Name:__________________________________
Date:___________________________________

5 Kepler's Laws

5.1 Introduction

Throughout human history, the motion of the planets in the sky was a mystery: why did some planets move quickly across the sky, while other planets moved very slowly? Even two thousand years ago it was apparent that the motion of the planets was very complex. For example, Mercury and Venus never strayed very far from the Sun, while the Sun, the Moon, Mars, Jupiter and Saturn generally moved from the west to the east against the background stars (at this point in history, both the Moon and the Sun were considered "planets"). The Sun appeared to take one year to go around the Earth, while the Moon only took about 30 days. The other planets moved much more slowly. In addition to this rather slow movement against the background stars was, of course, the daily rising and setting of these objects. How could all of these motions occur? Because these objects were important to the cultures of the time—even foretelling the future using astrology. Being able to predict their motion was considered vital.

The ancient Greeks had developed a model for the Universe in which all of the planets and the stars were embedded in perfect crystalline spheres that revolved around the Earth at uniform, but slightly different speeds. This is the "geocentric", or Earth-centered model. But this model did not work very well–the speed of the planet across the sky changed. Sometimes, a planet even moved backwards! It was left to the Egyptian astronomer Ptolemy (85 − 165 AD) to develop a model for the motion of the planets (you can read more about the details of the Ptolemaic model in your textbook). Ptolemy developed a complicated system to explain the motion of the planets, including "epicycles" and "equants", that in the end worked so well, that no other models for the motions of the planets were considered for 1500 years! While Ptolemy's model worked well, the philosophers of the time did not like this model–their Universe was perfect, and Ptolemy's model suggested that the planets moved in peculiar, imperfect ways.

In the 1540's Nicholas Copernicus (1473 − 1543) published his work suggesting that it was much easier to explain the complicated motion of the planets if the Earth revolved around the Sun, and that the orbits of the planets were circular. While Copernicus was not the first person to suggest this idea, the timing of his publication coincided with attempts to revise the calendar and to fix a large number of errors in Ptolemy's model that had shown up over the 1500 years since the model was first introduced. But the "heliocentric" (Sun-centered) model of Copernicus was slow to win acceptance, since it did not work as well as the geocentric model of Ptolemy.

Johannes Kepler (1571 − 1630) was the first person to truly understand how the planets in our solar system moved. Using the highly precise observations by Tycho Brahe (1546 −

1601) of the motions of the planets against the background stars, Kepler was able to formulate three laws that described how the planets moved. With these laws, he was able to predict the future motion of these planets to a higher precision than was previously possible. Many credit Kepler with the origin of modern physics, as his discoveries were what led Isaac Newton (1643 − 1727) to formulate the law of gravity. Today we will investigate Kepler's laws and the law of gravity.

5.2 Gravity

Gravity is the fundamental force governing the motions of astronomical objects. No other force is as strong over as great a distance. Gravity influences your everyday life (ever drop a glass?), and keeps the planets, moons, and satellites orbiting smoothly. Gravity affects everything in the Universe including the largest structures like super clusters of galaxies down to the smallest atoms and molecules. Experimenting with gravity is difficult to do. You can't just go around in space making extremely massive objects and throwing them together from great distances. But you can model a variety of interesting systems very easily using a computer. By using a computer to model the interactions of massive objects like planets, stars and galaxies, we can study what would happen in just about any situation. All we have to know are the equations which predict the gravitational interactions of the objects.

The orbits of the planets are governed by a single equation formulated by Newton:

$$F_{gravity} = \frac{GM_1M_2}{R^2} \tag{1}$$

A diagram detailing the quantities in this equation is shown in Fig. 5.1. Here $F_{gravity}$ is the gravitational attractive force between two objects whose masses are M_1 and M_2. The distance between the two objects is "R". The gravitational constant G is just a small number that scales the size of the force. **The most important thing about gravity is that the force depends only on the masses of the two objects and the distance between them.** This law is called an Inverse Square Law because the distance between the objects is *squared*, and is in the denominator of the fraction. There are several laws like this in physics and astronomy.

Today you will be using several online simulators. The TA should have already provided these links in an email. These Links are

- *Materials: Below website* program, a device, a ruler, and a calculator
- https://phet.colorado.edu/en/contributions/view/4613
- https://www.geogebra.org/m/wEbSe5ab
- https://ophysics.com/f6.html
- https://academo.org/demos/keplers-third-law/

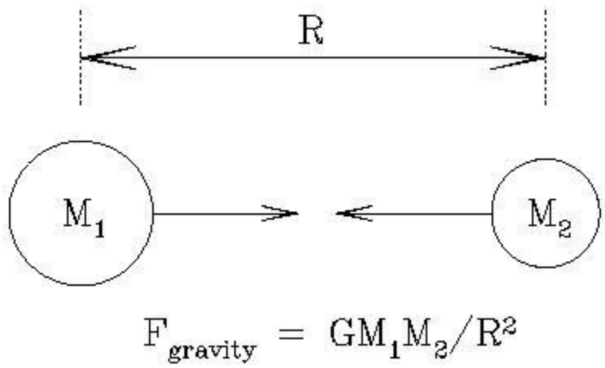

Figure 5.1: The force of gravity depends on the masses of the two objects (M_1, M_2), and the distance between them (R).

- *Goals:* to understand Kepler's three laws and use them in conjunction with the above programs to explain the orbits of objects in our solar system and beyond

5.3 Kepler's Laws

Before you begin the lab, it is important to recall Kepler's three laws, the basic description of how the planets in our Solar System move. Kepler formulated his three laws in the early 1600's, when he finally solved the mystery of how planets moved in our Solar System. These three (empirical) laws are:

I. "The orbits of the planets are ellipses with the Sun at one focus."

II. "A line from the planet to the Sun sweeps out equal areas in equal intervals of time."

III. "A planet's orbital period squared is proportional to its average distance from the Sun cubed: $P^2 \propto a^3$"

Let's look at the first law, and talk about the nature of an ellipse. What is an ellipse? An ellipse is one of the special curves called a "conic section". If we slice a plane through a cone, four different types of curves can be made: circles, ellipses, parabolas, and hyperbolas. This process, and how these curves are created is shown in Fig. 5.2.

Before we describe an ellipse, let's examine a circle, as it is a simple form of an ellipse. As you are aware, the circumference of a circle is simply 2πR. The radius, R, is the distance between the center of the circle and any point on the circle itself. In mathematical terms, the center of the circle is called the "focus". An ellipse, as shown in Fig. 5.3, is like a flattened circle, with one large diameter (the "major" axis) and one small diameter (the "minor" axis). A circle is simply an ellipse that has identical major and minor axes. Inside of an ellipse, there are two special locations, called "foci" (foci is the plural of focus, it is pronounced "fo-sigh"). The foci are special in that the sum of the distances between the foci and any points on the ellipse are always equal. Fig. 5.4 is an ellipse with the two foci identified, "F_1"

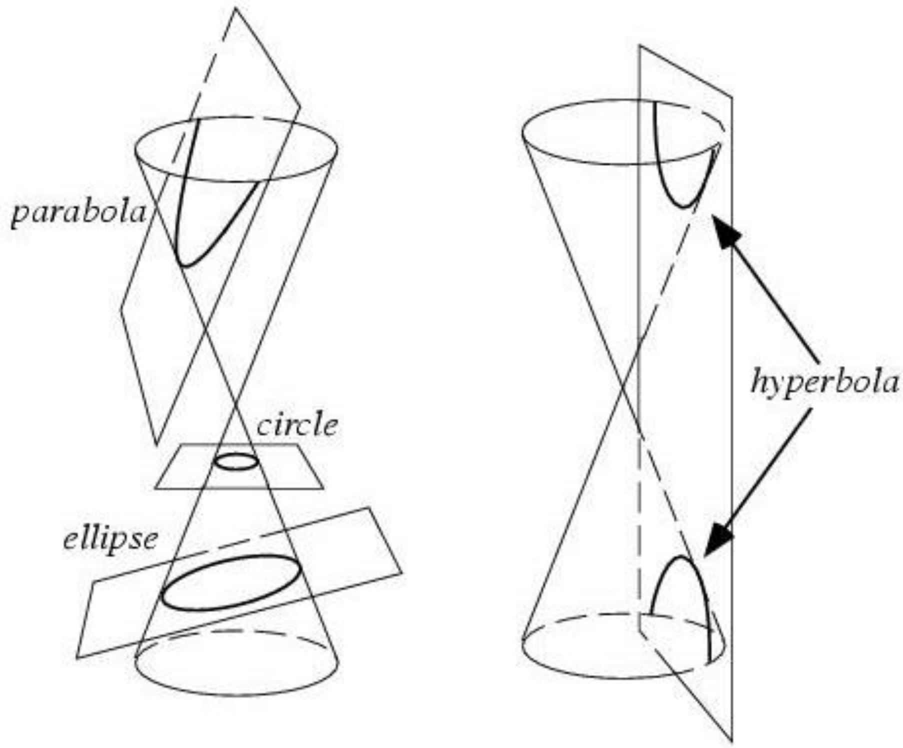

Figure 5.2: Four types of curves can be generated by slicing a cone with a plane: a circle, an ellipse, a parabola, and a hyperbola. Strangely, these four curves are also the allowed shapes of the orbits of planets, asteroids, comets and satellites!

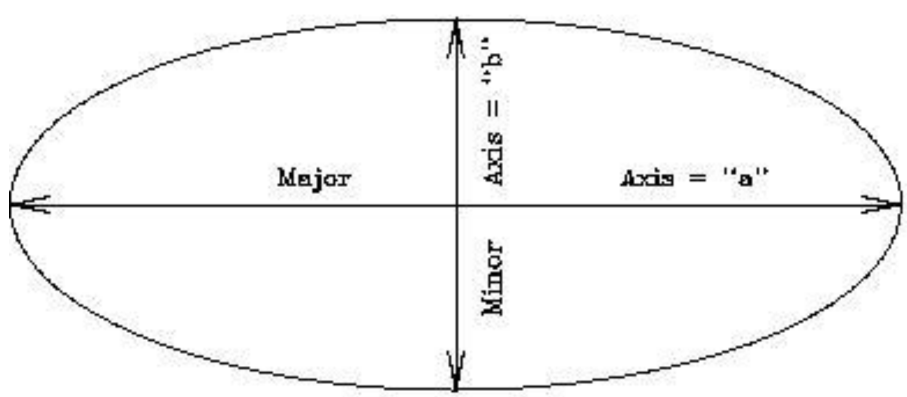

Figure 5.3: An ellipse with the major and minor axes identified.

and “F_2”.

Exercise #1: On the ellipse in Fig. 5.4 are two X’s. Confirm that that sum of the distances between the two foci to any point on the ellipse is always the same by measuring the distances between the foci, and the two spots identified with X’s. Show your work. (**3 points**)

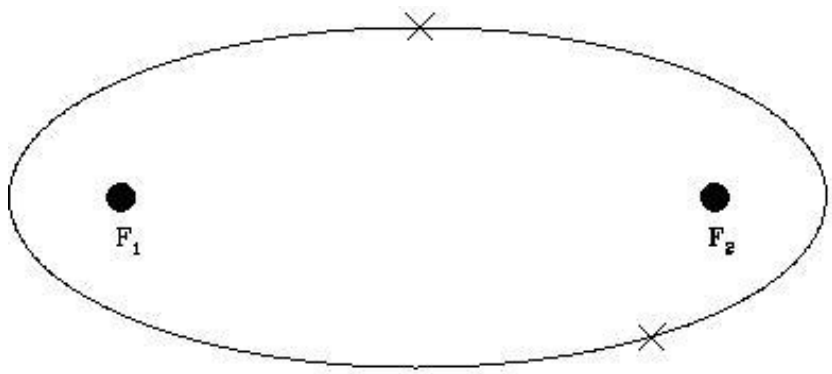

Figure 5.4: An ellipse with the two foci identified.

Exercise #2: In the ellipse shown in Fig. 5.5, two points (“P_1” and “P_2”) are identified that are not located at the true positions of the foci. Repeat exercise #1, but confirm that P_1 and P_2 are not the foci of this ellipse. (**3 points**)

We will now use various online simulators to explore Kepler’s First Law of planetary motion. Each simulator will focus on a particular topic; however, as we go through this lab, try to connect each topic learned to the next (and previous!) simulator.

Your TA should have already sent an email with links to make loading up the links easier. - https://www.geogebra.org/m/wEbSe5ab - We will start exploring Kepler’s first Law using this program.

Exercise #3: Kepler’s first law -“Each planet’s orbit around the Sun is an ellipse, with the sun at one foc”

Once the simulator is loaded up, Click on the “start” play button to begin the motion of the planet moving. There are three sliders that will allow you to change how the planet

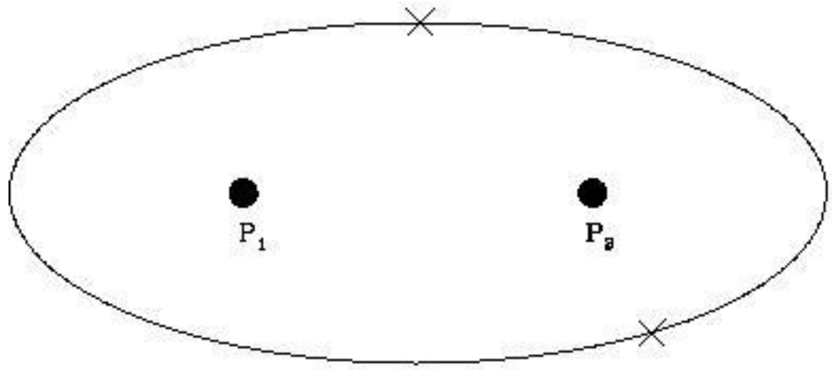

Figure 5.5: An ellipse with two non-foci points identified.

is moving around the sun - Speed, Time span, and "e" (eccentricity). The default "e" value should be set to 0.55. If not, make sure it is set to 0.55.

1. Describe how the orbit of the planet looks for the default values. Where are the two foci? (**3 points**).

   Now we want to explore another ellipse. In the slider, we will want to change the eccentricity (how elliptical the orbit is). We can change the "e" slider by dragging it left and right.

2. Now, move the slider for the "eccentricity" value such that it is higher than 0.55 (but not 1.0!). What value did you choose? How does this change the orbit? What happened to the foci? (**3 points**)

3. Now, move the slider for "eccentricity" such that the slider is all the way at the max value of "1". What is the shape of this orbit now? And where are the two foci located?

Do you think this is physically possible (why or why not?)? (**3 points**)

4. Now, move the slider for "eccentricity" such that the slider is all the way at the minimum value of "0". What is happening here? The orbit is now a circle. Where are the two foci located? In this case, what is the name that describes the distance between the focus point and the orbit? (**4 points**)

   The point in the orbit where the planet is closest to the Sun is called "perihelion", and that point where the planet is furthest from the Sun is called "aphelion". For a circular orbit, the aphelion is the same as the perihelion, and can be defined to be anywhere!

Exercise #4: Kepler's Second Law: "A line from a planet to the Sun sweeps out equal areas in equal intervals of time."

We will use this simulator for Kepler?s Second Law - https://ophysics.com/f6.html .

"Show Kepler's 2nd Law Trace" should be already checked - if not, click it so that a checkmark appears. And hit "Run"

1. Describe what is happening here (and what the lines and "sweeps" indicate). Does this confirm Kepler's second law? How and why? When the planet is at perihelion, is it moving slowly or quickly? Why do you think this happens (**4 points**)

Look back to the equation for the force of gravity. You know from personal experience that the harder you hit a ball, the faster it moves. The act of hitting a ball is the act of applying a force to the ball. The larger the force, the faster the ball moves (and, generally, the farther it travels). In the equation for the force of gravity, the amount of force generated depends on the masses of the two objects, and the distance between them. But note that it depends on one over the square of the distance: $1/R^2$. Let's explore this "inverse square law" with some calculations.

- If R = 1, what does $1/R^2$ = ______________________?
- If R = 2, what does $1/R^2$ = ______________________?
- If R = 4, what does $1/R^2$ = ______________________?

2. What is happening here? As R gets bigger, what happens to $1/R^2$? Does $1/R^2$ decrease/increase quickly or slowly? (**3 points**)

The equation for the force of gravity has a $1/R^2$ in it, so as R increases (that is, the two objects get further apart), does the force of gravity *felt* by the body get larger, or smaller? Is the force of gravity stronger at perihelion, or aphelion? Newton showed that the speed of a planet in its orbit depends on the force of gravity through this equation:

$$V = \sqrt{(G(M_{\text{sun}} + M_{\text{planet}})(2/r - 1/a))} \tag{2}$$

where "r" is the radial distance of the planet from the Sun, and "a" is the mean orbital radius (the semi-major axis).

3. Do you think the planet will move faster, or slower when it is closest to the Sun? Test this by assuming that r = 0.5a at perihelion, and r = 1.5a at aphelion, and that a=1! [Hint, simply set $G(M_{\text{sun}} + M_{\text{planet}}) = 1$ to make this comparison very easy!] Does this explain Kepler's second law? (**4 points**)

4. What do you think the motion of a planet in a circular orbit looks like? Is there a definable perihelion and aphelion? Make a prediction for what the motion is going to look like–how are the triangular areas seen for elliptical orbits going to change as the planet orbits the Sun in a circular orbit? Why? (**3 points**)

5. Going back to - https://www.geogebra.org/m/wEbSe5ab - and running the simulation for a circular orbit by setting "e" to 0, what happens, were your predictions correct? (**3 points**)

Exercise #5: Kepler's Third Law: "A planet's orbital period squared is proportional to its average distance from the Sun cubed: $P^2 \propto a^3$".

As we have just learned, the law of gravity states that the further away an object is, the weaker the force. We have already found that at aphelion, when the planet is far from the Sun, it moves more slowly than at perihelion. Kepler's third law is merely a reflection of this fact–the further a planet is from the Sun ("a"), the more slowly it will move. The more slowly it moves, the longer it takes to go around the Sun ("P"). The relation is $P^2 \propto a^3$, where P is the orbital period in years, while a is the average distance of the planet from the Sun, and the mathematical symbol for proportional is represented by "$\propto$". However, if we use units of 'years' for P and 'AU' for a we can replace the proportional sign with an equal sign:

$$P^2 = a^3 \tag{3}$$

Let's use equation (3) to make some predictions. If the average distance of Jupiter from the Sun is about 5 AU, what is its orbital period? Set-up the equation:

$$P_J^2 = a_J^3 = 5^3 = 5 \times 5 \times 5 = 125 \tag{4}$$

So, for Jupiter, $P^2 = 125$. How do we figure out what P is? We have to take the square root of both sides of the equation:

$$\sqrt{P^2} = P = \sqrt{125} = 11.2 \ \ years \tag{5}$$

The orbital period of Jupiter is approximately 11.2 years. Your turn:

1. If an asteroid has an average distance from the Sun of 4 AU, what is its orbital period? Show your work. (**3 points**)

   Load up - https://academo.org/demos/keplers-third-law/ - To confirm the previous question, set R1 to 50 an R2 to a large value (such as 1,000) (you might have to zoom out!). And then click animate - What do you notice about the motion of the inner planet compared to the motion of the outer planet?

2. Please describe the motion of the inner planet compared to the motion of the outer planet. What do you notice about the motion and how does this support into Kepler's Laws. (**3 points**)

3. The below table has the orbital periods of the planets

Table 5.1: The Orbital Periods of the Planets

Planet	a (AU)	P (yr)
Mercury	0.387	0.24
Venus	0.72	0.62
Earth	1.000	1.000
Mars	1.52	1.88
Jupiter	5.20	12
Saturn	9.54	29.5
Uranus	19.22	84.3
Neptune	30.06	164.8
Pluto	39.5	248.3

   Notice that the further the planet is from the Sun, the slower it moves, and the longer it takes to complete one orbit around the Sun (its "year").

4. Neptune was discovered in 1846, and Pluto was discovered in 1930 (by Clyde Tombaugh, a former professor at NMSU). How many orbits (or what fraction of an orbit) have Neptune and Pluto completed since their discovery? (**4 points**)

5.4 Going Beyond the Solar System

One of the basic tenets of physics is that all natural laws, such as gravity, are the same everywhere in the Universe. Thus, when Newton used Kepler's laws to figure out how gravity worked in the solar system, we suddenly had the tools to understand how stars interact, and how galaxies, which are large groups of billions of stars, behave: the law of gravity works the same way for a planet orbiting a star that is billions of light years from Earth, as it does for the planets in our solar system. Therefore, we can use the law of gravity to construct simulations for all types of situations—even how the Universe itself evolves with time! For the remainder of the lab we will investigate binary stars, and planets in binary star systems.

First, what is a binary star? Astronomers believe that about one half of all stars that form, end up in binary star systems. That is, instead of a single star like the Sun, being orbited by planets, a pair of stars are formed that orbit around each other. Binary stars come in a great variety of sizes and shapes. Some stars orbit around each other very slowly, with periods exceeding a million years, while there is one binary system containing two white dwarfs (a white dwarf is the end product of the life of a star like the Sun) that has an orbital period of 5 minutes!

We will now use a simulator to try and model to the best of our ability stars and planets beyond our own solar system. We will be using a simulator from PheT - https://phet.colorado.edu/sims/html/gravity-and-orbits/latest/gravity-and-orbits_en.html

Once you go to the link, click on the "model version". This model version of the simulator stars with our Sun?s mass and Earth?s mass. Using this, we can start to see how different masses of stars and planets affect the gravity and orbits around other stars.

In Fig. 5.6 is a diagram explaining the center of mass. If you think of a teeter-totter, or a simple balance, the center of mass is the point where the balance between both sides occurs. If both objects have the same mass, this point is halfway between them. If one is more massive than the other, the center of mass/balance point is closer to the more massive object.

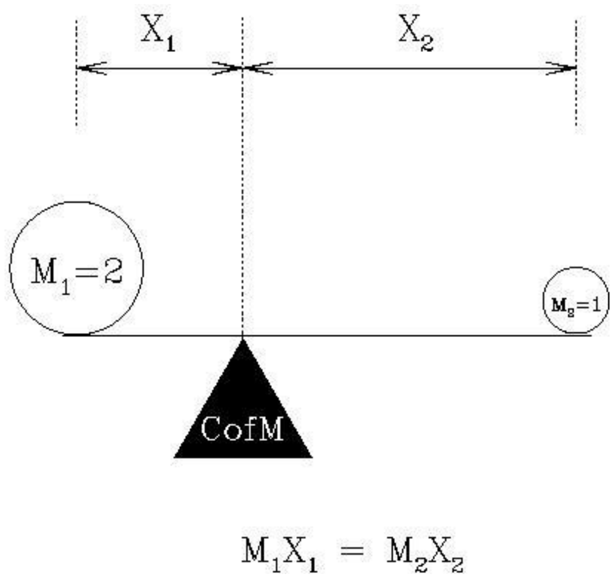

Figure 5.6: A diagram of the definition of the center of mass. Here, object one (M_1) is twice as massive as object two (M_2). Therefore, M_1 is closer to the center of mass than is M_2. In the case shown here, $X_2 = 2X_1$.

Most binary star systems have stars with similar masses ($M_1 \approx M_2$), but this is not always the case. In the first (default) binary star simulation, $M_1 = 2M_2$. The "mass ratio" ("q") in this case is 0.5.

Mass ratio is defined to be

$$q = \frac{M_2}{M_1} \tag{6}$$

Here, $M_2 = 1$, and $M_1 = 2$, so $q = M_2/M_1 = 1/2 = 0.5$. This is the number that appears in the "Mass Ratio" window of the simulation.

Exercise #6:

Starting with our Sun's Mass and Earth's Mass - (In the right hand control box, make sure the sliders of "star mass" and "planet mass" are set to "Our Sun" and "Earth" respectively. Additionally, in the control box, make sure that "path" and "grid" are also checked.) Hit the play button.

1. Describe what is happening with this orbit. Describe its shape. Does it have an obvious perihelion or aphelion? Thinking back to the previous section, why might this be? (**4 points**)

   Recall Kepler's First law of Planetary motion - the orbits of planets are elliptical. Even though the simulation looks very circular, we can see (thanks to the grid!) that the orbit is not a perfect circle (and is elliptical!, as we would expect!).

   While the simulation is still running, change the Star Mass to "1.5" in the slider.

2. How does the new star mass affect the orbit of the planet? Draw the orbit and label the perihelion and aphelion sections. Based on what you know about Kepler?s Laws, why did the orbit change when the mass of the star changed? (**5 points**)

   While running the simulation, change the "star mass" to 1.5 and the planet mass to

0.5. Additionally, check "gravity force" and "velocity" in the right control box. We now see a couple arrows on the screen. The green arrow shows us the velocity of the moving planet, and the blue arrow shows us the strength of the gravitational force. The longer the arrow is, the stronger the force.

3. At what point in the orbit is the planet moving the fastest (the green arrow is the largest?) and what point is the planet moving the slowest (the green arrow is the smallest?) (**5 points**)

Next, change the "Star Mass" to 0.5 while keeping the planet mass the same.

4. What happens to the planet? As the planet moved, described what happened to the gravity (blue arrow) and the green arrow (velocity). Based on those, why do you think the planet wandered off from the sun?. (**5 points**)

Name:______________________________
Date:______________________________

5.5 Take Home Exercise (35 points total)

On a clean sheet of paper, please summarize the important concepts of this lab. Use complete sentences, and proofread your summary before handing in the lab. Your response should include:

- Describe the Law of Gravity and what happens to the gravitational force as *a)* as the masses increase, and *b)* the distance between the two objects increases
- Describe Kepler's three laws *in your own words*, and describe how you tested each one of them.
- Mention some of the things which you have learned from this lab
- Astronomers think that finding life on planets in binary systems is unlikely. Why do they think that? Use your simulation results to strengthen your argument.

5.6 Possible Quiz Questions

1. Describe the difference between an ellipse and a circle.
2. List Kepler's three laws.
3. How quickly does the strength ("pull") of gravity get weaker with distance?
4. Describe the major and minor axes of an ellipse.

5.7 Extra Credit (ask your TA for permission before attempting this, 5 points)

Derive Kepler's third law ($\mathrm{P}^2 = C \times a^3$) for a circular orbit. First, what is the circumference of a circle of radius a? If a planet moves at a constant speed "v" in its orbit, how long does it take to go once around the circumference of a circular orbit of radius a? [This is simply the orbital period "P".] Write down the relationship that exists between the orbital period "P", and "a" and "v". Now, if we only knew what the velocity (v) for an orbiting planet was, we would have Kepler's third law. In fact, deriving the velocity of a planet in an orbit is quite simple with just a tiny bit of physics (go to this page to see how it is done: http://www.go.ednet.ns.ca/~larry/orbits/kepler.html). Here we will simply tell you that the speed of a planet in its orbit is $v = (\mathrm{GM}/a)^{1/2}$, where "G" is the gravitational constant mentioned earlier, "M" is the mass of the Sun, and a is the radius of the orbit. Rewrite your orbital period equation, substituting for v. Now, one side of this equation has a square root in it–get rid of this by squaring both sides of the equation and then simplifying the result. Did you get $\mathrm{P}^2 = C \times a^3$? What does the constant "C" have to equal to get Kepler's third law?