ASTRONOMY 698 EXAM 1: Solutions

1. Lines in your \( z_{em} = 3.5 \) QSO spectrum. This problem requires only 10 calculations.

- All but the Mg\( \text{II} \) emission lines will be observed; it will be beyond the red limit of your spectrum. For those that will appear in the spectrum, the wavelength locations for each are

\[
\begin{align*}
\lambda_{em}(\text{Ly}\beta) &= (1 + 3.5) \times 1025.72 = 4615.75 \\
\lambda_{em}(\text{O} \text{vi}) &= (1 + 3.5) \times 0.5(1031.93 + 1037.62) = 4656.49 \\
\lambda_{em}(\text{Ly} \alpha) &= (1 + 3.5) \times 1215.67 = 5470.52 \\
\lambda_{em}(\text{C} \text{iv}) &= (1 + 3.5) \times 0.5(1548.20 + 1550.77) = 6972.68
\end{align*}
\]

where all wavelengths are in Å. These locations are plotted in the top panel of Figure 1.

- Absorbers are only seen blueward of the corresponding emission line. Thus, O\( \text{vi} \), Ly\( \alpha \), and C\( \text{iv} \) have upper redshift limits of \( z_{rabs} = 3.5 \). The lower redshift limits are given by the blue limit of the observed spectrum, \( \lambda_b = 3500 \) Å. These are given by

\[
\begin{align*}
z^b_{abs}(\text{O} \text{vi}) &= 3500/1031.93 - 1 = 2.39 \\
z^b_{abs}(\text{Ly} \alpha) &= 3500/1215.67 - 1 = 1.88 \\
z^b_{abs}(\text{C} \text{iv}) &= 3500/1548.20 - 1 = 1.26
\end{align*}
\]

The Mg\( \text{II} \) doublet can be seen for all observed wavelengths, which corresponds to the redshift range, 3500–9000 Å,

\[
\begin{align*}
z_{abs}(\text{Mg} \text{II}) &= 3500/2796.35 - 1 = 0.25 \\
z_{abs}(\text{Mg} \text{II}) &= 9000/2803.53 - 1 = 2.21 \\
\end{align*}
\]

where we have accounted for capturing both members of the doublet for \( z_{abs}^r(\text{Mg} \text{II}) \). These wavelength ranges are plotted on the center panel of Figure 1 and the redshift ranges are labeled.

- The wavelength range of Ly\( \alpha \) only lines is simply the region between the Ly\( \alpha \) and Ly\( \beta \) emission lines. The upper wavelength (redshift) of the range has been computed above to be \( \lambda 5470.52 \) (\( z = 3.5 \)). The lower range is given by the Ly\( \alpha \) absorber having a wavelength corresponding to the location of the Ly\( \beta \) emission line, \( \lambda 4615.75 \), which corresponds to a Ly\( \alpha \) absorption redshift of

\[
z_{abs}(\text{Ly} \alpha) = \lambda_{em}(\text{Ly} \beta)/1215.67 - 1 = 2.80
\]

These ranges are plotted on the lower panel of Figure 1.
2. Absorption Distance

- There are two effects due to the expansion of the universe. The first is along the line of sight. The term \( E(z) \) accounts for the “stretching” of the line of sight with decreasing redshift. The second is the transverse expansion of the universe. This is an decrease in the effective “area” probed. Actually, for fixed absorber cross sections, the absorbers move “away” from the line of sight in the direction perpendicular to the line of sight. This is described by the term \((1 + z)^2\) in the numerator. At higher redshift, the universe was more compact, so the probability of intersection from transverse expansion increases as the described by the numerator.

- A non-evolving population should have properties that are constant with absorption distance, \( X(z) \). Any departure that is significant within uncertainties, say 3\( \sigma \), would indicate evolution in the studied properties. Typically, the property that is constrained is \( N(z) = n(z)\sigma(z) \), or \( \Omega_{\text{gas}} \).

3. Doppler Parameters

- For an atomic weight, \( A \), the \( b \) parameter is
  \[
  b^2 = \frac{2kT}{Am_p} = \frac{2k}{m_p} \left( \frac{T}{A} \right) = 0.0165 \cdot \frac{T}{A} \quad \text{km s}^{-1})^2,
  \]
  where there are \( 10^5 \) cm/km. For \( T = 30,000 \) we have
  \[
  b(\text{Mg} \text{II}) = \left( 0.0165 \cdot \frac{30,000}{24} \right)^{1/2} = 4.55 \quad \text{km s}^{-1},
  \]
  and
  \[
  b(\text{Fe} \text{II}) = \left( 0.0165 \cdot \frac{30,000}{56} \right)^{1/2} = 2.98 \quad \text{km s}^{-1}.
  \]

- The turbulent component is given by
  \[
  b_{\text{turb}}^2 = \frac{b^2 - (m_1/m_2)b_1^2}{1 - (m_1/m_2)},
  \]
  where \( m_1/m_2 = 24/56 = 0.43 \); we have
  \[
  b_{\text{turb}}^2 = \frac{(10.5)^2 - (0.43)(11.0)^2}{1 - (0.43)} = 10.1 \quad \text{km s}^{-1}.
  \]

4. Photoionization Models

- (i) \((\log U, \log N_{\text{HI}}) \simeq (-1.5, 16.7)\) to \((-1.5, 17.4)\) to \((\log U, \log N_{\text{HI}}) \simeq (-4.5, 18.0)\) to \((-4.5, 16.4)\); (ii) Optically thick means LLS systems. For \( \log U \leq -2.5 \), pretty much \( \log N(\text{Mg} \text{II}) = 13.2 \text{ cm}^{-2} \). For \( \log U \geq -2.5, \ 12.6 \leq \log N(\text{Mg} \text{II}) \leq 13.2 \text{ cm}^{-2} \). (iii) For \( \log U = -2 \) and \( \log N_{\text{HI}} = 17.0 \text{ cm}^{-2}, \log N_{\text{HI}} = 20.4 \text{ cm}^{-2} \). The average ionization fraction of H1 is then \( f_{\text{HI}} = 10^{17.0-20.4} = 10^{-3.4} = 3.98 \times 10^{-4} \).
Changing the grid to solar metallicity is equivalent to sliding the vertically grid upward by an order of magnitude along the $N(\text{Mg} \, \text{II})$ axis. This results in there being less neutral hydrogen for the fixed observed Mg\,\text{II} column densities. Thought of another way (for heuristic purposes only), this can be done by sliding the $N(\text{Mg} \, \text{II})$ axis vertically downward, or by replacing 12 with 13, 11 with 12, etc, on the Mg\,\text{II} axis. (i) $(\log U, \log N_{\text{HI}}) \simeq (-1.5, 16.4)$ to $(-1.5, 16.9)$ to $(\log U, \log N_{\text{HI}}) \simeq (-3.0, 15.0)$ to $(-3.0, 16.2)$ to $(\log U, \log N_{\text{HI}}) \simeq (-4.5, 15.3)$ to $(-4.5, 16.4)$. Note the maximum in the curve for $\log U = -3.0$; (ii) Optically thick means LLS systems. For $Z/Z_{\odot}$, no model clouds are optically thick; the highest $N_{\text{HI}}$ is at $\log U = -1.5$ and is $\log N_{\text{HI}} = 16.9 \text{ cm}^{-2}$. iii) The ionization fraction is the same as for the 0.1 metallicity model.

Figure 1. — Solution to Problem 1.