

Name: \_\_\_\_\_  
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# 1 Discovering Exoplanets

## 1.1 Introduction

One of the most exciting discoveries in Astronomy over the last twenty years was the conclusive detection of planets orbiting other stars. At last count, we are closing-in on having discovered *two thousand* planets orbiting other stars. Planets orbiting other stars are called “exoplanets.” These exoplanets range in size from similar to the Earth, to larger than Jupiter. With much hard work, we now know that small exoplanets are much more common than big exoplanets, and some astronomers believe that Earth-sized planets orbit nearly every normal star. The current goal of astronomers is to find exoplanets that are most similar to Earth (same mass, radius, orbiting their host star at 1 AU, etc.). With improvements in technology, we will one day be able to determine whether such exoplanets support life. In the distant future, maybe we will be able to send a space probe to those exoplanets to investigate the life found there.

Astronomers have been studying the sky with advanced instruments for more than 100 years, but it was only in the early 1990’s that the first real exoplanets were found. Why did it take so long? The answer is that compared to their host stars, exoplanets are tiny, and hard to see. We will quantify how hard it is to see them shortly. First though, how might we discover such objects? There are three main techniques: direct imaging, transits (mini-eclipses), and “radial velocity” measurements. As its name suggests, direct imaging is simply taking a picture of a star and looking for its planets. The big problem is that the star is very bright (it generates its own energy), while an exoplanet shines by reflected light from the star. This is by far the hardest method to find exoplanets. To be effective, we will need to launch special telescopes into space where our image-disturbing atmosphere does not exist, allowing us to see much, much more clearly.

The transit method is much easier in that what we monitor is the light output from a star, and if an exoplanet crosses in front of the star, the light briefly dims. As we will learn, this technique also tells us the *diameter* of the exoplanet. The radial velocity method uses the Doppler effect to detect the orbital motion of the planet. The radial velocity technique allows us to determine the *mass* of the exoplanet. If we can combine the transit and radial velocity techniques, we can get the size and mass of a planet, and thus measure its *density*, and therefore constrain its composition. We will investigate all three methods in this lab, and then learn how we can characterize the properties of these objects.

## 2 Why are Exoplanets so hard to see?

In our first experiment, we are simply going to demonstrate how hard it is to directly see an exoplanet. First, however, a diagram to remind you how small the Earth and Jupiter are compared to the Sun (Figure 1).

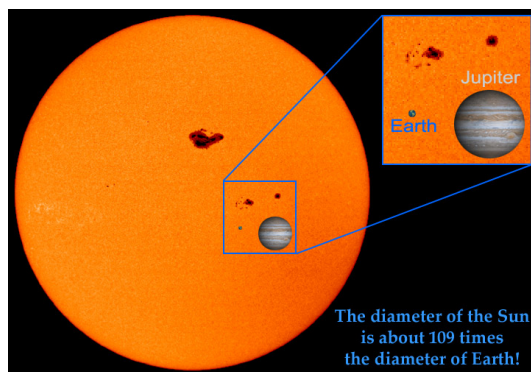


Figure 1: Comparison of the size of the Earth and Jupiter to the Sun.

Let's look at some numbers. The radius of the Sun is 695,550 km, the radius of Jupiter is 69,911 km, and the radius of the Earth is 6,371 km. Note that these objects are all spheres, and thus when we look at them from space, they all appear to be circles ("disks"). What is the area of a circle?  $A_{\text{circle}} = \pi R^2$ .

**Exercise #1:** Calculate the areas of the circular disks of the Sun, Jupiter, and Earth (if you want make the calculation simpler, just set  $R_{\text{Sun}} = 700,000$  km,  $R_{\text{Jupiter}} = 70,000$  km, and  $R_{\text{Earth}} = 6000$  km). **(3 points)**.

(Area of Earth) = \_\_\_\_\_  $\text{km}^2$

(Area of Jupiter) = \_\_\_\_\_  $\text{km}^2$

(Area of Sun) = \_\_\_\_\_  $\text{km}^2$

If we are going to take a picture of a exoplanet around a star, we have two problems: how much light will the exoplanet reflect compared to its star, and how close-in is it? Let's tackle the first question.

**Exercise #2:** We are going to keep everything very simple, and just estimate how much sunlight the Earth or Jupiter would reflect compared to that emitted by the Sun. We will assume that these planets reflect 100% of the light that hits them, and we are going to ignore the fact that the amount of sunlight at the orbits of each of these planets is less

than at the surface of the Sun (remember, the amount of light passing through a sphere surrounding a light source drops off as  $1/R^2$ ). In this unrealistic scenario, the maximum amount of light that a planet can reflect is simply the ratio of its area to that of the star it orbits. Calculate the following: **(2 points)**.

(Area of Earth)/(Area of Sun) = \_\_\_\_\_

(Area of Jupiter)/(Area of Sun) = \_\_\_\_\_

These already small numbers are actually way too big. As we noted, Earth can only reflect the amount of light it intercepts at the distance it is from the Sun. In fact, the Earth only intercepts  $1.67 \times 10^{-9}$  of the Sun's light output, and the amount of visible light it reflects (its "albedo") is 40%. So, seen from distant space, the Earth is only *one billionth* as bright as the Sun! Jupiter is obviously much bigger than the Earth, but remember, Jupiter is at 5.2 AU, so it actually receives  $1/27^{\text{th}}$  the amount of sunlight as does the Earth. Thus, to an observer outside our solar system, Jupiter is only 4.4 times more luminous than the Earth.

Directly detecting exoplanets is going to be hard, besides being very faint, they are located very close to their host stars. We need a way to "turn off" the star. One way to do this is to block its light out with a small, opaque metal disk. As shown in Figure 2, we now have the capability to do this, but only for finding big planets located far from their host stars (in fact, to date, only Jupiter-sized planets located at large distances from their host stars have been directly imaged). There is a more complex technique called "nulling interferometry" where you use the star's own light to cancel itself out, but not its planets, that lets astronomers search for planets closer to the host star. While it can be done from the ground, it is better from space. You can more read about this method by searching for the canceled NASA mission "Terrestrial Planet Finder" on the web (it was killed due to budget cuts).

### 3 Exoplanet Transits

The exoplanet transit method of discovery is simple to envision, and the easiest to carry-out. As shown in Figure 3, a transit occurs when an exoplanet crosses the disk of its host star as seen by observers on Earth. Since the planet does not emit any light (we are looking at the "nighttime side"), it is completely dark. Thus, the amount of light from the star will dim as the planet blocks out ("eclipses") a small portion of the star's light-emitting disk. The plot of the brightness of a star versus time is called a "light curve". The light curve of the transit is shown below the cartoon of the star and exoplanet in Figure 3.

#### Exercise #3: Simulating an Exoplanet transit

As part of the materials set out for you to use during today's lab is a device to simulate an exoplanet transit. Take a look at the wooden device. It has a light meter attached to the

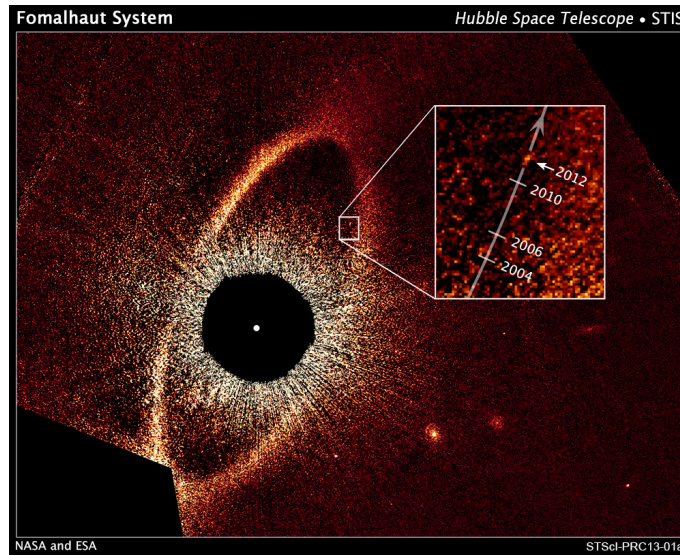


Figure 2: A planet orbiting the star Fomalhaut (inside the box, with the arrow labeled “2012”). This image was obtained with the Hubble Space Telescope, and the star’s light has been blocked-out using a small metal disk. Fomalhaut is also surrounded by a dusty disk of material—the broad band of light that makes a complete circle around the star. This band of dusty material is about the same size as the Kuiper belt in our solar system. The planet, “Fomalhaut B”, is estimated to take 1,700 years to orbit once around the star. Thus, using Kepler’s third law ( $P^2 \propto a^3$ ), it is roughly about 140 AU from Fomalhaut (remember that Pluto orbits at 39.5 AU from the Sun).

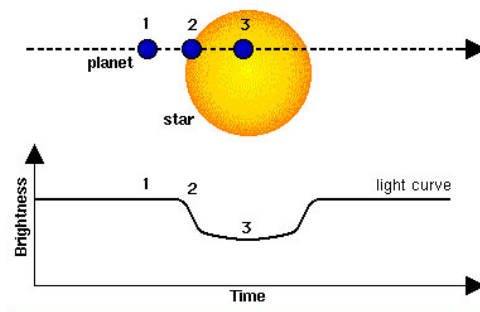


Figure 3: The diagram of an exoplanet transit. The planet, small, dark circle/ disk, crosses in front of the star as seen from Earth. In the process, it blocks out some light. The light curve shown on the bottom, a plot of brightness versus time, shows that the star brightness is steady until the exoplanet starts to cover up some of the visible surface of the star. As it does so, the star dims. It eventually returns back to its normal brightness only to await the next transit.

back, and three rods that dangle down in front of the light meter. We will use the desk lamp as our light source (“star”), and move the rods across the light meter. Note that the dowel rod on top has five notches. The two furthest from the center represent where we

will be at the start and end of our simulation. At these positions, no light is blocked (similar to position #1 in Figure 3). Note also that we have two planets, one big, one small, and a bare rod without a planet. What do you think the latter is used for? Yes, our planets need to be attached to *something* to allow us to perform this experiment. Thus, this planet-less rod allows us to measure how much light just the bare rod blocks out. We will have to take this into account when we plot our light curves.

The light meter itself is rather simple, it has power, mode, and hold buttons. We only will use the power and mode buttons. Hit the power button (note that to extend battery life, the device automatically shuts off after 45 seconds). At the bottom of the device display window there will either be a “LUX” or “FC” displayed. We want the unit to be in LUX, so click the mode button until LUX is displayed.

**Setting things up:** Move all of the metal rods to the left end of the dowel rod so that nothing is blocking the lamp light from illuminating the white circle. Turn on the light meter. Note that the number bounces around—this is due to electronic noise. Every electronic device has this type of noise, and it takes much hard work (and expense) to minimize this noise (one way is to chill the device to low temperatures). Here we have to live with it, but this is just like what an astronomer would have to deal with in a real observation. You are going to have to make a mental average of the values at each measurement point. For example, in five seconds, if the numbers are 78, 81, 79, 82, and 78, we would just estimate the count rate as “80”. Note: the light meter is very sensitive, so you must keep yourself and your hands well away from the front of the device when making a measurement (the meter will detect light reflected off of *you*, making it hard to figure out what is going on!).

With the room lights turned off, set the desk lamp about two feet in front of the transit device. Power on the light and the light meter. With no rods in front of the glass disk, adjust the *height and direction* of the desk lamp to maximize the number of counts. Make sure the light bulb in the lamp is at roughly the same height as the round, glass disk in front of the light meter. One way to do this is move the big planet in front (putting the rod in the centermost notch) and make sure its shadow hits the center of the glass disk. Move all of the rods out of the way, and then move the transit device closer to the lamp until it gives a reading above 200 counts.

Now we are simply going to move each of the three rods (Bare Rod, Small Planet, Large Planet) into the five notches on the top dowel rod, and write down the average value of the light meter measurement at each position into Table 1. We do this one rod at a time. Once done, move that rod to the far right side of the dowel rod to start the process for the next rod. The rods may swing around a bit, just let them stop moving, back away from the front of the device, and take your measurement. It sometimes takes a few seconds for the light meter to settle to the correct value, so give it a few seconds, and then make a estimate of the average light value at this position. Note: if you accidentally bump the lamp or transit device you have to start over! Small changes in the separation or lamp height will result in bad data.

Now we have to account for the dimming effect of the rod. First, add the bare rod

Table 1: Exoplanet Transit Data

Position	Bare Rod	$\delta$	Small P.	S.P.+ $\delta$	Large P.	L.P.+ $\delta$
#1		0.0				
#2						
#3						
#4						
#5		0.0				

measurements at positions #1 and #5 together and divide by 2 to create the average unobscured value. Now, in the column labeled “ $\delta$ ”, fill in the differences between the average unobscured value you just calculated, and your bare rod measurements at positions 2 through 4 ( $\delta = \text{Ave.} - \#2$ , etc.). Then in columns 5 (S.P. +  $\delta$ ) and 7 (L.P. +  $\delta$ ), add the value in column 2 to the measurements in columns 4 and 6, respectively for all five measurements [obviously, you add the value of  $\delta$  at position #2 to the value of Small P. at position #2 to get the value of (S.P. +  $\delta$ ) at position #2]. **(14 points)**

### Making Light Curves

Now we want to plot the data in Table 1 to make a light curve for our two planets. Plot your data on the graph paper in the next two windows. We have filled-in the X axis with notation for the five positions you measured. You will have to put values on the Y axis that allow the entire light curve to be plotted. For example if the unobscured value was near 285 (positions #1 or #5), the top Y axis grid line might be set to 300. If the value at position #3 was 223, the bottom of the Y axis could have a value of 200. It depends on your light meter, and how bright the light source was. You will have to decide how to label the Y axis! Plot the data for both planets in Figures 4 and 5. **(8 points)**.

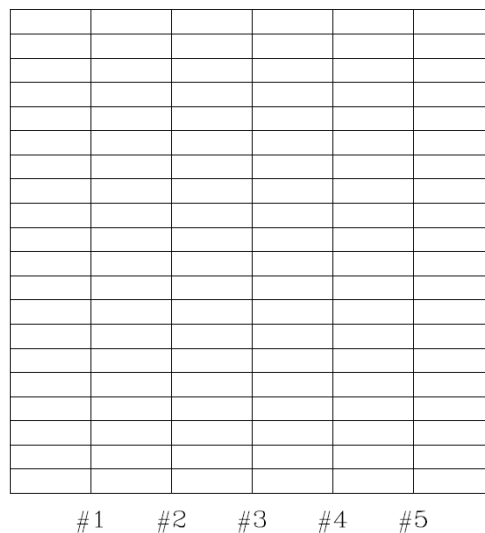


Figure 4: The light curve of the transit of the small planet.

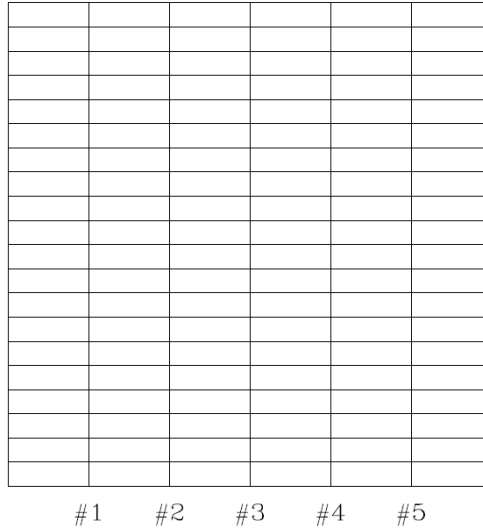


Figure 5: The light curve of the transit of the large planet.

### 3.1 Real exoplanet transits

Now that we have seen how one might observe a real exoplanet transit, and construct its light curve. We now want to examine how hard this really is. You probably have already found the dimming signal due to the small planet was quite small. Let's calculate how much the light dimmed in our simulations so we can compare them to real exoplanet transits. First we need to find the difference between the unobscured value, and the value at position #3 for both planets: **(2 points)**

Total dimming small planet = (Position #1) – (Position #3) = \_\_\_\_\_ counts

Total dimming large planet = (Position #1) – (Position #3) = \_\_\_\_\_ counts

Now let's put this in the fractional amount of dimming (“ $\Delta F/F$ ”):

Fractional dimming small planet = (Total dimming small planet)/(Position #1) = \_\_\_\_\_

Fractional dimming large planet = (Total dimming large planet)/(Position #1) = \_\_\_\_\_

How does this compare to the real world? You actually already calculated the percent dimming for the Earth and Jupiter in Exercise #2. In that exercise we calculated the ratio of the areas of the planets relative to the Sun—this ratio is in fact how much the light from the Sun would dim (in fractional terms) when the Earth or Jupiter transited it as seen from a very distant point in space (or as some alien would measure watching those crazy exoplanets

transit the star we call the Sun!).

### Questions:

1) Compare the percent dimming of our simulated exoplanets to the values for the Earth and Jupiter found in Exercise #2. Was our simulation very realistic? **(2 points)**

2) Let's imagine an alien pointed their telescope at our Sun to watch a transit of the Earth. If his light meter was measuring 25,000 counts from the Sun before the Earth transited (i.e., Point #1), what would it read at mid-transit (i.e., Point #3)? Show your math. [Hint: remember the dimming is very small, so the mid-transit number will be very close to the unobscured value.] **(2 points)**

As you have now seen, detecting planets around other stars is very hard. The amount of dimming during a transit is only about 1% for a Jupiter-sized exoplanet that orbits another star. To make such high precision measurements, especially to see Earth-sized planets, requires us to get above the Earth's atmosphere and use special detectors that have very low noise. Note that we also have to observe for a very long time—the Earth only has one transit per year! Jupiter would have one every 12 years! These events only last a few hours, so we also have to observe the star continuously so we do not miss the transit. This requires a dedicated instrument, and this need was the genesis of the Kepler mission launched by NASA several years ago. Kepler detected over 1,000 transiting exoplanets during its four year mission. Unfortunately, Kepler is no longer fully functional, and it will not be able to continue searching for Earth-like planets.

Before we leave the subject of transits behind, we want to talk a little more about how we can use light curves to get actual information on the exoplanet. In Figure 6 is plotted an



exoplanet transit and light curve, with all of the math (scary, eh?) that needs to be taken into account to decipher exactly what is going on (actually the math is not real scary, as it is derived from Kepler's laws). In the preceding we have assumed that the planet crosses the center of the star—but this almost never happens. The orbit is tilted a little bit, so the transit path is shortened. There are ways to figure all of this out, as demonstrated by the many math equations in this figure. But we want to focus your attention on the most important result that a transit tells you: the radius of the exoplanet. In the top corner of Figure 6 there is a simple equation:  $\Delta F/F = (R_p/R_*)^2$ . As we have calculated above, the depth of the eclipse,  $\Delta F/F$ , allows you to determine the radius of an exoplanet. As all of the math in this figure shows you, if you can estimate the stellar parameters ( $R_*$ ,  $M_*$ ), you can also determine other characteristics of the exoplanet orbit (semi-major axis, orbital inclination). It is fairly simple to estimate  $R_*$  and  $M_*$ . In fact, if we measure the period of the orbiting planet, we can measure the mass of the host star using Kepler's laws (the  $P^2 = 4\pi^2/GM_*$  equation). Thus observing transits provides much insight into the nature of an exoplanet, its orbit, and the host star.

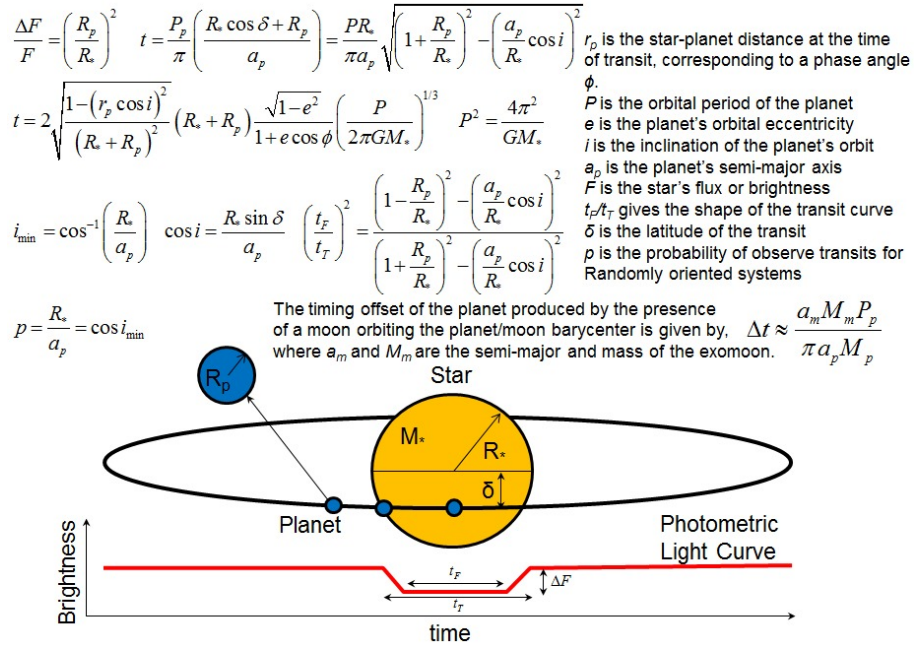


Figure 6: An exoplanet transit light curve (bottom) can provide a useful amount of information. As we have shown, the most important attribute is the radius of the exoplanet. But if you know the mass and radius of the exoplanet host star, you can determine other details about the exoplanet's orbit. As the figure suggests, by observing multiple transits of an exoplanet, you can actually determine whether it has a moon! This is because the exoplanet and its moon orbit around the center of mass of the system ("barycenter"), and thus the planet appears to wobble back and forth relative to the host star. We will discuss center of mass, and the orbits of stars and exoplanets around the center of mass, in the next section.

## 4 Exoplanet Detection by Radial Velocity Variations

The final method we are going to investigate today, the technique of radial velocity variations, is the most difficult to understand as we have to talk about “center of mass”, the Doppler effect, and spectroscopy. You have probably heard about all three of these during the lectures over this past semester, but we are sure you need to have a review of these topics.

You are certainly aware of the concept center of mass, even if you never knew what it was called. Take the pencil or pen that you have with you today and try to balance it across the tip of your finger. The point on the pencil/pen where it balances on your finger tip is its center of mass. A teeter totter is another good way to envision the center of mass. If a small kid and a big kid are playing on the teeter totter, the balance is not good, and it is hard to have fun. You need to either adjust the balance point of the teeter totter, or have two kids with the same weight use it.

A diagram for defining the center of mass for two objects with different masses is shown in Figure 7. If the two objects had the same mass, the center of mass would be halfway between them. If one object has a much bigger mass, than the center of mass will be located closer to it. You have a device today that clearly demonstrates this type of system.



Figure 7: Center of mass, “ $x_{CM}$ ”, for two objects that have unequal masses. The center of mass can be thought as being the point where the system would balance on a “fulcrum” if connected by a rod.

### Exercise #4: Defining the Center of Mass for a Two Body System

As part of the materials for today’s lab, you were given a center of mass demonstrator. It consists of a large black mass connected to a small white mass by a long rod. There is also a wooden handle with a small pin at one end.

Remove the wooden handle from the long rod. Using the meter stick, estimate the length of the entire device, from the **center** of the black sphere (we will call it “ $M_1$ ”), to the **center** of the white sphere (“ $M_2$ ”). What is this number in cm? (1 point):

Ok, now find the halfway point from the center of one ball to the next. You need to divide the length you just measured by two, and measure in from one of the balls and note its location (if necessary, use a piece of tape). Is there a hole there? If you try to balance the device on the tip of your finger at this center point/hole, what happens? **(2 points)**

Now put the device on the tip of your finger and find the balance point of the device. There is also a hole there. Use the meter stick to estimate (and write down) the distance between the center of the black ball to this point (we will call this “ $X_1$ ”), and the distance between the center of the white ball and this point (we will call this “ $X_2$ ”). This exercise is best done by two people. **(2 points)**

$X_1 =$  \_\_\_\_\_ cm

$X_2 =$  \_\_\_\_\_ cm

This spot on the rod is “the center of mass”. The center of mass point is important, as it allows us to determine the “mass ratio”, and if we know the mass of one of the objects, we can figure out the mass of the other object. The equation for center of mass is this:

$$M_1X_1 = M_2X_2$$

and the mass ratio is:

$$M_1/M_2 = X_2/X_1$$

Determine the mass ratio for the center of mass device. **(2 points)**

$M_1/M_2 =$

If  $M_1 = 250$  grams, what is the mass of  $M_2$ ? **(2 points)**

$M_2 =$  \_\_\_\_\_ grams

Now that we have explored the concept of center of mass, let’s see how it applies to objects that orbit each other. Inserting the pin on the wooden handle into the center point of the rod (not the center of mass hole!), hold the wooden handle and try to spin the device.

Now, move the wooden handle to the center of mass hole. Spin the device. Explain what happened at both locations: **(2 points)**

Any two objects in orbit around each other actually orbit the center of mass of the system. This is diagrammed in Figure 8<sup>1</sup>. Thus, the Earth and Sun orbit each other around their center of mass, and Jupiter and the Sun orbit each other around their center of mass, etc. In fact, the motion of the Sun is a complex combination of the orbits of all of the planets in our solar system. For now, we are going to ignore the other planets, and figure out where the center of mass is for the Sun–Earth system.

The Sun has a mass of  $M_{\text{Sun}} = 2.0 \times 10^{30}$  kg, while the Earth has a mass of  $M_{\text{Earth}} = 6.0 \times 10^{24}$  kg. We will save you some math and just tell you that the approximate mass ratio is:

$$M_{\text{Sun}}/M_{\text{Earth}} = 330,000$$

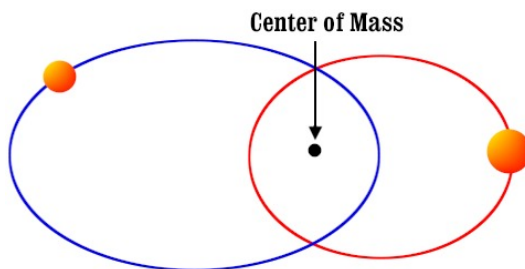


Figure 8: If two stars are orbiting around each other, or a planet is orbiting a star, they both *actually* orbit the center of mass. If the two objects have the same mass, the center of mass is exactly halfway between the two objects. Otherwise, the orbits have different sizes.

To determine where the center of mass is for the Earth-Sun system, we have to do a little bit of algebra. Remember that the mean distance between the Earth and the Sun is 1 AU. Thus, using our notation from above:

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<sup>1</sup>An animation of this can be found at <http://astronomy.nmsu.edu/tharriso/ast105/Orbit3.gif>

$$1 \text{ AU} = X_1 + X_2$$

Therefore,

$$X_1 = 1 \text{ AU} - X_2 \quad (\text{Equation \#1})$$

Does that make sense to you?  $X_1$  and  $X_2$  are the distance from the Sun to the center of mass, and the Earth to the center of mass, respectively. As the center of mass demonstration device shows you, the center of mass is located somewhere on the line that connects the two objects. Thus,  $X_1 + X_2 = \text{distance between the two masses}$ . For the Earth and Sun,  $X_1 + X_2 = 1 \text{ AU}$ . Now, going back to our center of mass equation:

$$M_1 X_1 = M_2 X_2 \quad (\text{Equation \#2})$$

We can substitute the result for  $X_1$  in equation #1 into Equation #2:

$$M_1(1 \text{ AU} - X_2) = M_2 X_2 \quad (\text{Equation \#3})$$

Dividing both sides of Equation #3 by  $M_1$  gives:

$$1 \text{ AU} - X_2 = (M_2/M_1)X_2 \quad (\text{Equation \#4})$$

But  $(M_2/M_1) = 1/330,000$  for the Earth-Sun system, and now we can solve to find  $X_2$ :

$$1 \text{ AU} = (M_2/M_1)X_2 + X_2 = (1/330,000 + 1)X_2$$

Thus,

$$X_2 = 1.0 / (1 + 1/330,000) = 0.999997 \text{ AU}$$

Essentially, the Earth is 1 AU from the center of mass, how far away is the Sun from the Earth-Sun center of mass? Go back to equation #1:

$$X_1 = 1 - 0.999997 = 0.000003 \text{ AU}$$

The Sun is very close to the center of mass of the Earth-Sun system.

### **Exercise #5: Determining the Size and Velocity of the Sun’s “Reflex Motion”**

We are going to calculate the size of the Sun’s orbit around the center of mass for the Sun-Earth system, and then determine how fast the Sun is actually moving. The motion of the Sun (or any star) due to an orbiting planet is called the “reflex motion”. Like the name suggests, it is the response of the star to the gravitational pull of the planet. Since AU per year is not a normal unit with which to measure velocity, we need to convert the numbers we have just calculated to something more useful.

1 AU = 149,597,871 km. How far from the center of mass is the Sun in km? **(1 point):**

$$X_1 \text{ (km)} = X_1 \text{ (AU)} \times 149,597,871 \text{ (km/AU)} = \underline{\hspace{2cm}} \text{ km}$$

Hopefully, you noticed how the units of length canceled in the last equation.

So, we now have the distance of the Sun from the center of mass. Note that this number puts the center of mass of the Earth-Sun system well inside the Sun (actually very close to its core). We now want to figure out what the length of the orbit is that the Sun executes over one year (remember, the Earth takes one year to orbit the Sun, so the “orbital period” of the Sun around the Earth-Sun center of mass will be one year). Referring back to the center of mass device, if you put the handle in the center of mass hole and spin the system, what path do the masses trace? That’s right, a circle. Do you remember how to calculate the circumference of a circle?  $C = 2\pi R$ , where  $R$  is the radius of the circle and  $\pi = 3.14$ .

What is the circumference of the orbit circle (in km) that is traced-out by the Sun? **(2 points):**

This is how far the Sun travels each year, thus we can turn this into a velocity (km/hr = kph) by dividing the distance traveled (in km) by the number of hours in a year. Show your math **(2 points):**

$$V_{\text{Sun}} \text{ (km/hr)} = C \text{ (km)} \div (\# \text{ hours in year}) = ???$$

1) Comment on the size of the reflex velocity ( $V_{\text{Sun}}$ ) of the Sun. Note that the Earth travels much, much further during the year, so its velocity is much, much higher: 107,000 km/hr! **(3 points)**:

Because all of the math above involved simple, “linear” equations, we can quickly estimate the reflex velocity of the Sun if we replaced the Earth by something more massive. For example, if we put an object with 10 Earth masses in an orbit with  $R = 1$  AU, the reflex velocity of the Sun would be 10 times that which you just calculated for the Earth.

2) Jupiter has a mass that is 318 times that of the Earth. If Jupiter orbited the Sun at 1 AU, what would the reflex velocity of the Sun be? **(2 points)**:

Since Jupiter is at 5.2 AU, and its orbital period is 11.9 yr, the reflex motion of Jupiter is actually:  $V_{\text{Jupiter}} = 318 \times V_{\text{Earth}} \times 5.2 \div 11.9 \approx 45$  km/hr.

### **Exercise #6: Understanding the Sizes of the Reflex Motions**

For the final exercise of today’s lab, we want to demonstrate how big these reflex motions are by comparing them to the velocities that you can generate. To do so, we are going to be using radar guns just like those used by the police to catch speeders. These devices are very expensive, so please be extremely careful with them. The radar guns are a bit technical to set-up, so your TA will put them in the correct mode for measuring velocities in km/hr.

Your lab group should head out of the classroom, and into the hallway (or outside) to get a long enough path to execute this part of the lab. The idea is to have one of the lab members move down the hallway, and act as the “speeding car”. Note that if there are other people moving around in the hallway, the radar gun might get a confusing signal and not read correctly. So, make sure only one person is moving when doing this.

1) One lab member hold the radar gun, have another lab member walk towards the radar gun. Hold down the trigger a few seconds and then let go. Do this several times to get a good reading. What is the average velocity of the walking speed of this lab member? (**2 points**):

2) Now, we are going to measure the running speed. **BE CAREFUL!**. Have everyone participate, and see who can run the fastest. What are the velocities for the various lab members? (**2 points**):

3) Compare your walking and running velocities to the Sun's reflex velocity caused by the Earth that you calculated above. How massive a planet (in Earth masses) would it take to get your walking reflex motion to be executed by the Sun? How about your running reflex motion? (**5 points**).



## 5 Radial Velocity and the Doppler Effect

Earlier we called this final exoplanet discovery technique “the radial velocity” method. What do we mean by this term? The radial velocity is a measurement of how fast something is coming towards you, or going away from you. If an object is moving across your line of sight (like the cars on the road as you wait to cross a street at the pedestrian crossing), it has no radial velocity (formally, they would have a “tangential velocity” only). If we were an alien watching the Sun, the Sun would sometimes have a radial velocity coming towards us (normally defined to be a negative number), and a radial velocity going away from us (normally defined as a positive number), due to the reflex motions imparted on it by the planets in our solar system. This gives rise to something called a “radial velocity curve”.

So how do we detect the radial velocity of a star? We use something called the Doppler effect. The Doppler effect is the change in frequency of a sound or light *wave* due to motion of the source. Think of an ambulance. When the ambulance is coming towards you, the siren has a high pitch. As it passes by you, the pitch drops (for audio examples, go here: <http://www.soundsnap.com/search/audio/doppler/score?page=1>). This is shown in Figure 9. The radar guns you just used emit microwaves that are Doppler shifted by moving objects. Stars are too far away to use radar. Fortunately, the same process happens with all types of electromagnetic radiation. Astronomers use visible light to search for Exoplanets. In a source coming towards us the light waves get compressed to higher frequency. When it is receding the light waves are stretched to lower frequency. Compressing the frequency of light adds energy, so it “blueshifts” the light. Lowering the frequency removes energy, so it “redshifts” the light. For an object orbiting the center of mass, sometimes the light is blueshifted (at point #4 in Figure 10), sometimes it is redshifted (at point #2 in Figure 10).

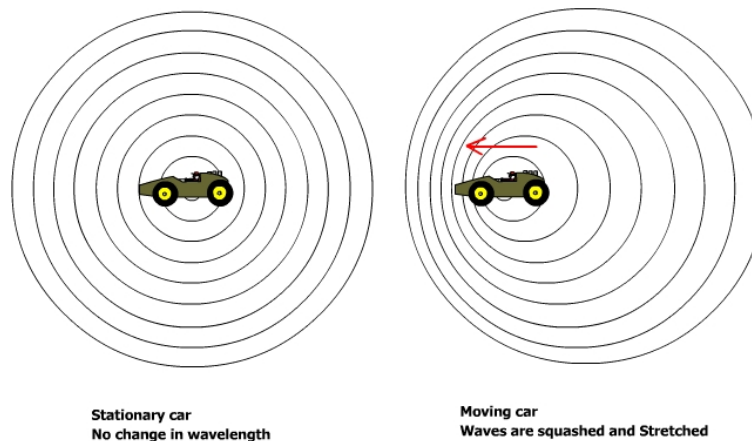


Figure 9: For a stationary vehicle emitting sound, there is no Doppler effect. As the vehicle begins to move, however, the sound is compressed in the direction it is moving, and stretched-out in the opposite direction.

This is how astronomers discover exoplanets, they monitor the spectrum of a star and look for a changing radial velocity like that shown in Figure 10. What they see is that the

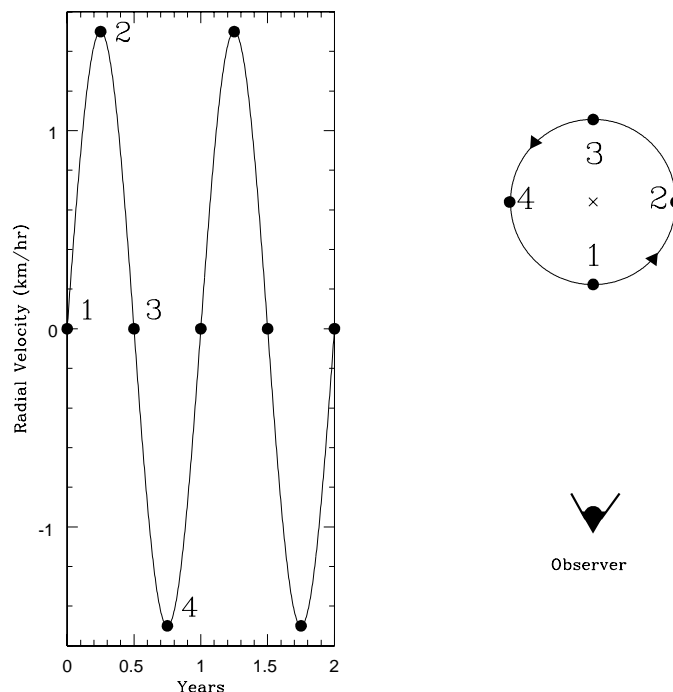


Figure 10: A radial velocity curve (left) for a planet with a one year orbit like Earth, but that imparts a reflex velocity of 1.5 km/hr on its host star. When the motion is directly away from us, #2, we have the maximum amount of positive radial velocity. When the motion of the object is directly towards us, #4, we have the maximum negative radial velocity. At points #1 and #3, the object is not coming towards us, or going away from us, thus its radial velocity is 0 km/hr. The orbit of the object around the center of mass (“X”) is shown in the right hand panel, where the observer is at the bottom of the diagram. The numbered points represent the same places in the orbit in both panels.

absorption lines in the spectrum of the exoplanet host star shift back and forth, red to blue to red to blue. Measuring the shift gives them the velocity. Measuring the time it takes to go from maximum blueshift to maximum redshift and back to maximum blueshift, is the exoplanet’s orbital period. Remember, the exoplanet is too faint to detect directly, it is only the reflex velocity of the host star that can be observed. And, now you should understand how we measure the mass of the exoplanet. The amount of reflex velocity is directly related to the mass of the exoplanet and the size of its orbit. We can use the orbital period and Kepler’s laws to figure out the size of the exoplanet’s orbit. We then measure the radial velocity curve, and if we can estimate the host star’s mass, we can directly measure the mass of the exoplanet using the techniques you have learned today.

Here is how it is done. To determine the mass of an exoplanet, we first must figure out the semi-major axis of its orbit (for the Earth, the semi-major axis =  $R = 1$  AU). We return to Kepler’s laws:

$$R^3 = \frac{GM_{star}}{4\pi^2} P^2 \quad (Equation \#5)$$

In this equation, “G” is the gravitational constant. P is the orbital period. In physics equations like these, the *system* of units used must be the same for each parameter. Such as centimeter-gram-second, or meter-kilogram-second. We call these the “cgs” and “mks” systems, respectively. You cannot mix and match. Thus, there have to be two flavors of G for this equation:  $G_{cgs} = 6.67 \times 10^{-8}$ , and  $G_{mks} = 6.67 \times 10^{-11}$ . The equation above is just Kepler’s third law  $P^2 \propto a^3$  you learned about at the beginning of the semester. What Isaac Newton did was figure out what is needed to change the “ $\propto$ ” into the “=” sign. If we know “R” and the exoplanet host star mass ( $M_{star}$ ) we can figure out the exoplanet’s mass. So using equation #5 above, we find R. Since we know the orbital period (P), we can estimate the exoplanet’s orbital velocity:

$$V_{pl} = \frac{2\pi R}{P} \quad (Equation \#6)$$

The mass of the planet is simply:

$$M_{pl} = \frac{M_{star} V_{star}}{V_{pl}}$$

In this equation  $V_{star}$  is the host star reflex velocity like those we calculated above for the Earth-Sun, and Jupiter-Sun systems. The biggest unknown when making such mass measurements is estimating the host star mass. There are ways to do this, but they are beyond the scope of today’s lab. We will use these equations in the take-home part of this lab, so make sure you understand what is going on here before leaving today.

Name: \_\_\_\_\_  
Date: \_\_\_\_\_

## 5.1 Take-Home Exercise (35 points total)

1) Discuss what you have learned about Exoplanets today. How hard are they to detect, and what are the main techniques astronomers use to find them? **(10 points)**

2) You have obtained a radial velocity curve for a transiting exoplanet that orbits the star 18 Scorpii, which is referred to as a “solar twin” (that is, identical in every way to the Sun). The planet has an orbital period of 400 days. The maximum reflex velocity of the star 18 Scorpii is 1 m/s. What is the mass of this exoplanet? You are going to need to use equations #4 and #5, and the mass (listed in Exercise #4) and radius (from Exercise #1) of the Sun. Remember that you must use a consistent set of units. In equation #4 “**G**” is listed in “mks”. Thus, the period of the exoplanet must be converted from days to seconds, and the mass and radius of the Sun must be in kilograms and meters, respectively, to correctly use equations #4 and #5. Compare the mass of this exoplanet to the mass of the Earth ( $6.0 \times 10^{24}$  kg). Show your work. **(10 points)**

3) As we noted, the exoplanet around 18 Scorpii is also a transiting system, and  $\Delta F/F = 4.8 \times 10^{-4}$ . Calculate the radius of this planet (like those in Exercise #3). Compare it to the radius of the Earth. What is the density of this planet in  $\text{kg/m}^3$ ? [Hint: density = mass/volume. What is the volume of a sphere?] Compare this to the density of the Earth ( $5,511 \text{ kg/m}^3$ ), and Jupiter ( $1,326 \text{ kg/m}^3$ ). Is “18 Scorpii B” more like a Terrestrial planet, or a Jovian planet? Show your work. **(15 points)**

## 5.2 Possible Quiz Questions

- 1) What is an Exoplanet?
- 2) Name one of the techniques used to find Exoplanets
- 3) Why are Exoplanets so hard to discover?

## 5.3 Extra Credit (make sure you get permission from your TA before attempting, 5 points)

Using the web, search for an article on an “Earth-like exoplanet” and write a one page discussion of this object, and what makes it “Earth-like”. Note that there are quite a few such objects, just pick the one you find most interesting (and one that has sufficient discussion to allow you to write a short paper).