Layered Accretion in Protoplanetary Disks

Interplay between the magneto-rotational and baroclinic instabilities

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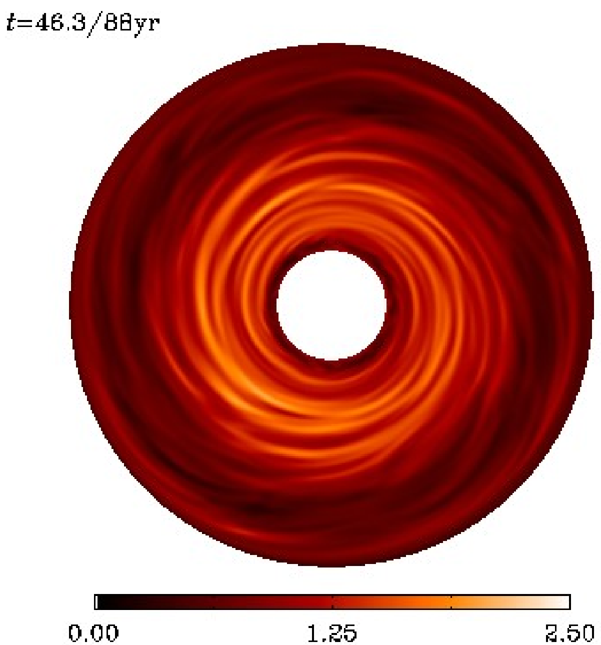
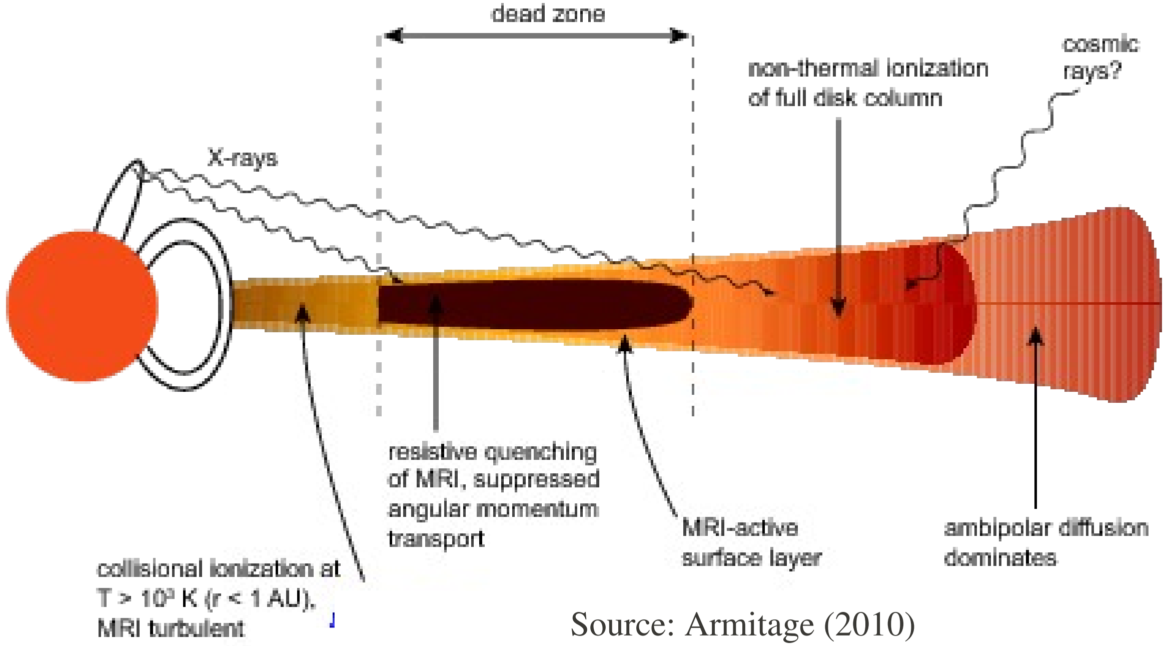
Dead zones are robust features of accretion disks

Turbulence in disks is enabled by
the Magneto-Rotational Instability



Source: Lyra et al. (2008)

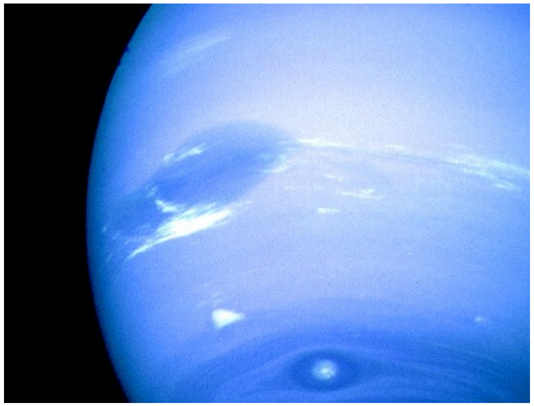


Source: Armitage (2010)

However.....

A substantial portion of the disk
is of **low ionization**
and therefore **dead** to the MRI

A possibility: Baroclinic Instability

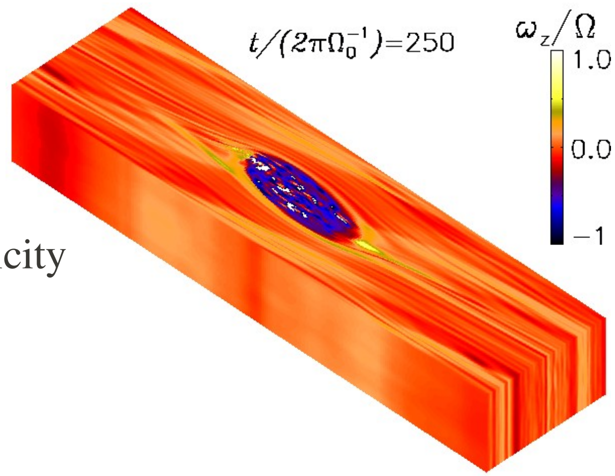
  

- Well known in planetary atmospheres
- Leads to the formation of **vortices**

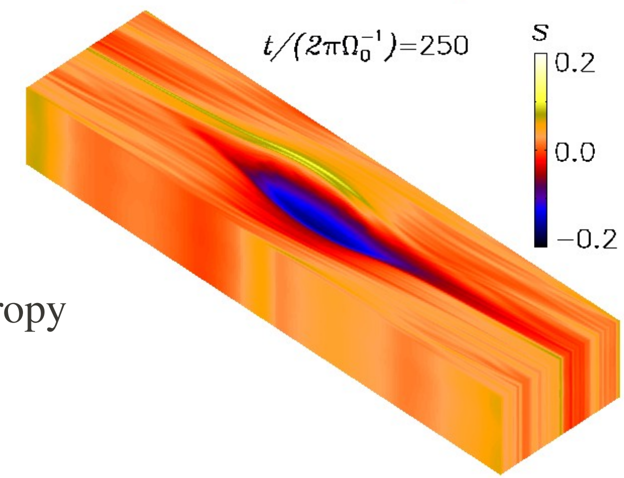
And vortices are:

- A solution of the NS equations: persistent structures
- Very interesting for planet formation

Baroclinic Instability - Excitation and self-sustenance of vortices



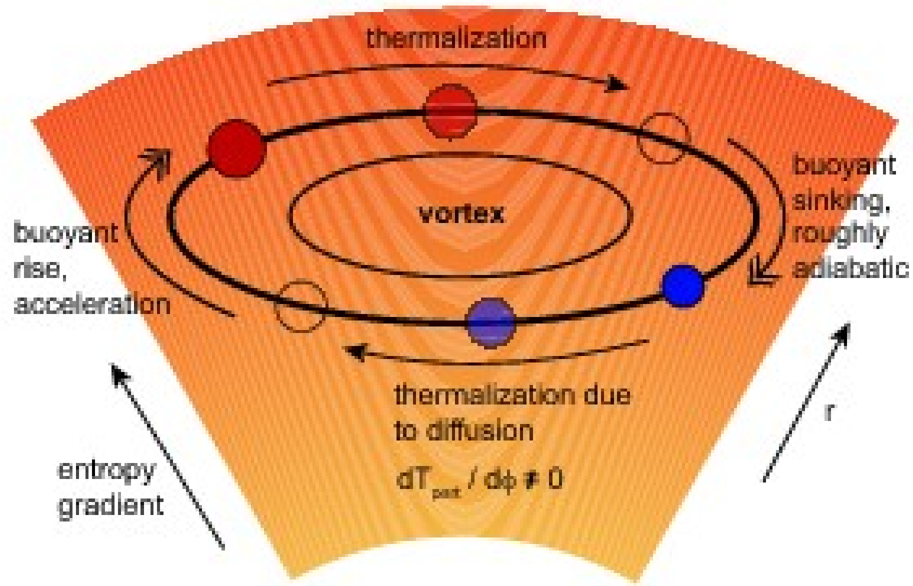
Vorticity



Entropy

Source: Lyra & Klahr (2010)

Sketch of the
Baroclinic Instability

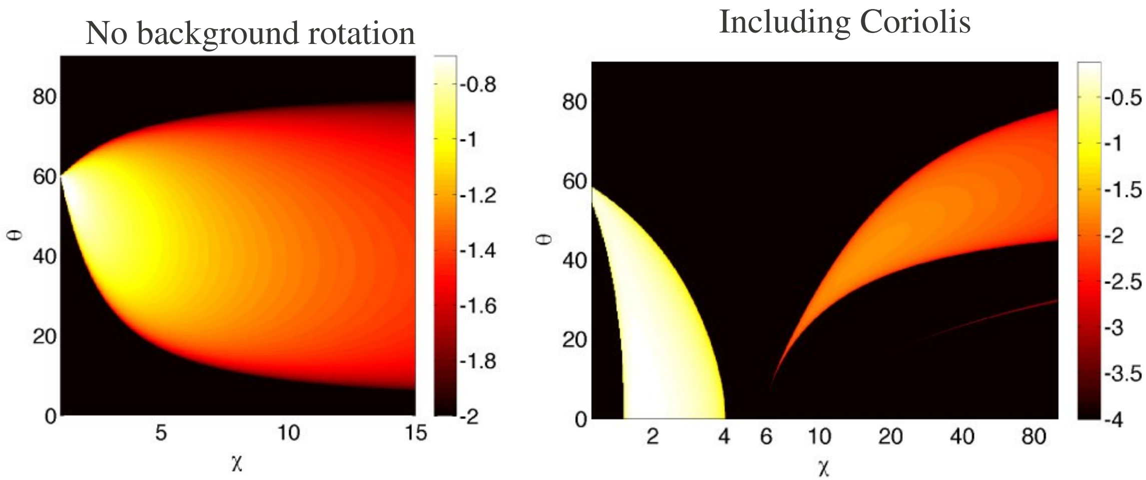


Source: Armitage (2010)

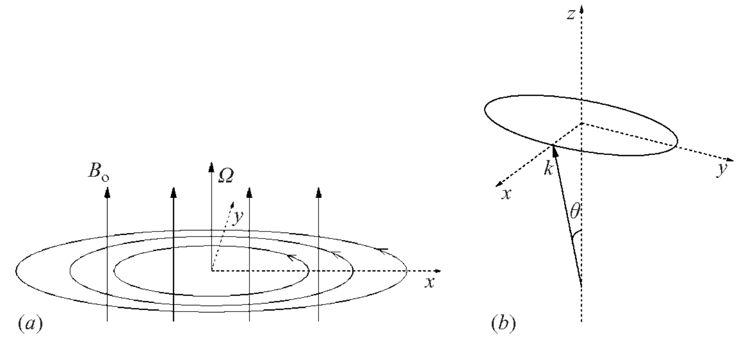
$$\frac{\partial \boldsymbol{\omega}}{\partial t} = -(\boldsymbol{u}\cdot\nabla)\,\boldsymbol{\omega} - \boldsymbol{\omega}\,(\nabla\cdot\boldsymbol{u}) + (\boldsymbol{\omega}\cdot\nabla)\,\boldsymbol{u} + \frac{1}{\rho^2}\nabla\rho\times\nabla p + \nu\nabla^2\boldsymbol{\omega}$$

advection — compression — stretching — baroclinicity — dissipation

Elliptic Instability



Source: Lesur & Papaloizou (2009)



Source: Mizerski & Bajer (2009)

Instability of elliptic streamlines

* In the **non-rotating** case:

- **Resonance** between

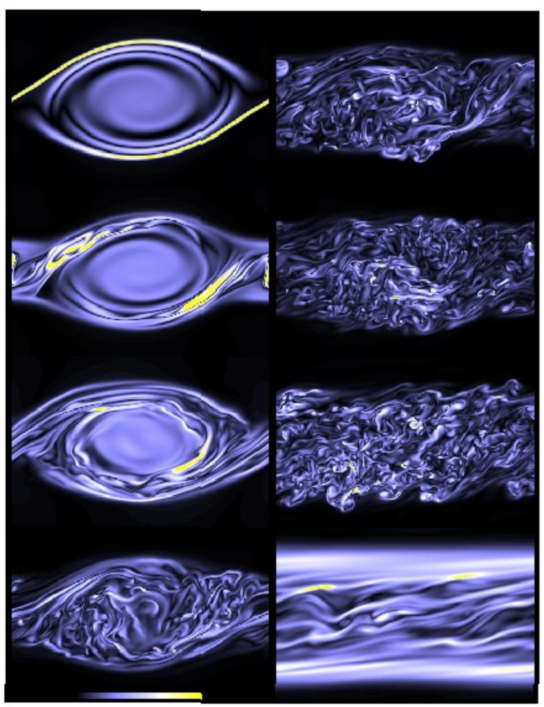
  vortex turnover frequency and **inertial waves**

* In the **rotating** case:

- Strong "horizontal" (theta=0) unstable mode:

  Exponential growth of epicyclic disturbances

Vortex coherence is destroyed
Energy cascades forward and dissipates

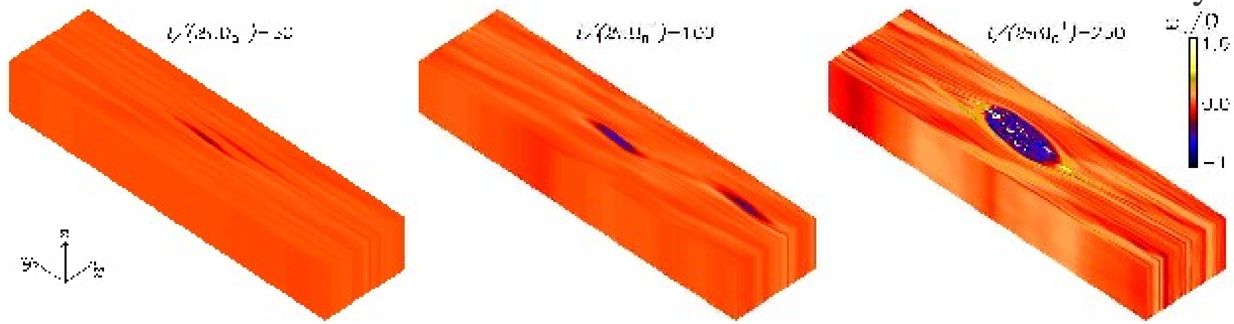


Source: McWilliams (2010)

Baroclinic Instability – Excitation and self-sustenance of vortices

The Baroclinic Instability in three dimensions

Vorticity

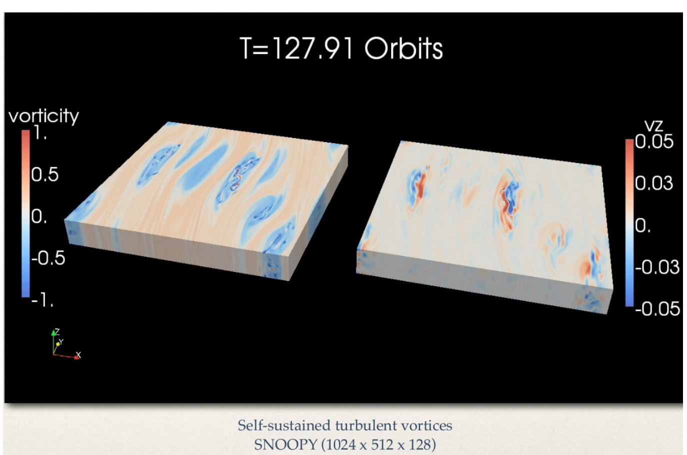


Source: Lyra & Klahr (2010)

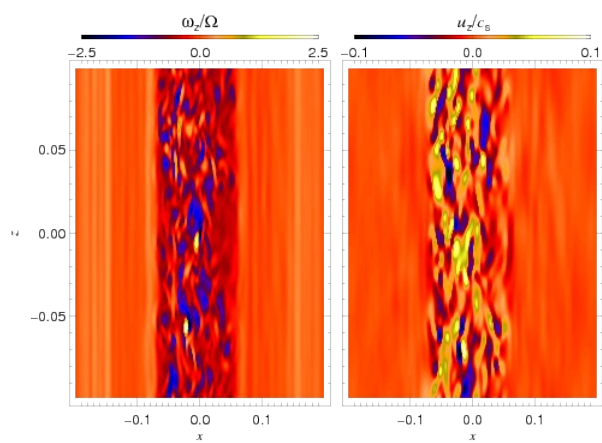
Despite the elliptical instability,

baroclinity keeps the vortex coherent.

The result is "core turbulence" only



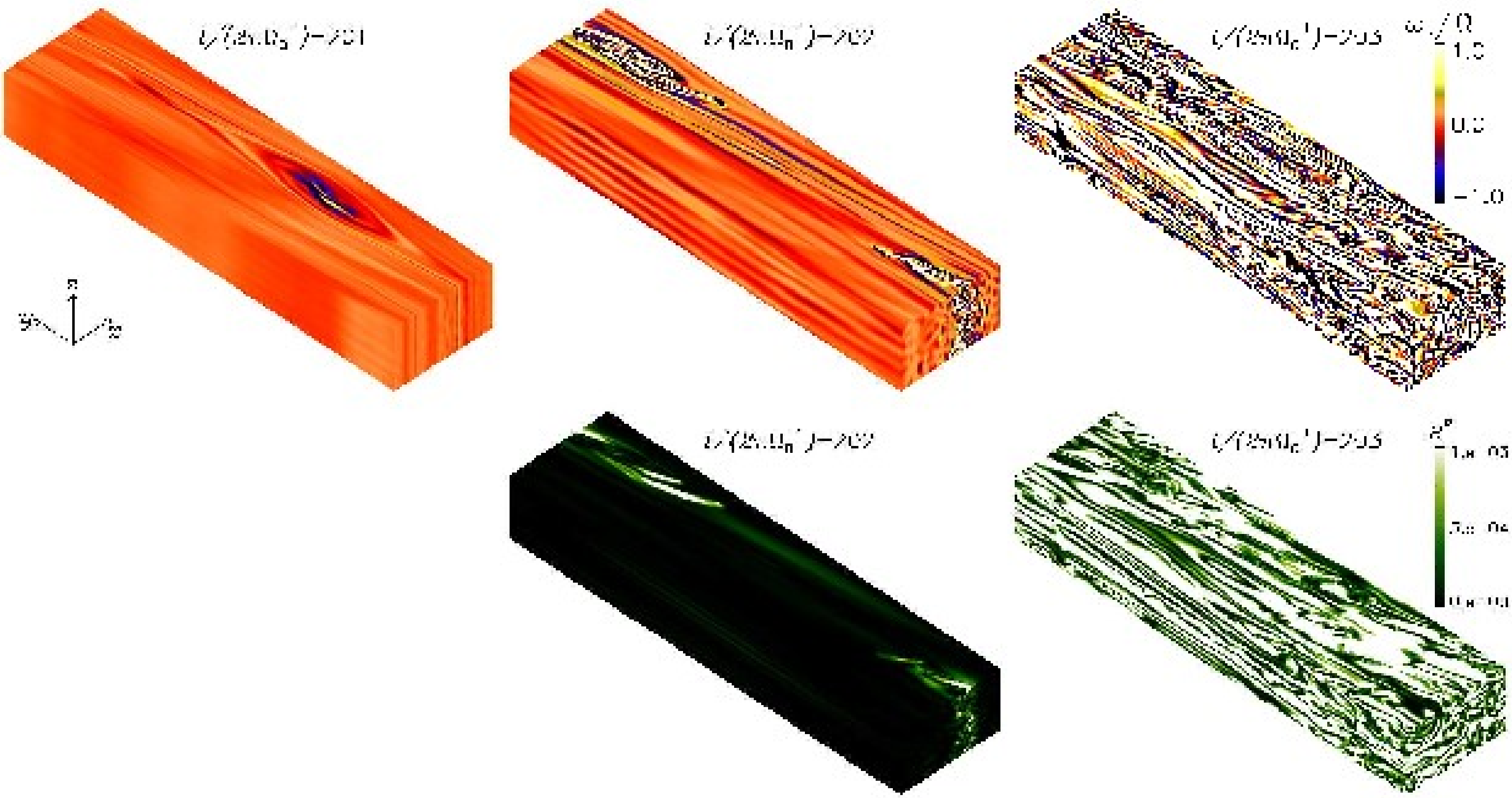
Source: Lesur & Papaloizou (2010)



Vorticity Kin. Energy

Interaction of Baroclinic and Magneto-Rotational Instabilities

What happens when the vortex is magnetized?



Vorticity

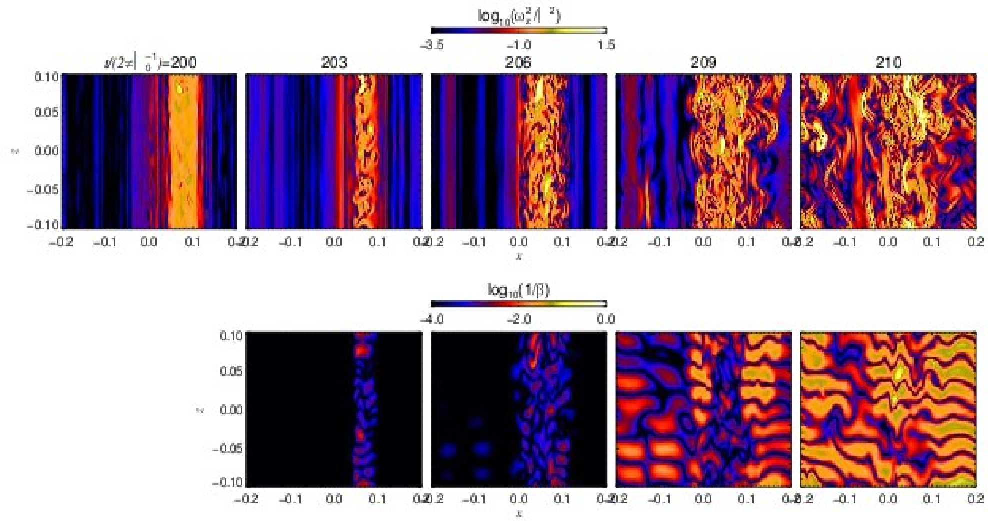
Magnetic Energy

Vortex gone!

Vortex MHD instability?

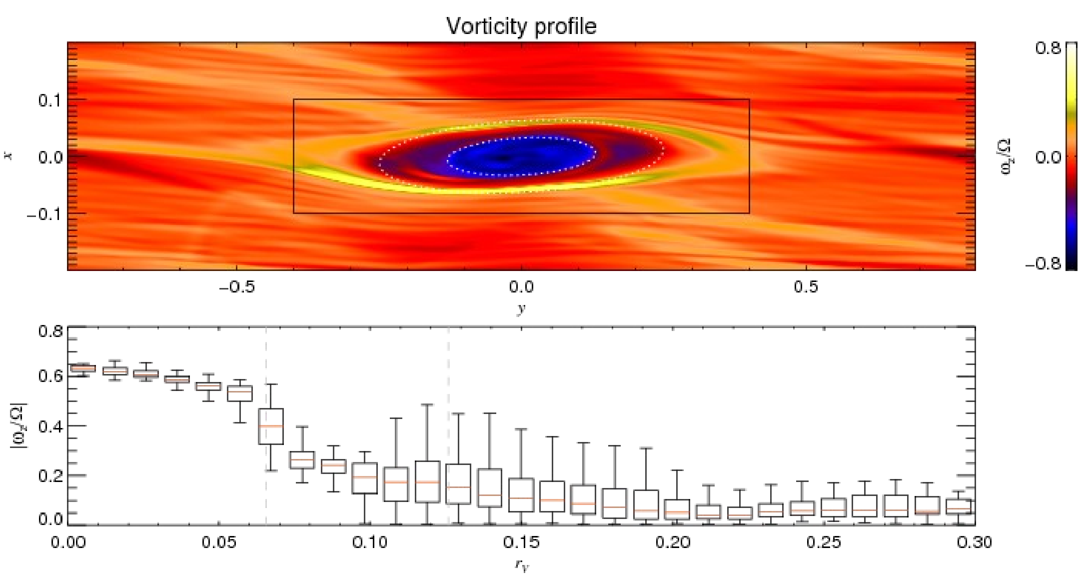
Notice that the vortex goes turbulent ***before*** the box.

Is this the MRI?



Enstrophy

Mag. Energy



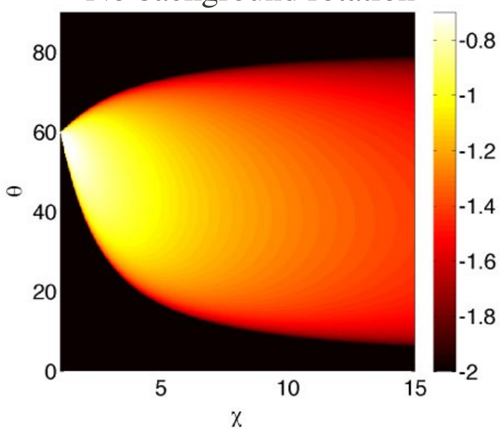
The MRI needs shear.
Yet, the core rotates close to rigid.

*So, **NO**, this is **not** the MRI.*

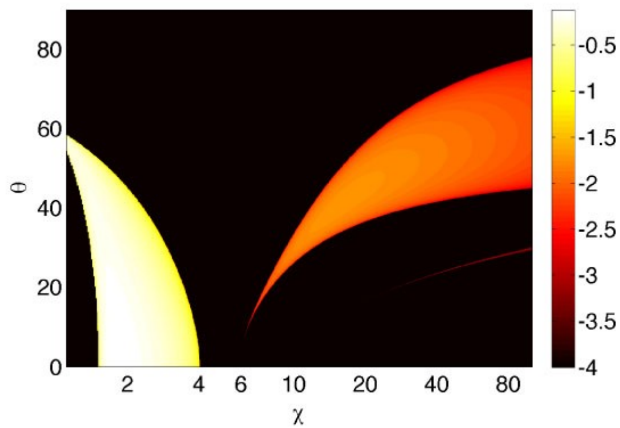
A magnetoelliptic mode?

Magneto-Elliptic Instability

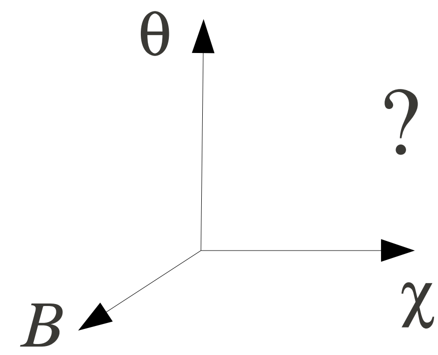
No background rotation



Including Coriolis



Including Lorentz



Mizerski & Bajer (2009, Journal of Fluid Mechanics)

"The presence of magnetic fields widens the range of existence of the horizontal instability to an unbounded interval of aspect ratios when

$$Ro^{-1} < -\frac{b^2}{4}$$

$$0 < k < 2|Ro|^{1/2}\frac{\Omega_K}{v_A}$$

$b = q/Ro$

$q = k/k_{BH}$

$Ro = \frac{\Omega_V \delta}{\Omega_K}$

$k_{BH} = \frac{\Omega_K}{v_A}$

$\delta = \frac{1}{2}(\chi + \chi^{-1})$

Magneto-Elliptic Instability

$$0 < k/k_{BH} < 2|Ro|^{1/2}$$

$$Ro = \frac{\Omega_V \delta}{\Omega_K}$$

Both the vortex and the shear flow contribute to the Rossby number.

$$u_x = -\Omega_V \, y/\chi$$

$$u_y = \Omega_V \, x \, \chi$$

$$\omega_T = \Omega_V(\chi + \chi^{-1}) = 2\,\Omega_V \delta$$

$$= \omega_V + \omega_{box}$$

$$= \omega_V - 3/2\,\Omega_K$$

$$\Omega_V \delta = \frac{\omega_V}{2} - \frac{3}{4}\Omega_K$$

$$Ro = \frac{\omega_V}{2\,\Omega_K} - \frac{3}{4}$$

In the no-vortex limit (ω_v=0) , Ro=-3/4

$$0 < k/k_{BH} < \sqrt{3}$$

Magneto-Elliptic Instability – No vortex limit

$$0 < k / k_{BH} < \sqrt{3}$$

MRI

Magneto-Elliptic Instability – No vortex limit.

Note on consistency $$Ro=\frac{\Omega_V\,\delta}{\Omega_K}=\frac{\omega_V}{2\,\Omega_K}-\frac{3}{4}$$

In the no-vortex limit (ω_v=0) , Ro=-3/4

Kida solution: $$\Omega_V=-\frac{3\,\Omega_K}{2(\chi-1)}$$

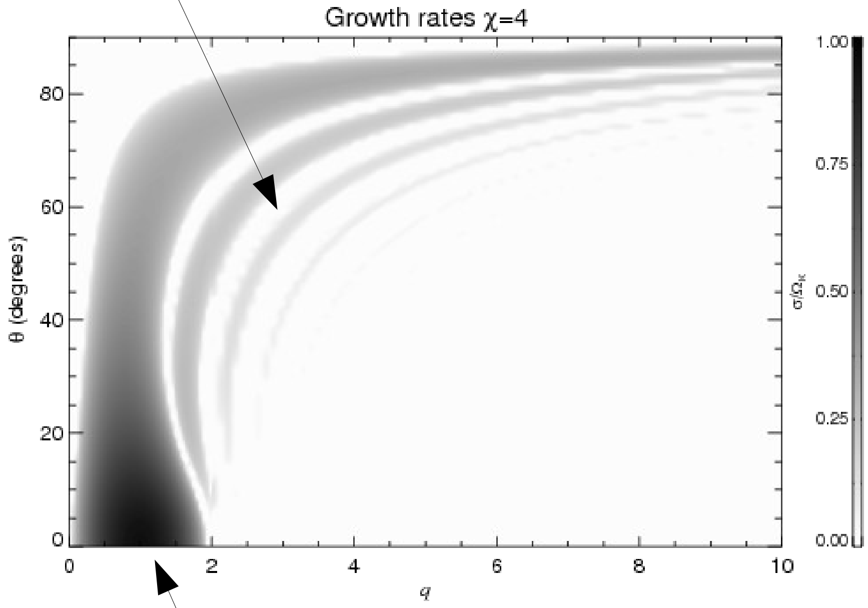
$$Ro=-\frac{3}{4}\frac{\chi^2-1}{\chi(\chi-1)} \longrightarrow \lim_{\chi\to\infty} Ro=-\frac{3}{4}$$

$$u_x=-\Omega_V\,y/\chi \qquad u_y=\Omega_V\,x\,\chi \longrightarrow \lim_{\chi\to\infty} u_x=0 \qquad \lim_{\chi\to\infty} u_y=-3/2\,\Omega_K\,x$$

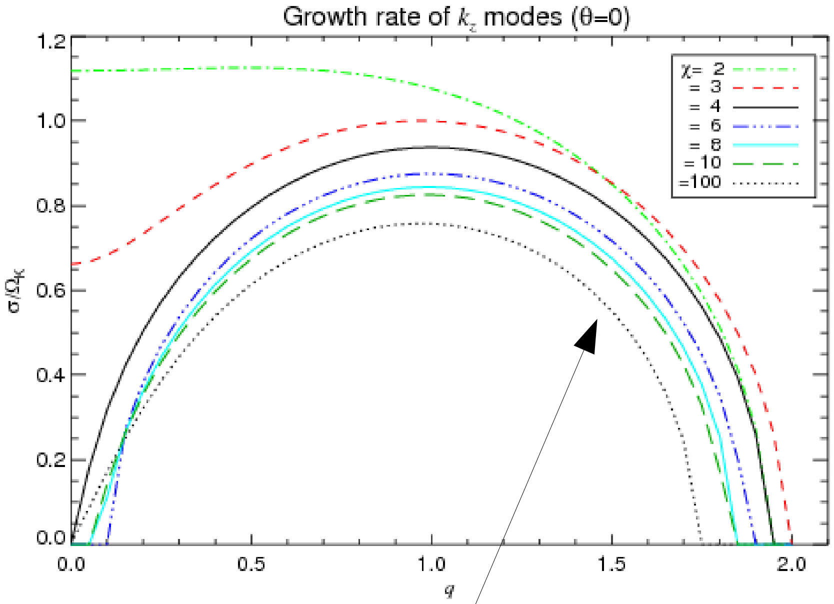
A vortex of infinite aspect ratio is equivalent to a shear flow

Growth rates

Vortex turnover resonance
with Alfvén waves

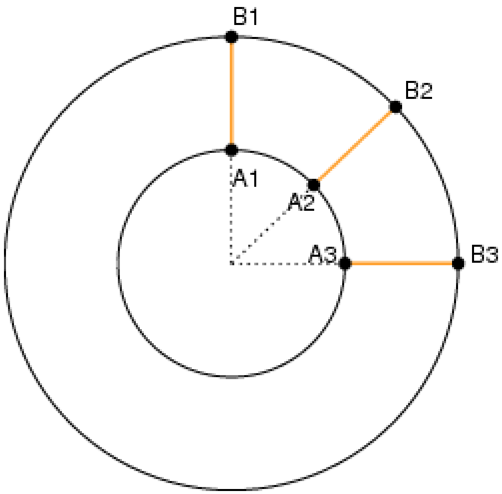


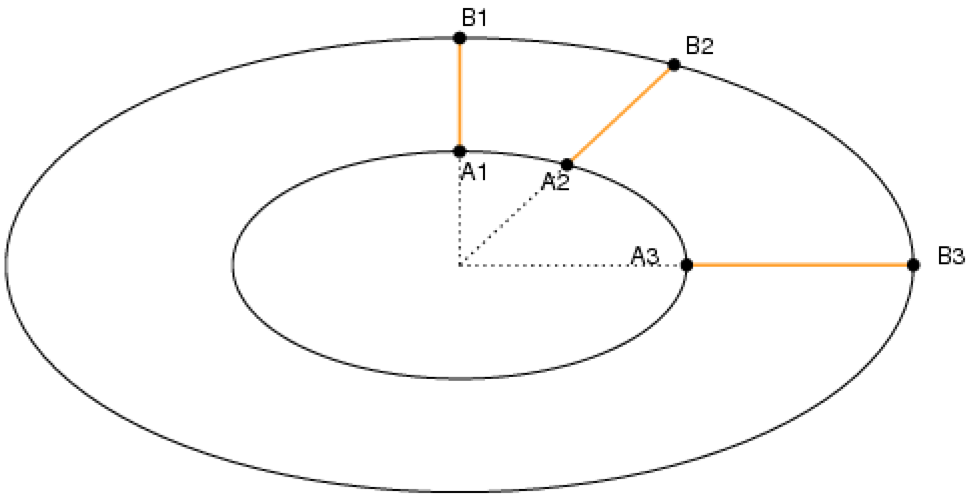
"Horizontal" magneto-elliptic
instability



MRI

Common ground between MRI and MEI



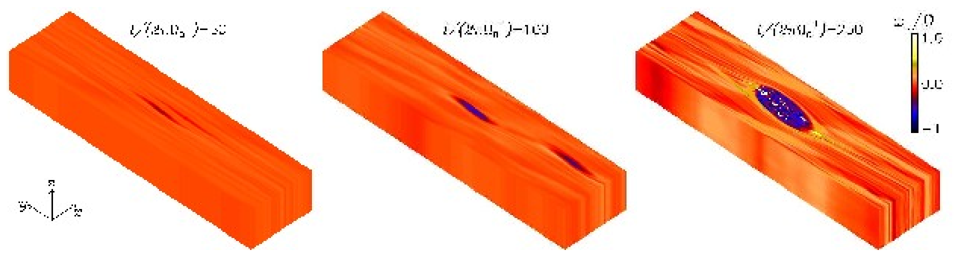


Elliptic streamlines have shear
even in uniform rotation

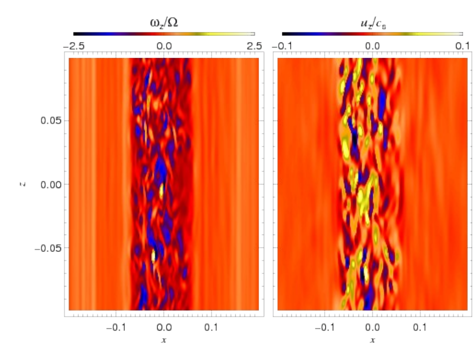
Conclusions

- 3D non-magnetized vortices reach a steady state
  * Unstable yet coherent
  * Balance between baroclinicity (+) and stretching (-)
  * Subsonic core turbulence (10% of sound speed)

- Vortices do not survive the MRI
  * Channel flows
  * Violent core turbulence
    - Magneto-elliptic instability
    - MRI is a limit of the MEI



- Fits neatly in the layered accretion paradigm.
  * Active layers are unmodified
  * Dead zone only is endowed with vortices



Open questions:
- Vertical stratification
- Realistic entropy gradients and thermal diffusion
- *Particles??*

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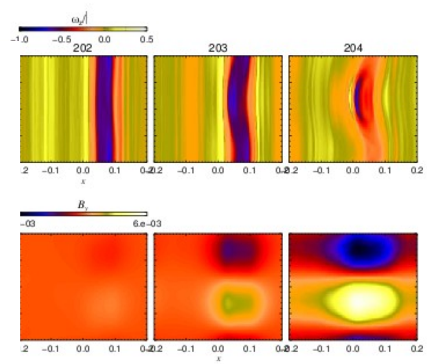

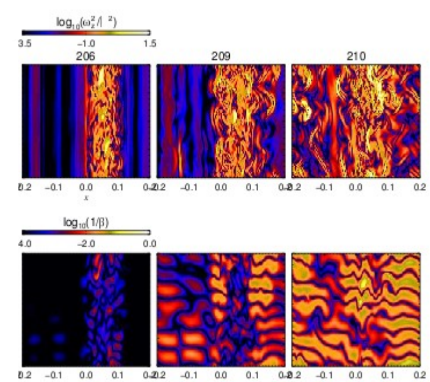

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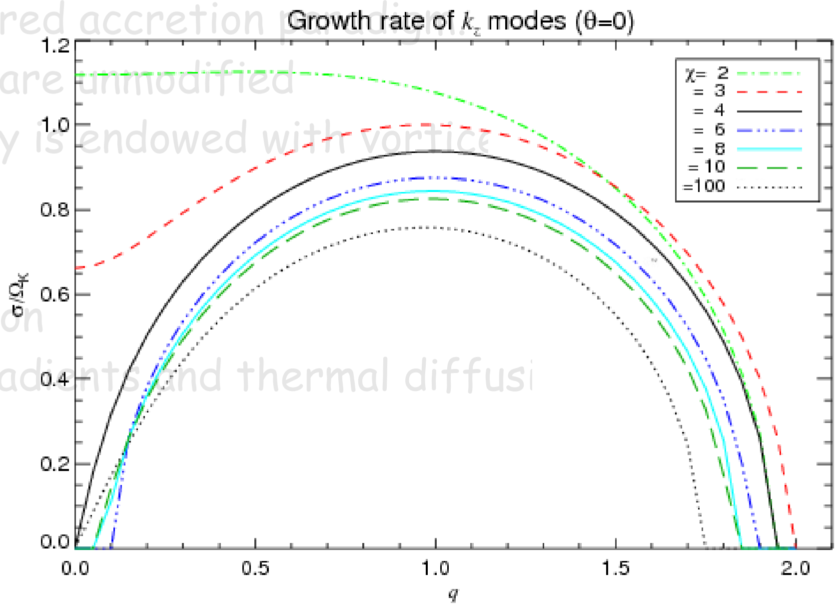

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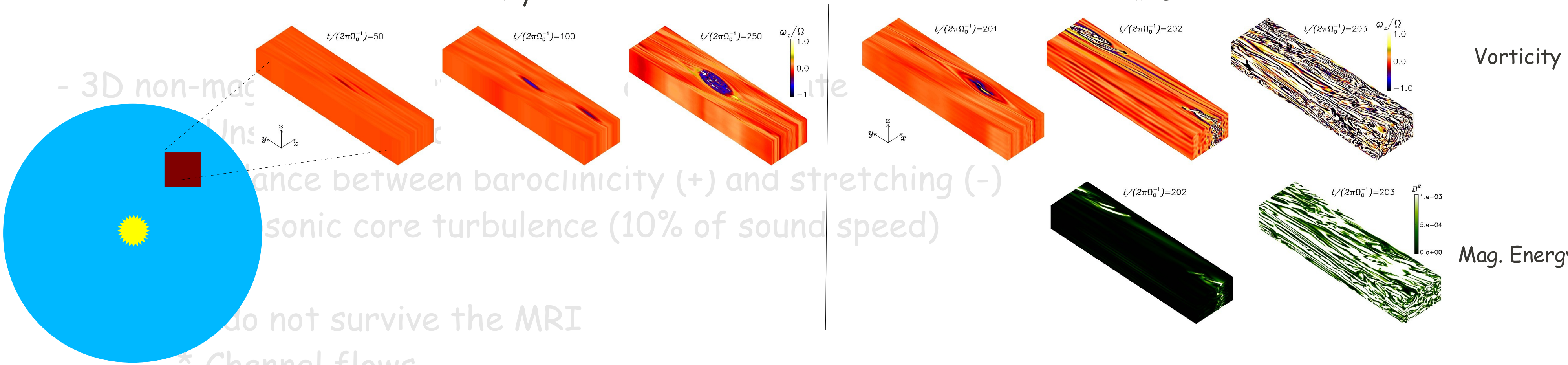

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Thanks for your attention

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Astronomy & Astrophysics

The baroclinic instability in the context of layered accretion

Self-sustained vortices and their magnetic stability in local compressible unstratified models of protoplanetary disks

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ABSTRACT

Context. Turbulence and angular momentum transport in accretion disks remains a topic of debate. With the realization that dead zones are robust features of protoplanetary disks, the search for hydrodynamical sources of turbulence continues. A possible source is the baroclinic instability (BI), which has been shown to exist in unmagnetized non-barotropic disks.
Aims. We aim to verify the existence of the baroclinic instability in 3D magnetized disks, as well as its interplay with other instabilities, namely the magneto-rotational instability (MRI) and the magneto-elliptical instability.
Methods. We performed local simulations of non-isothermal accretion disks with the Pencil Code. The entropy gradient that generates the baroclinic instability is linearized and included in the momentum and energy equations in the shearing box approximation. The model is compressible, so excitation of spiral density waves is allowed and angular momentum transport can be measured.
Results. We find that the vortices generated and sustained by the baroclinic instability in the purely hydrodynamical regime do not survive when magnetic fields are included. The MRI by far supersedes the BI in growth rate and strength at saturation. The resulting turbulence is virtually identical to an MRI-only scenario. We measured the intrinsic vorticity profile of the vortex, finding little radial variation in the vortex core. Nevertheless, the core is disrupted by an MHD instability, which we identify with the magneto-elliptic instability. This instability has nearly the same range of unstable wavelengths as the MRI, but has higher growth rates. In fact, we identify the MRI as a limiting case of the magneto-elliptic instability, when the vortex aspect ratio tends to infinity (pure shear flow). We isolated its effect on the vortex, finding that a strong but unstable vertical magnetic field leads to channel flows inside the vortex, which stretch it apart. When the field is decreased or resistivity is used, we find that the vortex survives until the MRI develops in the box. The vortex is then destroyed by the strain of the surrounding turbulence. Constant azimuthal fields and zero net flux fields also lead to vortex destruction. Resistivity quenches both instabilities when the magnetic Reynolds number of the longest vertical wavelength of the box is near unity.
Conclusions. We conclude that vortex excitation and self-sustenance by the baroclinic instability in protoplanetary disks is viable only in low ionization, i.e., the dead zone. Our results are thus in accordance with the layered accretion paradigm. A baroclinicly unstable dead zone should be characterized by the presence of large-scale vortices whose cores are elliptically unstable, yet sustained by the baroclinic feedback. Since magnetic fields destroy the vortices and the MRI outweighs the BI, the active layers are unmodified.

Key words. accretion, accretion disks – hydrodynamics – instabilities – magnetohydrodynamics (MHD)– turbulence – methods: numerical