i) accurately plot the redshift dependence of $N(\text{H}I)$ for density contrasts of $\delta = 1, 10, 100, 1000$. Cover the redshift range $0 \leq z \leq 4$, and assume $T_4 = 1, \Gamma_{12} = 1, \Omega_b h^2 = 0.02$, and $f_g = 0.16$. (ii) What approximate range of $N(\text{H}I)$ is observed for virialized halos, $50 \leq \delta \leq 100$, at $z = 0, 1, 2, 3, \text{ and } 4$? (iii) Qualitatively describe the behavior.

(b) Do Part (a) but include the redshift dependence of the photoionization rate, $\Gamma$. Use the parameterization from Haardt & Madau (1996, ApJ, 461, 20). The functional form is shown in their Figure 6 (upper panel). The parameterization is given by

$$\Gamma(z) = 6.7 \times 10^{-13} (1 + z)^{0.73} \exp \left[ -\frac{(z - 2.3)^2}{1.9} \right].$$

First, substitute this parameterization into Schaye’s Eq. 10 and derive the new functional form. How important do you find the evolution of the ultraviolet background to be in the formalism of Schaye?
HINTS FOR PROBLEM 1: Important equations are in the “book” (there is a typo in the expression for $\rho_c$, so be sure to use the above expression). Be sure to show which equations you are applying (write them out and show all calculations). Also, be sure to compute the uncertainty in $\Omega_g$ for both methods. Recall that $\Omega_g$ for this calculation is the mean gas density of neutral hydrogen only. Is your answer reasonable in terms of the known $\Omega_b$.

HINTS FOR PROBLEM 2: Be sure to get the form of the term $[(1+z)/4]^b$ and the leading constant correct. Then make the same accurate plot as in Part (a). Compare and discuss your results from the two methods.