0.1 Introduction

Throughout human history, the motion of the planets in the sky was a mystery: why did some planets move quickly across the sky, while other planets moved very slowly? Even two thousand years ago it was apparent that the motion of the planets was very complex. For example, Mercury and Venus never strayed very far from the Sun, while the Sun, the Moon, Mars, Jupiter and Saturn generally moved from the west to the east against the background stars (at this point in history, both the Moon and the Sun were considered "planets"). The Sun appeared to take one year to go around the Earth, while the Moon only took about 30 days. The other planets moved much more slowly. In addition to this rather slow movement against the background stars was, of course, the daily rising and setting of these objects. How could all of this motion occur? Because these objects were important to the cultures of the time, even foretelling the future using astrology, being able to predict their motion was considered vital.

The ancient Greeks had developed a model for the Universe in which all of the planets and the stars were imbedded in perfect crystaline spheres that revolved around the Earth at uniform, but slightly different speeds. This is the "geocentric", or Earth-centered model. But this model did not work very well–the speed of the planet across the sky changed. Sometimes, a planet even moved backwards! It was left to the Egyptian/Greek astronomer Ptolemy (85 – 165 AD) to develop a model for the motion of the planets (you can read more about the details of the Ptolemaic model in your textbook). Ptolemy developed a complicated system to explain the motion of the planets, including "epicycles" and "equants", that in the end worked so well, that no other models for the motions of the planets were considered for 1500 years! While Ptolemy's model worked well, the philosophers of the time did not like this model–their Universe was perfect, and Ptolemy's model suggested that the planets moved in peculiar, imperfect ways.

In the 1530's Nicholas Copernicus (1473 – 1543) published his work suggesting that it was much easier to explain the complicated motion of the planets if the Earth revolved around the Sun, and that the orbits of the planets were circular. While Copernicus was not the first person to suggest this idea, the timing of his publication coincided with attempts to revise the calendar and to fix a large number of errors in Ptolemy's model that had shown up over the 1500 years since the model was first introduced. But the "heliocentric" (Sun-centered) model of Copernicus was slow to win acceptance, since it did not work as well as the geocentric model of Ptolemy.

Johannes Kepler (1571 – 1630) was the first person to truly understand how the planets in our solar system moved. Using the highly precise observations by Tycho Brahe (1546 – 1601) of the motions of the planets against the background stars, Kepler was able to formulate three laws that described how the planets moved, and with these laws, he was able to predict the future motion of these planets to a higher precision than was previously possible. Many credit Kepler with the origin of modern physics, as his discoveries were what led Isaac Newton ($1643-1727$)to formulate the laws of gravity. Today we will investigate Kepler's laws and the laws of gravity.

0.2 Gravity

Gravity is the fundamental force governing the motions of astronomical objects. No other force is as strong over as great a distance. Gravity influences your everyday life (ever drop a glass?), and keeps the planets, moons, and satellites orbiting smoothly. Gravity affects everything in the Universe including the largest structures like super clusters of galaxies down to the smallest atoms and molecules. Experimenting with gravity is difficult to do. You can't just go around in space making extremely massive objects and throwing them together from great distances. But you can model a variety of interesting systems very easily using a computer. By using a computer to model the interactions of massive objects like planets, stars and galaxies, we can study what would happen in just about any situation. All we have to know are the equations which predict the gravitational interactions of the objects.

The orbits of the planets are governed by a single equation formulated by Newton:

$$F_{gravity} = \frac{GM_1M_2}{R^2} \tag{1}$$

A diagram showing detailing the quantities in this equation is shown in Fig. 1. Here $F_{gravity}$ is the gravitational attractive force between two objects whose masses are M_1 and M_2. The distance between the two objects is "R". The gravitational constant G is just a small number that scales the force. The most important thing about gravity is that the force depends only on the masses of the two objects and the distance between them. This law is called an Inverse Square Law because the distance between the objects is *squared*, and is in the denominator of the fraction. There are several laws like this in physics and astronomy.

Today you will be using a computer program called "Planets and Satellites" by Eugene Butikov to explore Kepler's laws, and how planets, the moons of planets, double stars, and planets in double star systems move. This program uses the law of gravity to simulate how celestial objects move.

- *Goals:* to understand Kepler's three laws and use them in conjunction with the computer program "Planets and Satellites" to explain the orbits of objects in our solar system and beyond
- *Materials: Planets and Satellites* program, a ruler, and a calculator

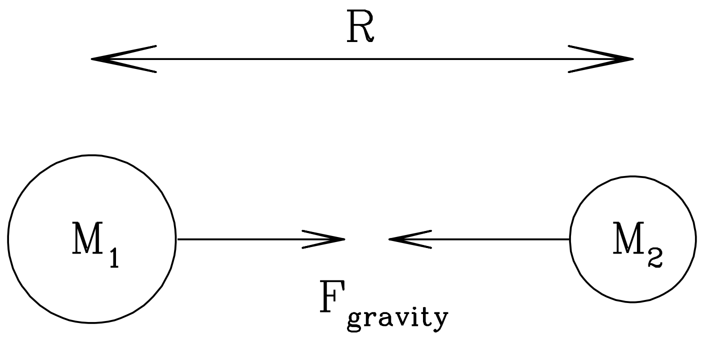

Figure 1: The force of gravity depends on the masses of the two objects (M_1, M_2), and the distance between them (R).

0.3 Kepler's Laws

Before you begin the lab, it is important to recall Kepler's three laws, the basics of orbital mechanics which govern the orbits of planets in our Solar System. Kepler formulated his three laws in the early 1600's, when he finally solved the mystery of how planets moved in our Solar System. These three laws are:

I. "The orbits of the planets are ellipses with the Sun at one focus."

II. "A line from the planet to the Sun sweeps out equal areas in equal intervals of time."

III. "A planet's orbital period squared is proportional to its average distance from the Sun cubed: P^2 (years) = A^3 (A.U.)"

Let's look at the first law, and talk about the nature of an ellipse. What is an ellipse? An ellipse is one of the special curves called a "conic section". If we slice a plane through a cone, four different types of curves can be made: circles, ellipses, parabolas, and hyperbolas. This process, and how these curves are created is shown in Fig. 2.

Before we talk about an ellipse, let's examine a circle, as it is a simple form of an ellipse. As you are aware, the circumference of a circle is simply $2\pi R$. The radius, R, is the distance between the center of the circle and any point on the circle itself. In mathematical terms, the center of the circle is called the "focus". An ellipse, as shown in Fig. 3 is like a flattened circle, with one large diameter (the "major" axis) and one small diameter (the "minor" axis). A circle is simply an ellipse that has identical major and minor axes. Inside of an ellipse, there are two special locations, called "foci" (foci is the plural of focus, it is pronounced "fo-sigh"). The foci are special in that the sum of the distances between the foci and any points on the ellipse are always equal. In Fig. 4 is an ellipse with the two foci indentified, "F_1" and "F_2".

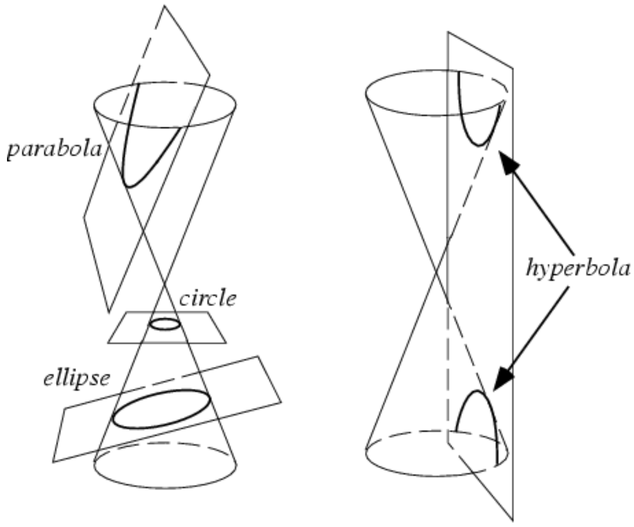

Figure 2: Four types of curves can be generated by slicing a cone with a plane: a circle, an ellipse, a parabola, and a hyperbola. Strangely, these four curves are also the allowed shapes of the orbits of planets, asteroids, comets and satellites!

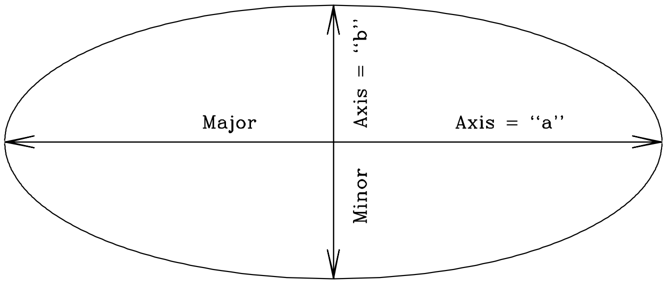

Figure 3: An ellipse with the major and minor axes identified.

Exercise #1: On the ellipse in Fig. 4 are two X's. Confirm that that sum of the distances between the two foci to any point on the ellipse is always the same by measuring the distances between the foci, and the two spots identified with X's. Show your work. (2 points)

Exercise #2: In the ellipse shown in Fig. 5, two points ("P_1" and "P_2") are identified that are not located at the true positions of the foci. Repeat exercise #1, but confirm that P_1 and P_2 are not the foci of this ellipse.

Now we will use the Planets and Satellites program to examine Kepler's laws. It is possible that the program will already be running when you get to your computer. If not, however, you will have to start it up. If your TA gave you a CDROM, then

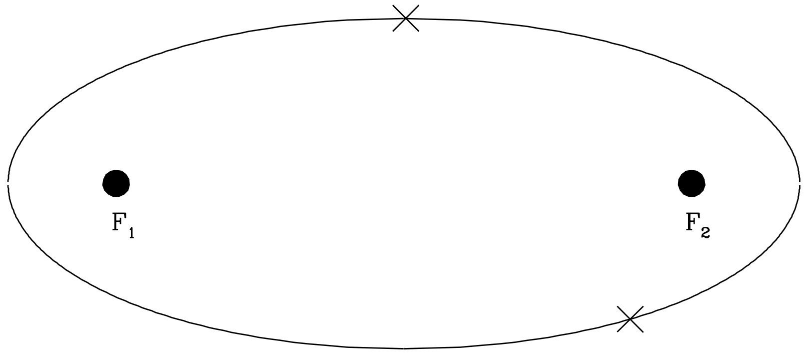

Figure 4: An ellipse with the two foci identified.

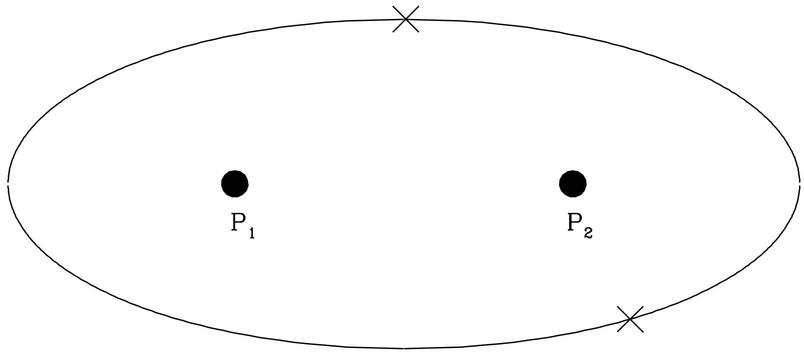

Figure 5: An ellipse with two non-foci points identified.

you need to insert the CDROM into the CDROM drive on your computer, and open that device. On the CDROM will be an icon with the program name. It is also possible that Planets and Satellites has been installed on the computer you are using. Look on the desktop for an icon, or use the start menu. Start-up the program, and you should see a title page window, with four boxes/buttons ("Getting Started", "Tutorial", "Simulations", and "Exit"). Click on the "Simulations" button. We will be returning to this level of the program to change simulations. Note that there are help screens and other sources of information about each of the simulations we will be running–do not hesitate to explore those options!

Exercise #3: Kepler's first law. Click on the "Kepler's Law button" and then the "First Law" button inside the Kepler's Law box. A window with two panels opens up. The panel on the left will trace the motion of the planet around the Sun, while the panel on the right sums the distances of the planet from the foci. Remember, Kepler's first law states "the orbit of a planet is an ellipse with the Sun at one focus". The Sun in this simulation sits at one focus, while the other focus is empty (but whose location will be obvious once the simulation is run!).

At the top of the panel is the program control bar. For now, simply hit the "Go" button. You can clear and restart the simulation by hitting "Restart" (do this as often as you wish). After hitting Go, note that the planet executes an orbit around the ellipse. The program draws the “vectors” from each focus to 25 different positions of the planet in its orbit. It draws a blue vector from the Sun to the planet, and a yellow vector from the other focus to the planet. The right hand panel sums the blue and yellow vectors. [Note: if your computer runs the simulation too quickly, or too slowly, simply adjust the “Slow down/Speed Up” slider for a better speed.]

Describe the results that are displayed in the right hand panel for this first simulation. (2 points).

Now we want to explore another ellipse. In the extreme left hand side of the control bar is a slider to control the “Initial Velocity”. At start-up it is set to “1.2”. Slide it up to the maximum value of 1.35 and hit Go.

Describe what the ellipse looks like at 1.35 vs. that at 1.2. Does the sum of the vectors (right hand panel) still add up to a constant? (3 points)

Now let’s put the Initial Velocity down to a value of 1.0. Run the simulation. What is happening here? The orbit is now a circle. Where are the two foci located? In this case, what is the distance between the focus and the orbit equivalent to? (4 points)

The point in the orbit where the planet is closest to the Sun is called "perihelion", and that point where the planet is furthest from the Sun is called "aphelion". For a circular orbit, the aphelion is the same as the perihelion. Exit this simulation (click on "File" and "Exit").

Exercise #4: Kepler's Second Law: "A line from a planet to the Sun sweeps out equal areas in equal intervals of time." From the simulation window, click on the "Second Law" after entering the Kepler's Law window. Move the Initial Velocity slide bar to a value of 1.2. Hit Go.

Describe what is happening here. Does this confirm Kepler's second law? How? When the planet is at perihelion, is it moving slowly or quickly? Why do you think this happens? (4 points)

Look back to the equation for the force of gravity. You know from personal experience that the harder you hit a ball, the faster it moves. The act of hitting a ball is the act of applying a force to the ball. The larger the force, the faster the ball moves (and, generally, the further it travels). In the equation for the force of gravity, the amount of force generated depends on the masses of the two objects, and the distance between them. But note that it depends on one over the square of the distance: $1/R^2$. Let's explore this "inverse square law" with some calculations.

- If $R = 1$, what does $1/R^2 =$ ______________________?
- If $R = 2$, what does $1/R^2 =$ ______________________?
- If $R = 4$, what does $1/R^2 =$ ______________________?

What is happening here? [Hint: As R increases, what happens to $1/R^2$?] (2 points)

Ok, the equation for the force of gravity has a $1/R^2$ in it, so as R increases (that is, the two objects get further apart), does the force of gravity get larger, or smaller? Is the force of gravity stronger at perihelion, or aphelion? If the force of gravity is stronger, do you think the planet will move faster, or slower? Does this help explain Kepler's second law? (5 points)

What do you think the motion of a planet in a circular orbit looks like? Is there a definable perihelion and aphelion? Make a prediction for what the motion is going to look like–how are the triangular areas seen for elliptical orbits going to change as the planet orbits the Sun in a circular orbit? Why? (3 points)

Now let's run a simulation for a circular orbit by setting the Initial Velocity to 1.0. What happened? Were your predictions correct? (3 points)

Exit out of the Second Law, and start-up the Third Law simulation.

Exercise 4: Kepler's Third Law: "A planet's orbital period squared is proportional to its average distance from the Sun cubed: $P^2 \propto a^3$". As we have just learned, the law of gravity states that the further away an object is, the weaker the force. We have already found that at aphelion, the planet moves more slowly than at perihelion. The force of gravity is stronger at perihelion, and the planet moves more quickly. Kepler's third law is merely a reflection of this fact–the further a planet is from the Sun ("a"), the more slowly it will move because the force of gravity is smaller (remember, $1/R^2$!). The more slowly it moves, the longer it takes to go around the Sun ("P"). The exact relation is $P^2 \propto a^3$, where P is the orbital period in years, while a is the average distance of the planet from the Sun, and the mathematical symbol for proportional is represented by "$\propto$". To turn the proportion sign into an equal sign requires the multiplication of the a^3 side of the equation by a constant: $P^2 = c \times a^3$. But we can get rid of this constant, "c", by making a ratio. We will do this below.

In this simulation, there will be two planets: one in a smaller orbit, which will represent the Earth (and has $a = 1$), and a planet in a larger orbit (where a is adjustable). Start-up the Third Law simulation and hit Go. You will see that the inner planet moves around more quickly, while the planet in the larger ellipse moves more slowly. Let's set-up the math to better understand Kepler's Third Law. We begin by constructing the ratio of of the Third Law equation ($P^2 = c \times a^3$) for an arbitrary planet divided by the Third Law equation for the Earth:

$$\frac{P_P^2}{P_E^2} = \frac{c \times a_P^3}{c \times a_E^3} \tag{2}$$

In this equation, the planet's orbital period and average distance are denoted by P_P and a_P, while the orbital period of the Earth and its average distance from the Sun are P_E and a_E. As you know from from your high school math, any quantity that appears on both the top and bottom of a fraction can be canceled out. So, we can get rid of the pesky constant "c", and Kepler's Third Law equation becomes:

$$\frac{P_P^2}{P_E^2} = \frac{a_P^3}{a_E^3} \tag{3}$$

But we can make this equation simpler by noting that if we use years for the orbital period, and Astronomical Units for the average distance of the Earth to the Sun, we get:

$$\frac{P_P^2}{1} = \frac{a_P^3}{1} \quad or \quad P_P^2 = a_P^3 \tag{4}$$

(Remember that the cube of 1, and the square of 1 are both 1!)

Let's use equation (4) to make some predictions. If the average distance of Jupiter from the Sun is about 5 AU, what is its orbital period? Set-up the equation:

$$P_J^2 = a_J^3 = 5^3 = 5 \times 5 \times 5 = 125 \tag{5}$$

So, for Jupiter, $P^2 = 125$. How do we figure out what P is? We have to take the square root of both sides of the equation:

$$\sqrt{P^2} = P = \sqrt{125} = 11.2 \quad years \tag{6}$$

The orbital period of Jupiter is approximately 11.2 years. Your turn:

If an asteroid has an average distance from the Sun of 4 AU, what is its orbital period? Show your work. (2 points)

In the Third Law simulation, there is a slide bar to set the average distance from the Sun for any hypothetical solar system body. At start-up, it is set to 4 AU. Run the simulation, and confirm the answer you just calculated. Note that for each orbit of the inner planet, a small red circle is drawn on the outer planet's orbit. Count up these red circles to figure out how many times the Earth revolved around the Sun during a single orbit of the asteroid. Did your calculation agree with the simulation? Describe your results. (2 points)

If you were observant, you noticed that the program calculated the number of orbits that the Earth executed for you (in the "Time" window), and you do not actually have to count up the little red circles. Let's now explore the orbits of the nine planets in our solar system. In the following table are the semi-major axes of the nine planets. Note that the "average distance to the Sun" (a) that we have been using above is actually a quantity astronomers call the "semi-major axis" of a planet. a is simply one half the major axis of the orbit ellipse. Fill in the missing orbital periods of the planets by running the Third Law simulator for each of them. (3 points)

Table 1: The Orbital Periods of the Planets

Planet	a (AU)	P (yr)
Mercury	0.387	0.24
Venus	0.72	
Earth	1.000	1.000
Mars	1.52	
Jupiter	5.2	
Saturn	9.54	29.5
Uranus	19.22	84.3
Neptune	30.06	164.8
Pluto	39.5	248.3

Notice that the further the planet is from the Sun, the slower it moves, and the longer it takes to complete one orbit around the Sun (its "year"). Neptune was discovered in 1846, and Pluto was discovered in 1930 (by Clyde Tombaugh, a former professor at NMSU). How many orbits (or what percentage of an orbit) have Neptune and Pluto completed since their discovery? (3 points)

0.4 Going Beyond the Solar System

One of the basic tenets of physics is that all natural laws, such as gravity, are the same everywhere in the Universe. Thus, when Newton used Kepler's laws to figure out how gravity worked in the solar system, we suddenly had the tools to understand how stars interact, and how galaxies, which are large groups of billions of stars, behave: the law of gravity works the same way for a planet orbiting a star that is billions of light years away, as it does for the planets in the solar system. Therefore, we can use the law of gravity to construct simulations for all types of situations—even how the Universe itself evolves with time! For the remainder of the lab we will investigate binary stars, and planets in binary star systems.

First, what is a binary star? Astronomers believe that about one half of all stars that form, end up in binary star systems. That is, instead of a single star like the Sun, being orbited by planets, a pair of stars are formed that orbit around each other. Binary stars come in a great variety of sizes and shapes. Some stars orbit around each other very slowly, with periods exceeding a million years. While there is one binary system containing two white dwarfs (a white dwarf is the end product of the life of a star like the Sun) that has an orbital period of 5 minutes!

To get to the simulations for this part of the lab, exit the Third Law simulation (if you haven't already done so), and click on button "7", the "Two-Body and Many-Body" simulations. We will start with the "Double Star" simulation. Click Go.

In this simulation there are two stars in orbit around each other, a massive one (the blue one) and a less massive one (the red one). Note how the two stars move. Notice that the line connecting them at each point in the orbit passes through one spot–this is the location of something called the "center of mass". In Fig. 6 is a diagram explaining the center of mass. If you think of a teeter-totter, or a simple balance, the center of mass is the point where the balance between both sides occurs. If both objects have the same mass, this point is halfway between them. If one is more massive than the other, the center of mass/balance point is closer to the more massive object.

Most binary star systems have stars with similar masses ($\mathrm{M}_1 \approx \mathrm{M}_2$), but this is not always the case. In the first (default) binary star simulation, $\mathrm{M}_1 = 2\mathrm{M}_2$. The "mass ratio" ("$q$") in this case is 0.5, where mass ratio is defined to be $q = \mathrm{M}_2/\mathrm{M}_1$. Here, $\mathrm{M}_2 = 1$, and $\mathrm{M}_1 = 2$, so $q = \mathrm{M}_2/\mathrm{M}_1 = 1/2 = 0.5$. This is the number that appears in the "Mass Ratio" window of the simulation.

Exercise 5: Binary Star systems. We now want to set up some special binary star orbits to help you visualize how gravity works. This requires

us to access the "Input" window on the control bar of the simulation window to enter-in data for each simulation. Clicking on Input brings-up a menu with the following

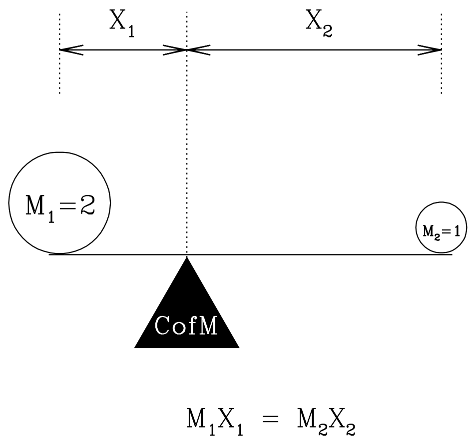

Figure 6: A diagram of the definition of the center of mass. Here, object one (M_1) is twice as massive as object two (M_2). Therefore, M_1 is closer to the center of mass than is M_2. In the case shown here, $X_2 = 2X_1$.

set-able parameters: Mass Ratio, "Transverse Velocity", "Velocity (magnitude)", and "Direction". Use the slide bar to set Transverse Velocity = 1.0, Velocity (magnitude) = 0.0, and Direction = 0.0. For now, we simply want to play with the mass ratio.

Use the slide bar (or type in the number) so that Mass Ratio = 1.0. Click "Ok". This now sets up your new simulation. Click Run. Describe the simulation. What are the shapes of the two orbits? Where is the center of mass located relative to the orbits? What does $q = 1.0$ mean? Describe what is going on here. (5 points)

Ok, now we want to run a simulation where only the mass ratio is going to be changed. Go back to Input and enter-in the correct mass ratio for a binary star system with $M_1 = 4.0$, and $M_2 = 1.0$. Run the simulation. Describe what is happening in this simulation. How are the stars located with respect to the center of mass? (4 points)

Finally, we want to move away from circular orbits, and make the orbit as elliptical as possible. You may have noticed from the Kepler's law simulation that the Transverse Velocity affected whether the orbit was round or elliptical. When the Transverse Velocity = 1.0, the orbit is a circle. Transverse Velocity is simply how fast the planet in an elliptical orbit is moving *at perihelion* relative to a planet in a circular orbit of the same orbital period. The maximum this number can be is about 1.3. If it goes much faster, the ellipse then extends to infinity and the orbit is a parabola. Go back to Input and now set the Transverse Velocity = 1.3. Run the simulation. Describe what is happening. When do the stars move the fastest? The slowest? Does this make sense? Why/why not? (4 points)

The final exercise explores what it would be like to live on a planet in a binary star system–not so fun! In the "Two-Body and Many-Body" simulations window, click on "Dbl. Star and a Planet" button. Here we simulate the motion of a planet going around the less massive star of a binary system. Click Go. Describe the simulation—what happened to the planet? Why do you think this happened? (4 points)

In this simulation, two more windows opened up to the right of the main one. These are what the simulation looks like if you were to sit on the surface of the two stars in the binary. For a while the planet orbits one star, and the goes away to orbit the other one, and returns. So, sitting on these stars gives you a different viewpoint than sitting high above the orbit. Let's see if you can keep the planet from wandering away from its parent star. Click on the "Settings" window. As you can tell, now that we have three bodies in the system, there are lots of parameters to play with. But let's confine ourselves to two of them: "Ratio of Stars Masses" and "Planet–Star Distance". The first of these is simply the q we encountered above, while the second changes the size of the planet's orbit. The default values of both at the start-up are $q = 0.5$, and Planet–Star Distance = 0.24. Run simulations with $q = 0.4$ and 0.6. Compare them to the simulations with $q = 0.5$. (4 points)

See if you can find the value of q at which larger values cause the planet to "stay home", while smaller values cause it to (eventually) crash into one of the stars (stepping up/down by 0.01 should be adequate). (2 points)

Ok, reset $q = 0.5$, and now let's adjust the Planet–Star Distance. In the Settings window, set the Planet–Star Distance = 0.1 and run a simulation. Note the outcome of this simulation. Now set Planet–Star Distance = 0.3. Run a simulation. What happened? Did the planet wander away from its parent star? Are you surprised?

Astronomers call orbits where the planet stays home, “stable orbits”. Obviously, when the Planet–Star Distance = 0.24, the orbit is unstable. The orbital parameters are just right that the gravity of the parent star is not able to hold on to the planet. But some orbits, even though the parent's hold on the planet is weaker, are stable–the force of gravity exerted by the two stars is balanced just right, and the planet can happily orbit around its parent and never leave. Over time, objects in unstable orbits are swept-up by one of the two stars in the binary. This can even happen in the solar system. If you have done the comet lab, then you saw some images where a comet ran into Jupiter (see Figure 3.6). The orbits of comets are very long ellipses, and when they come close to the Sun, their orbit can get changed by passing close to a major planet. This can cause the comet to crash into the Sun, or into a planet, or cause it to be ejected completely out of the solar system. (You can see an example of the latter process by changing the Planet–Star Distance = 0.4 in the current simulation.)