

Polydisperse Pebble Accretion

Lessons from the Kuiper belt

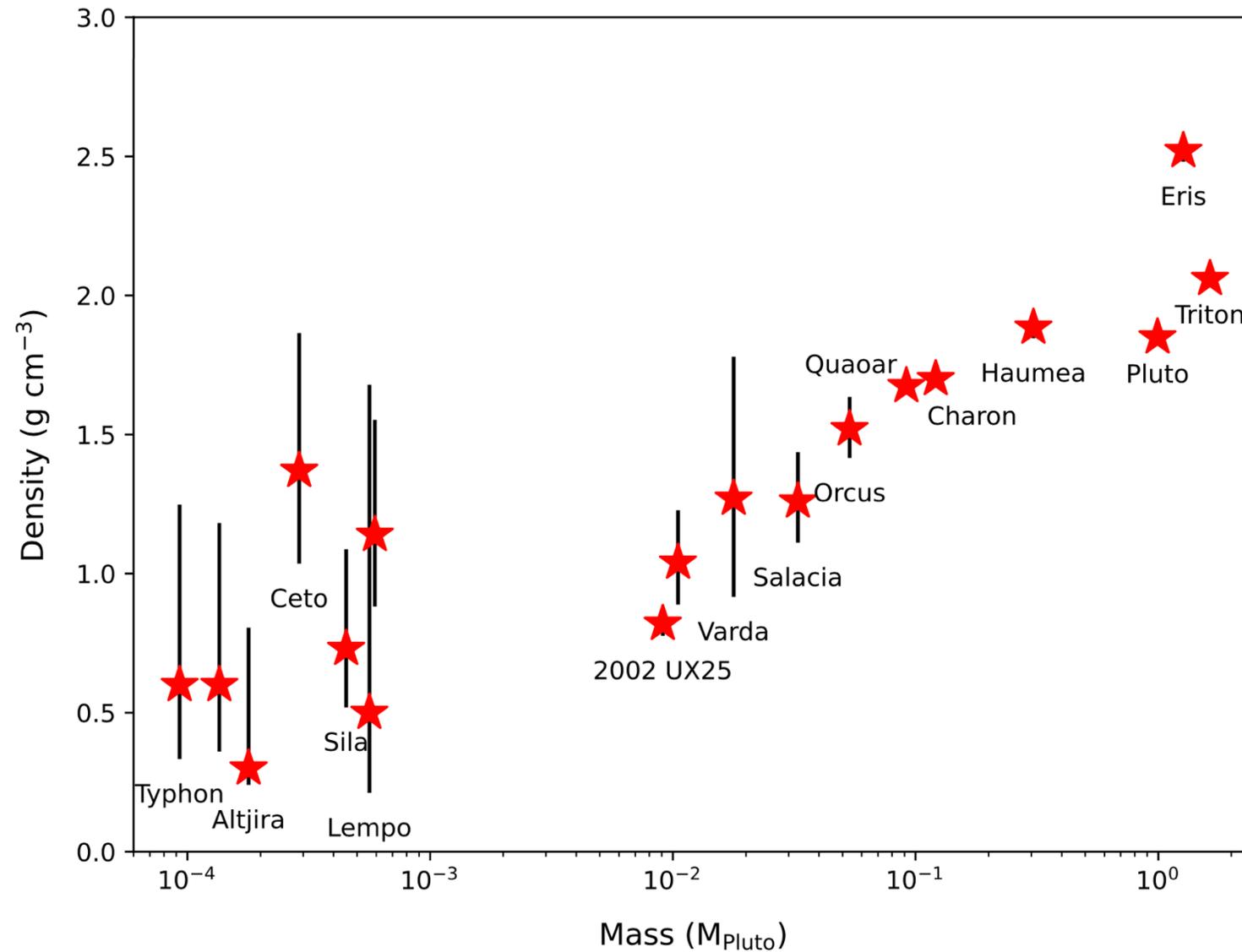
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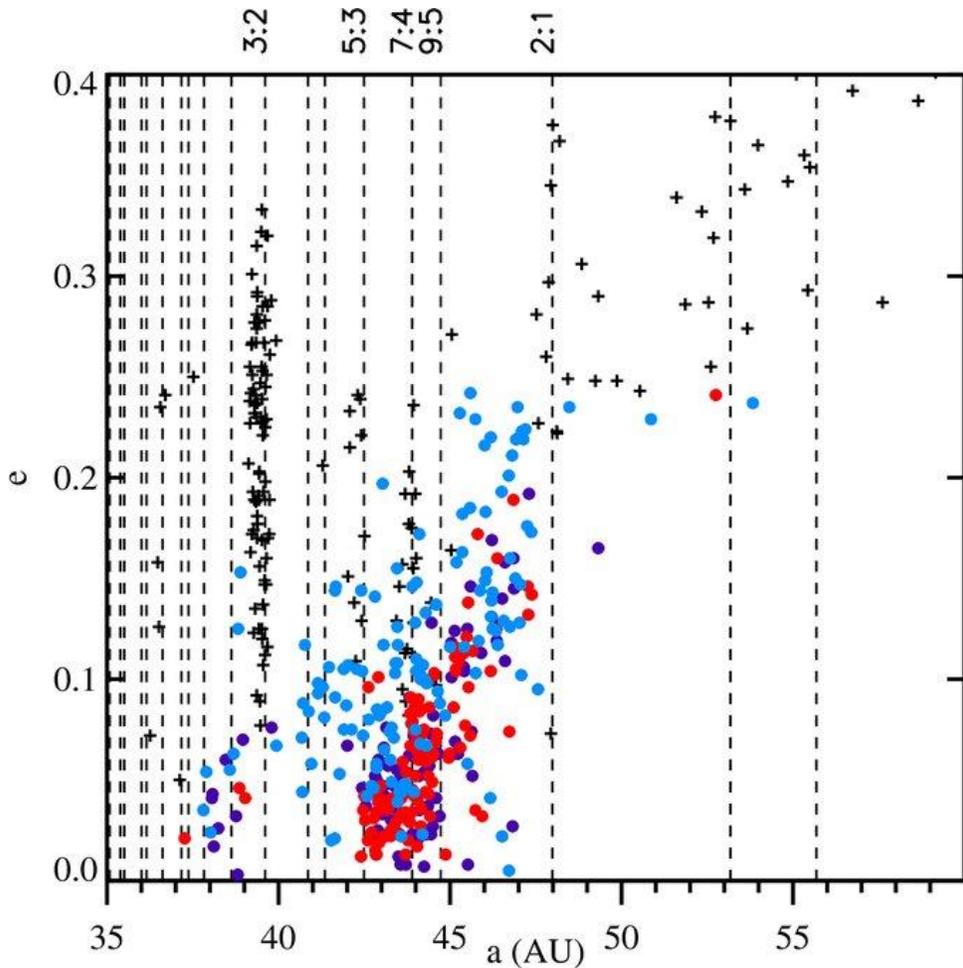
The size-density relationship of Kuiper Belt objects



Data; Thomas (2000), Stansberry et al. (2006), Grundy et al. (2007), Brown et al. (2011), Stansberry et al. (2012), Brown (2013), Fornasier et al. (2013), Vilenius, et al. (2014), Nimmo et al. (2016), Ortiz et al. (2017), Brown and Butler (2017), Grundy et al. (2019), Morgado et al. (2023), Pereira et al. (2023).

Cold Classicals Kuiper Belt Objects

Presumably pristine planetesimals



+ Resonant and Scattered

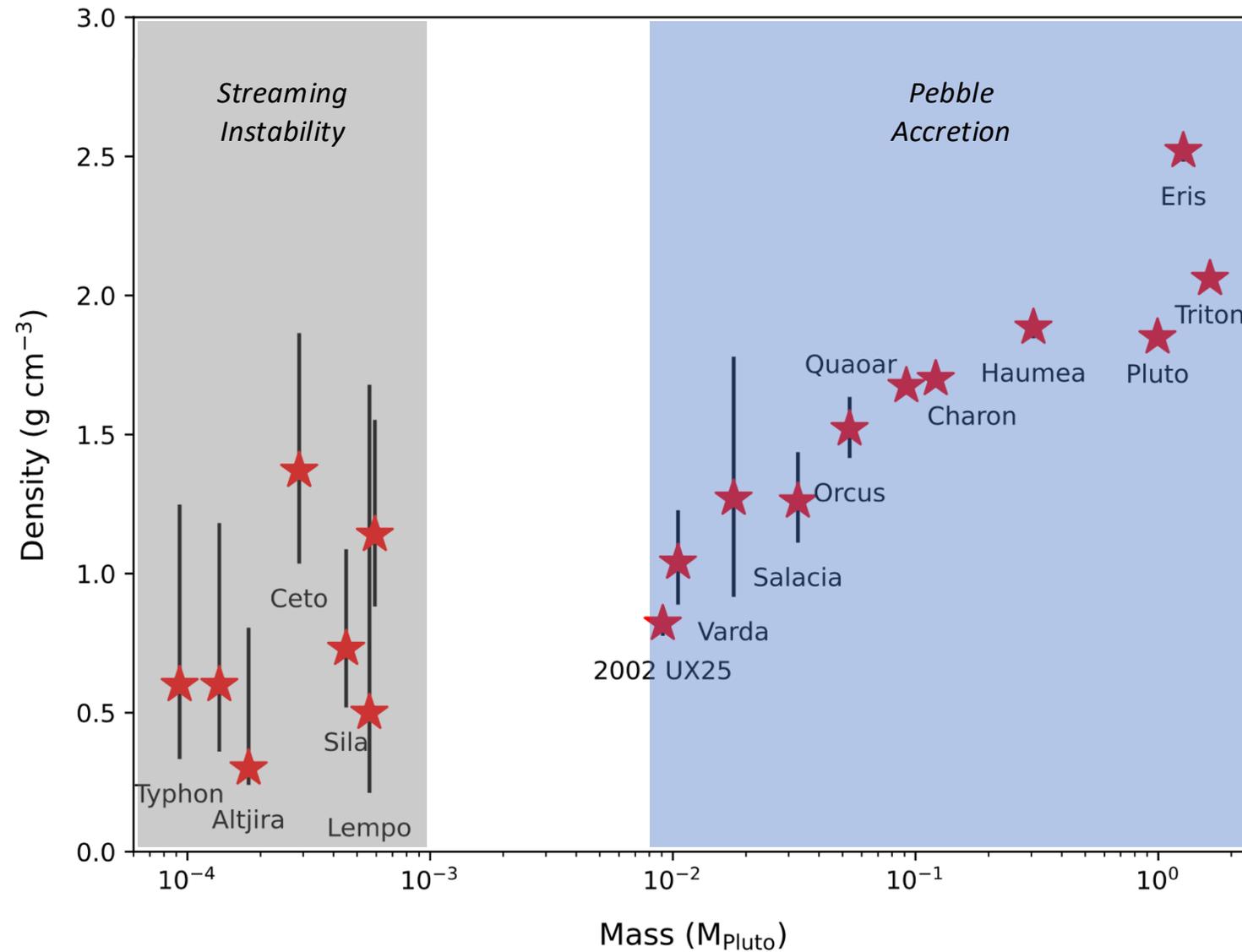
Cold Classical $i < 2^\circ$

"Ambiguous" $2^\circ < i < 6^\circ$

Hot Classicals $i > 6^\circ$



The size-density relationship of Kuiper Belt objects



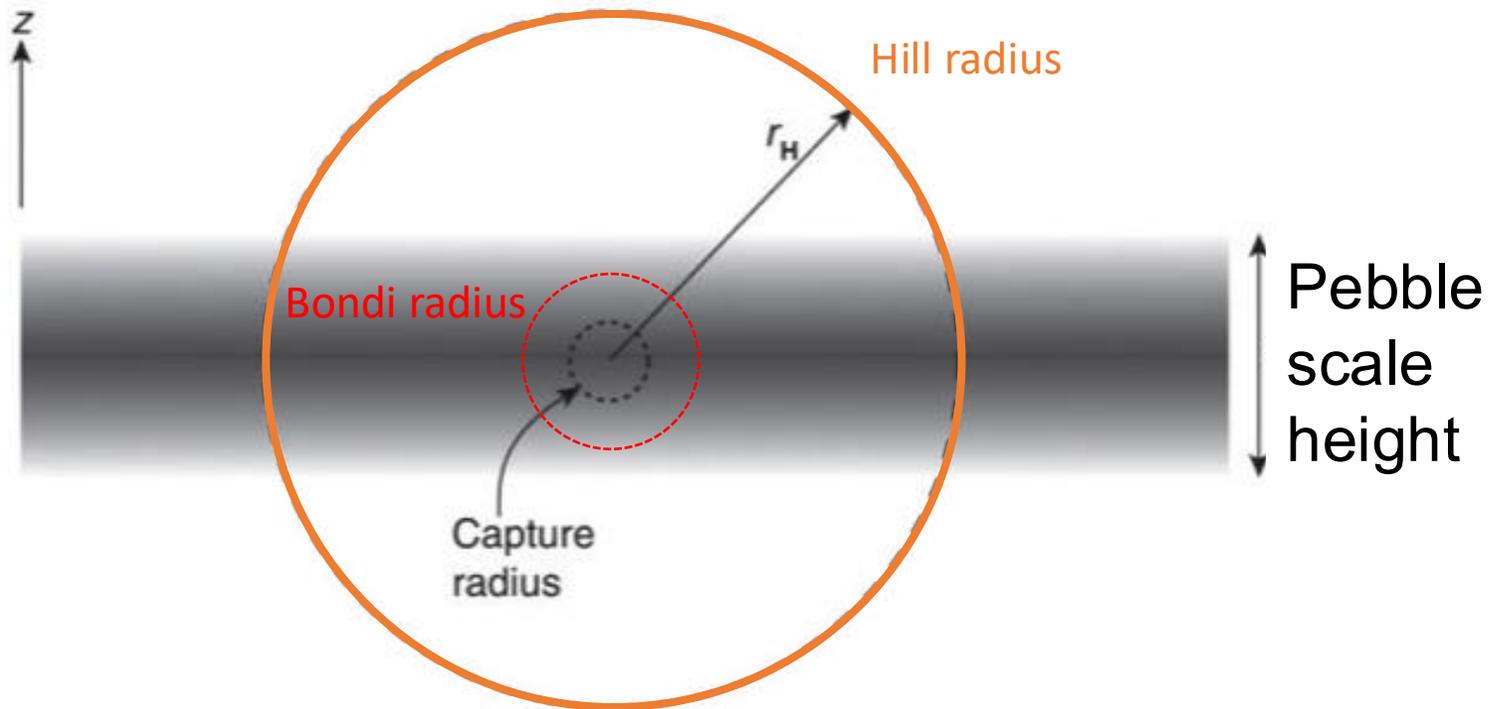
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Pebble Accretion: Geometric, Bondi, and Hill regime

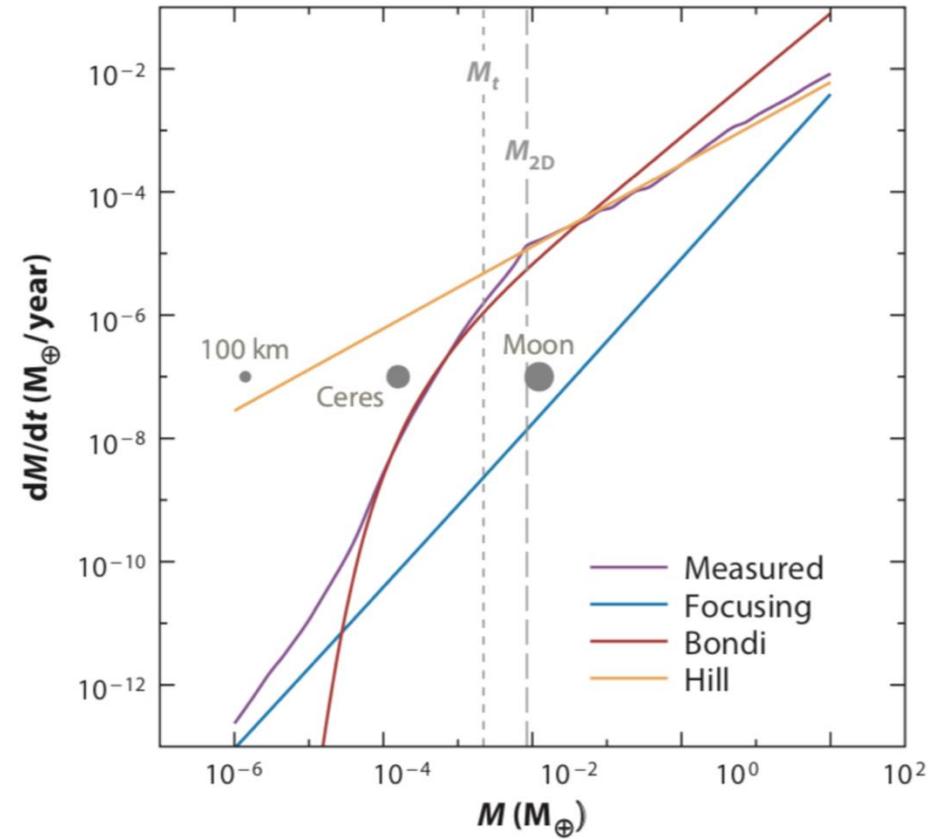
Bondi accretion - Bound against **headwind**

Hill accretion - Bound against **stellar tide**

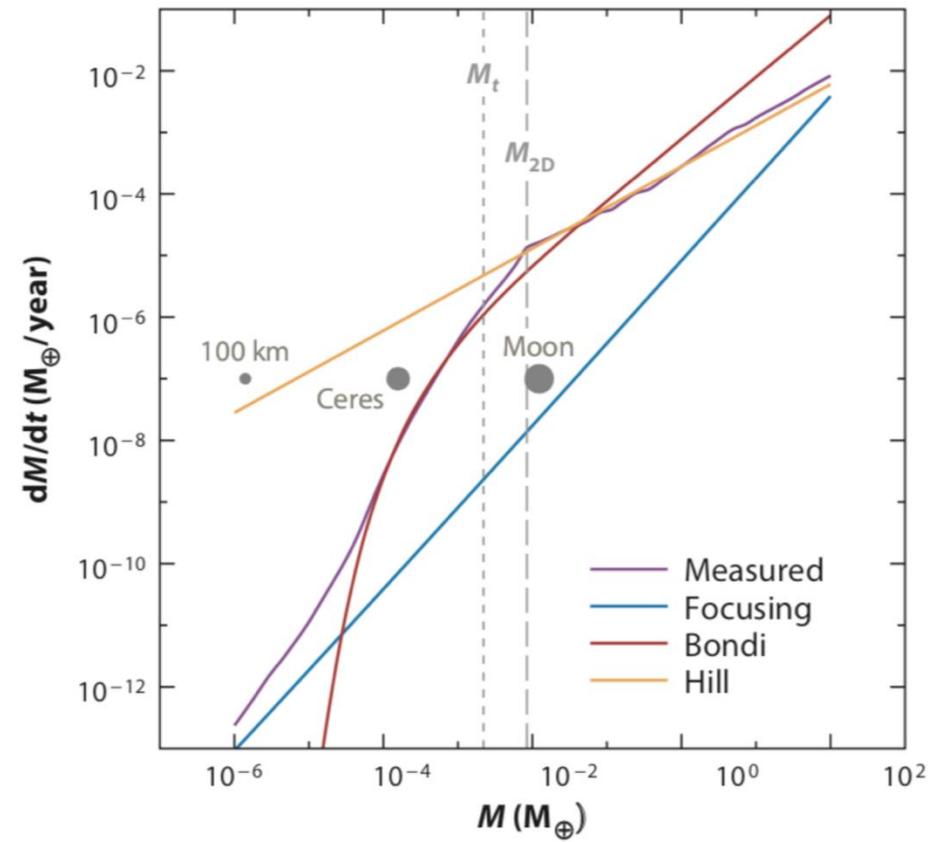
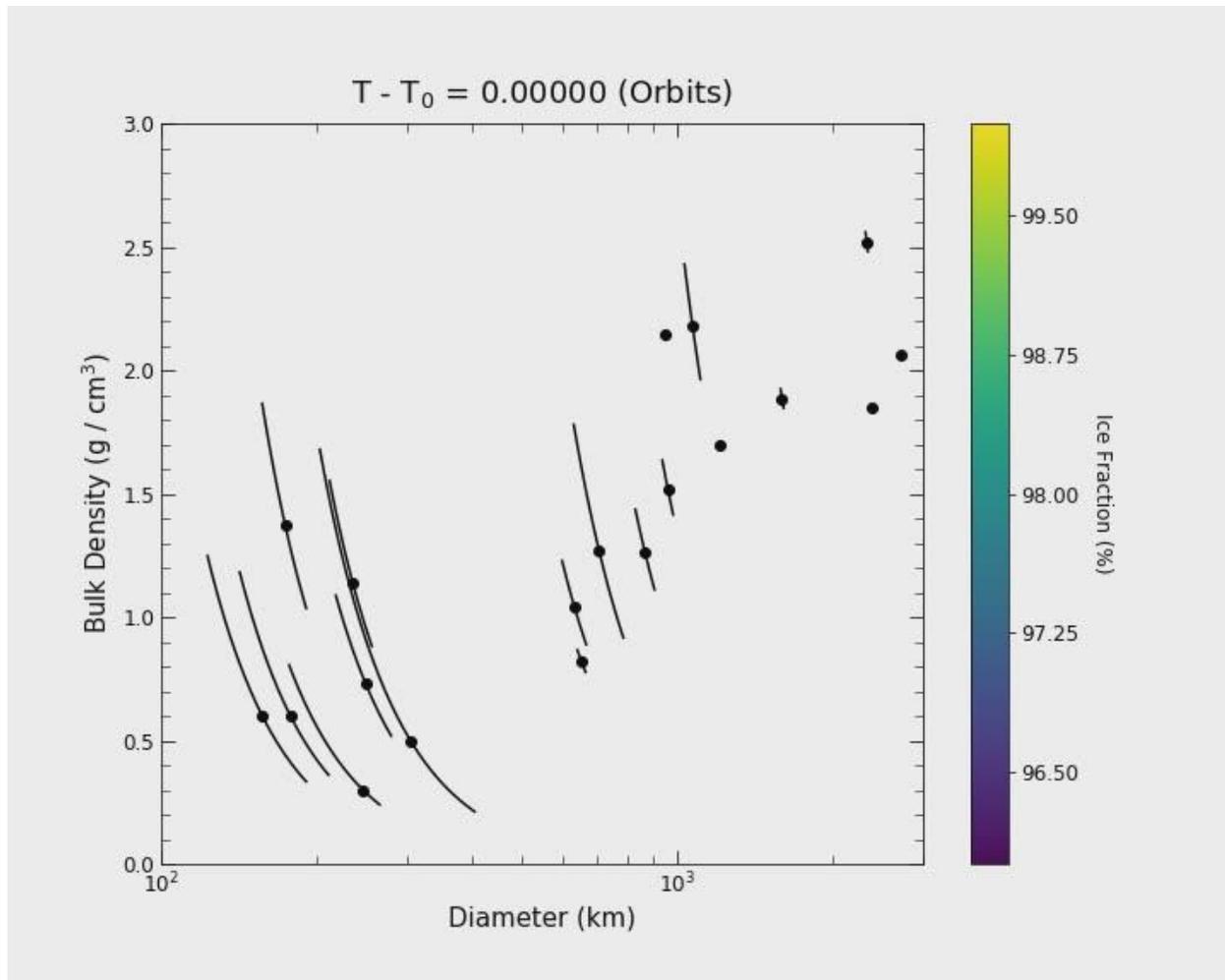
$$\xi \equiv \left(\frac{R_{\text{acc}}}{2H_d}\right)^2 \quad \begin{aligned} \dot{M}_{3D} &= \lim_{\xi \rightarrow 0} \dot{M} = \pi R_{\text{acc}}^2 \rho_{d0} \delta v, \\ \dot{M}_{2D} &= \lim_{\xi \rightarrow \infty} \dot{M} = 2R_{\text{acc}} \Sigma_d \delta v, \end{aligned}$$



Mass Accretion rates



Integrate pebble accretion

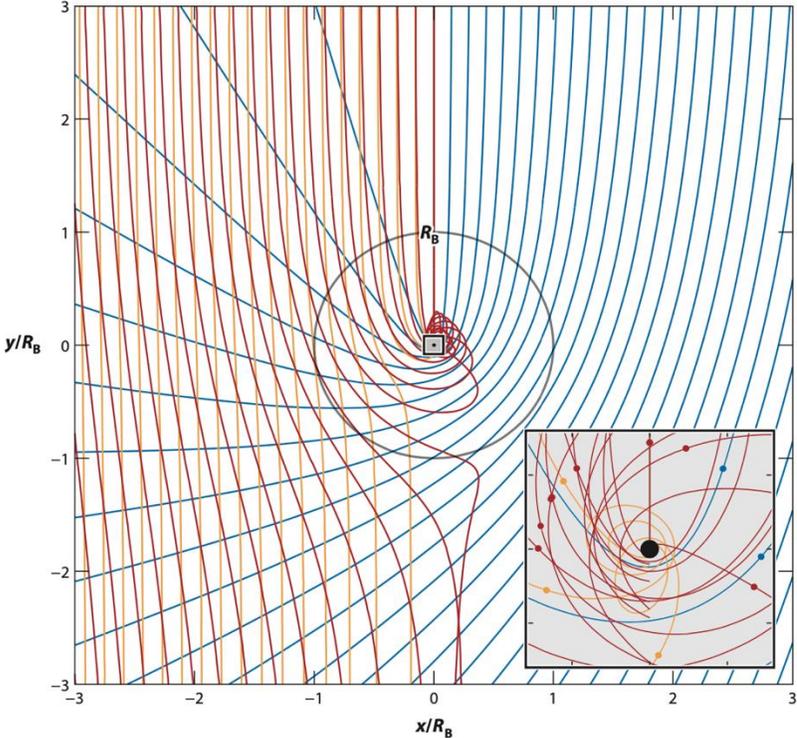


Pebble Accretion: Pebbles of different size accrete differently

“Goldilocks effect”

Bondi Regime

- Large ($t_s \gg t_b$)
- Medium ($t_s \sim t_b$)
- Small ($t_s \ll t_b$)



Pebble size that accretes optimally

Drag time $t_s \sim$ Time to cross Bondi sphere t_b

$$t_b = GM/\Delta v^3$$

Polydisperse (Multi-Species) Pebble Accretion

$$\rho_d(a, z) = \int_0^a m(a') F(a', z) da'.$$

$$F(a, z) \equiv f(a) e^{-z^2/2H_d^2},$$

$$f(a) = \frac{3(1-p)Z\Sigma_g}{2^{5/2}\pi^{3/2}H_g\rho_\bullet^{(0)}a_{\max}^{4-k}} \sqrt{1 + a \frac{\pi \rho_\bullet(a)}{2 \Sigma_g \alpha}} a^{-k}.$$

$$S \equiv \frac{1}{\pi R_{\text{acc}}^2} \int_{-R_{\text{acc}}}^{R_{\text{acc}}} 2\sqrt{R_{\text{acc}}^2 - z^2} \exp\left(-\frac{z^2}{2H_d^2}\right) dz,$$

$$W(a) = \frac{3(1-p)Z\Sigma_g}{4\pi\rho_\bullet^{(0)}a_{\max}^{4-k}} a^{-k},$$

$$\hat{R}_{\text{acc}}^{(\text{Bondi})} = \left(\frac{4\tau_f}{t_B}\right)^{1/2} R_B,$$

$$\hat{R}_{\text{acc}}^{(\text{Hill})} = \left(\frac{\text{St}}{0.1}\right)^{1/3} R_H,$$

$$\delta v \equiv \Delta v + \Omega R_{\text{acc}},$$

$$R_{\text{acc}} \equiv \hat{R}_{\text{acc}} \exp[-\chi(\tau_f/t_p)^\gamma],$$

$$\frac{\partial \Sigma_d(a)}{\partial a} \propto a^{-p};$$

$$\rho_\bullet \propto a^{-q};$$

$$t_p \equiv \frac{GM_p}{(\Delta v + \Omega R_H)^3}$$

$$\dot{M}(a) = \int_0^a \frac{\partial \dot{M}(a')}{\partial a'} da',$$

$$\frac{\partial \dot{M}(a)}{\partial a} = \pi R_{\text{acc}}^2(a) \delta v(a) S(a) m(a) f(a).$$

$$\dot{M}_{2\text{D, Hill}} = 2 \times 10^{2/3} \Omega R_H^2 \int_0^{a_{\max}} \text{St}(a)^{2/3} m(a) W(a) da.$$

$$\begin{aligned} \dot{M}_{3\text{D, Bondi}} &= \frac{4\pi R_B \Delta v^2}{\Omega} \\ &\times \int_0^{a_{\max}} \text{St} e^{-2\psi} m(a) f(a) \\ &\times \left[1 + 2 \left(\text{St} \frac{\Omega R_B}{\Delta v} \right)^{1/2} e^{-\psi} \right] da, \quad \psi \equiv \chi [\text{St}/(\Omega t_p)]^\gamma. \end{aligned}$$

Analytical theory of polydisperse (multi-species) pebble accretion

Monodisperse (single species)

$$\dot{M}_{3D} = \lim_{\xi \rightarrow 0} \dot{M} = \pi R_{\text{acc}}^2 \rho_{d0} \delta v,$$

$$\dot{M}_{2D} = \lim_{\xi \rightarrow \infty} \dot{M} = 2R_{\text{acc}} \Sigma_d \delta v,$$

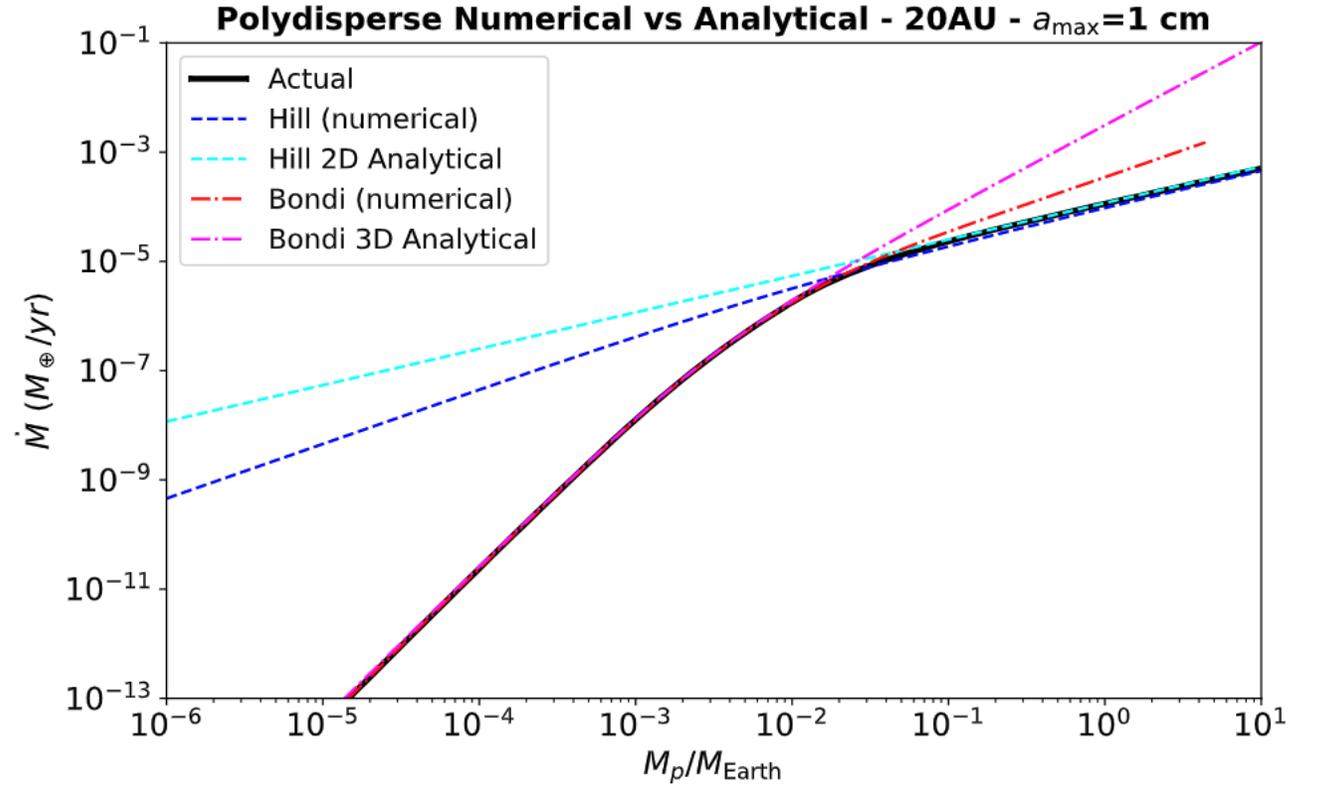
Lambrechts & Johansen (2012)

Polydisperse (multiple species)

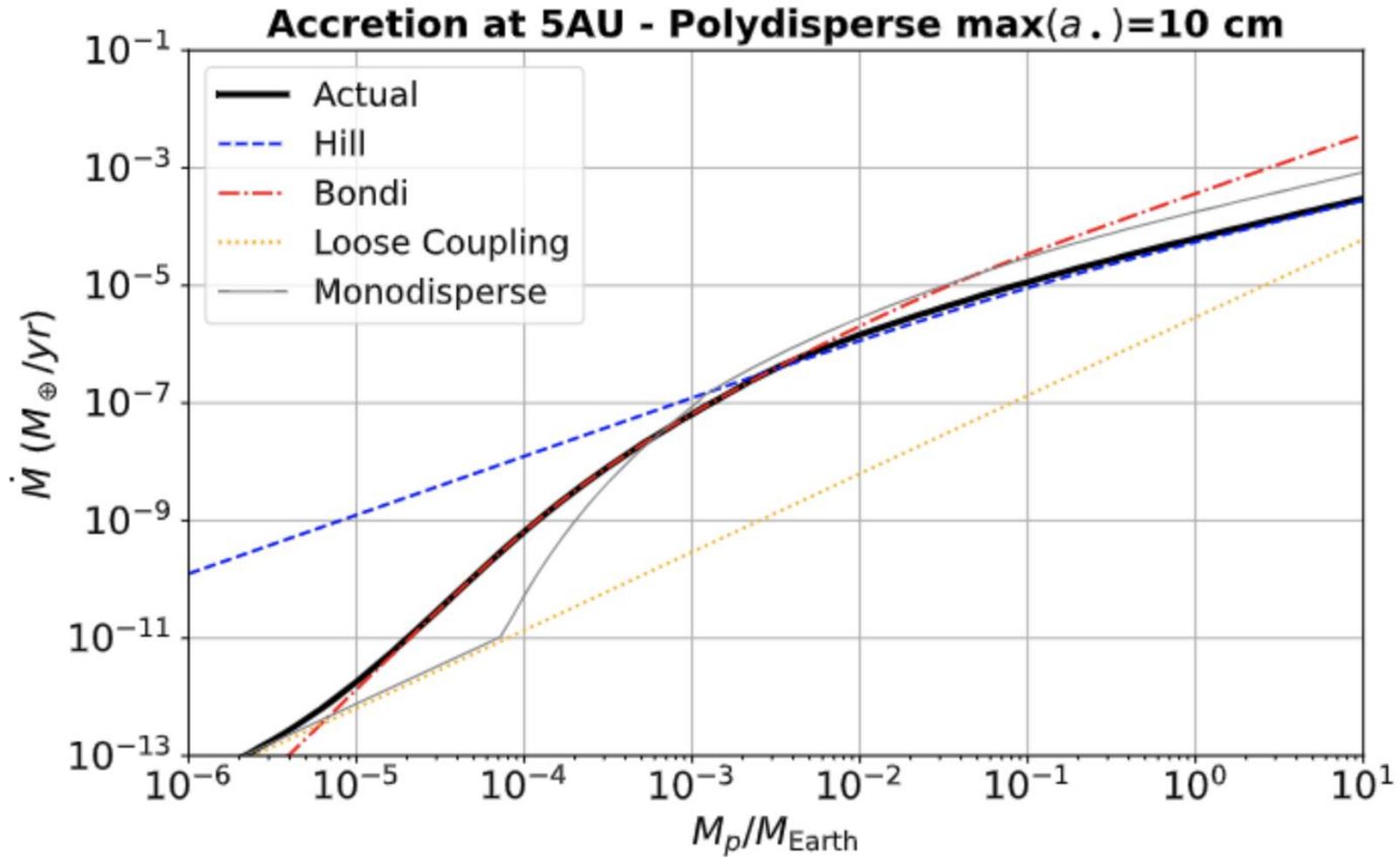
$$\dot{M}_{2D,\text{Hill}} = \frac{6(1-p)}{14-5q-3k} \left(\frac{\text{St}_{\text{max}}}{0.1} \right)^{2/3} \Omega R_H^2 Z \Sigma_g.$$

$$\dot{M}_{3D,\text{Bondi}} \approx C_1 \frac{\gamma_l \left(\frac{b_1+1}{s}, j_1 a_{\text{max}}^s \right)}{s J_1^{(b_1+1)/s}} + C_2 \frac{\gamma_l \left(\frac{b_2+1}{s}, j_2 a_{\text{max}}^s \right)}{s J_2^{(b_2+1)/s}} + C_3 \frac{\gamma_l \left(\frac{b_3+1}{s}, j_3 a_{\text{max}}^s \right)}{s J_3^{(b_3+1)/s}} + C_4 \frac{\gamma_l \left(\frac{b_4+1}{s}, j_4 a_{\text{max}}^s \right)}{s J_4^{(b_4+1)/s}},$$

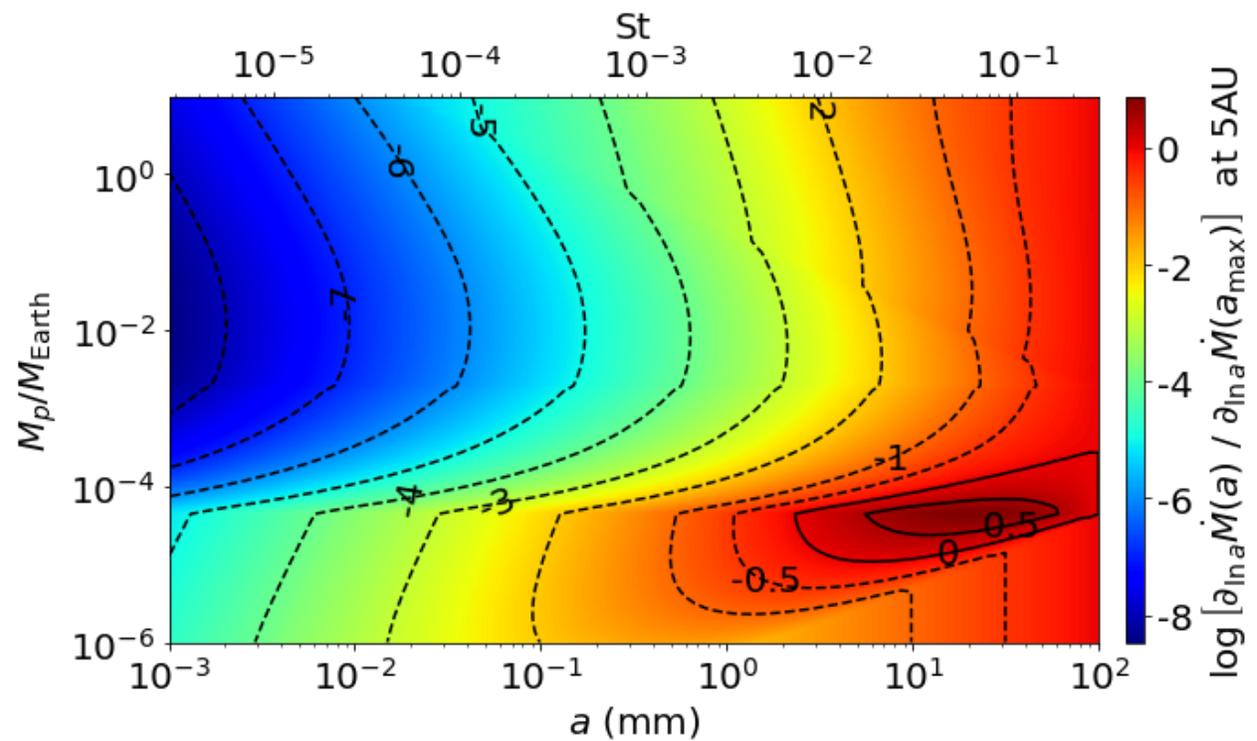
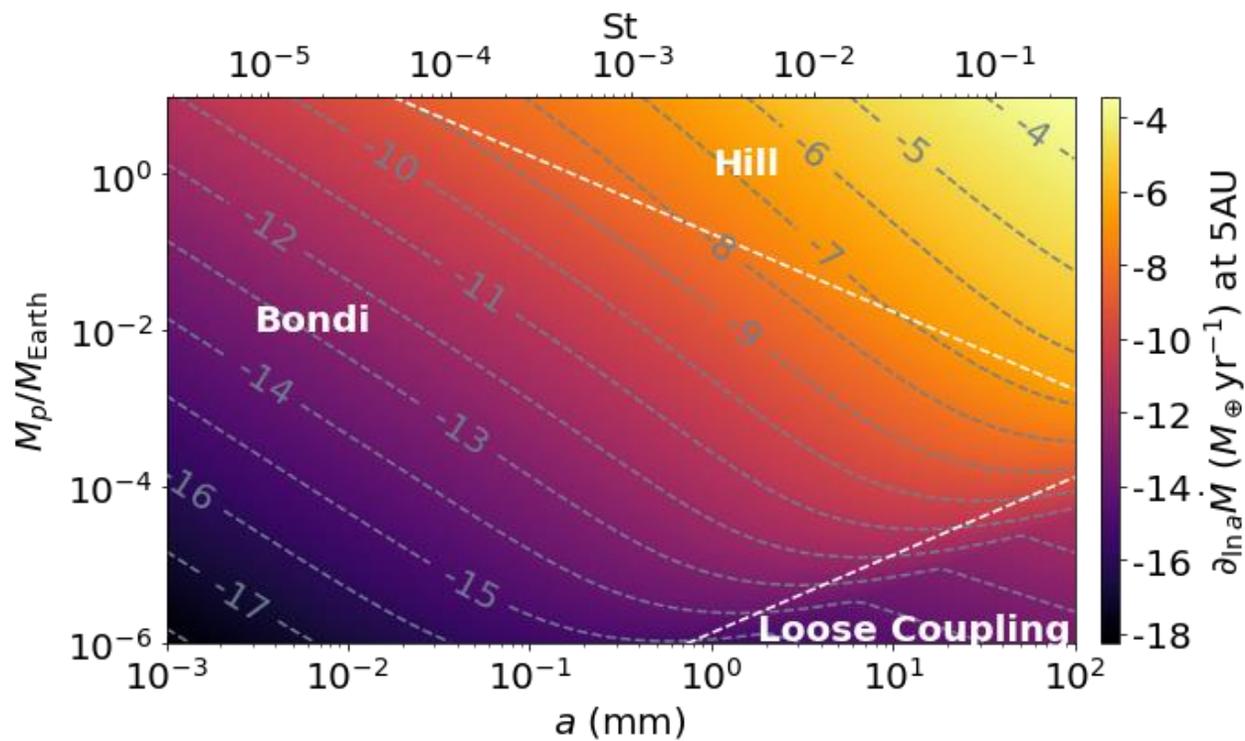
Lyra & Johansen + (2023)



Accretion Rates

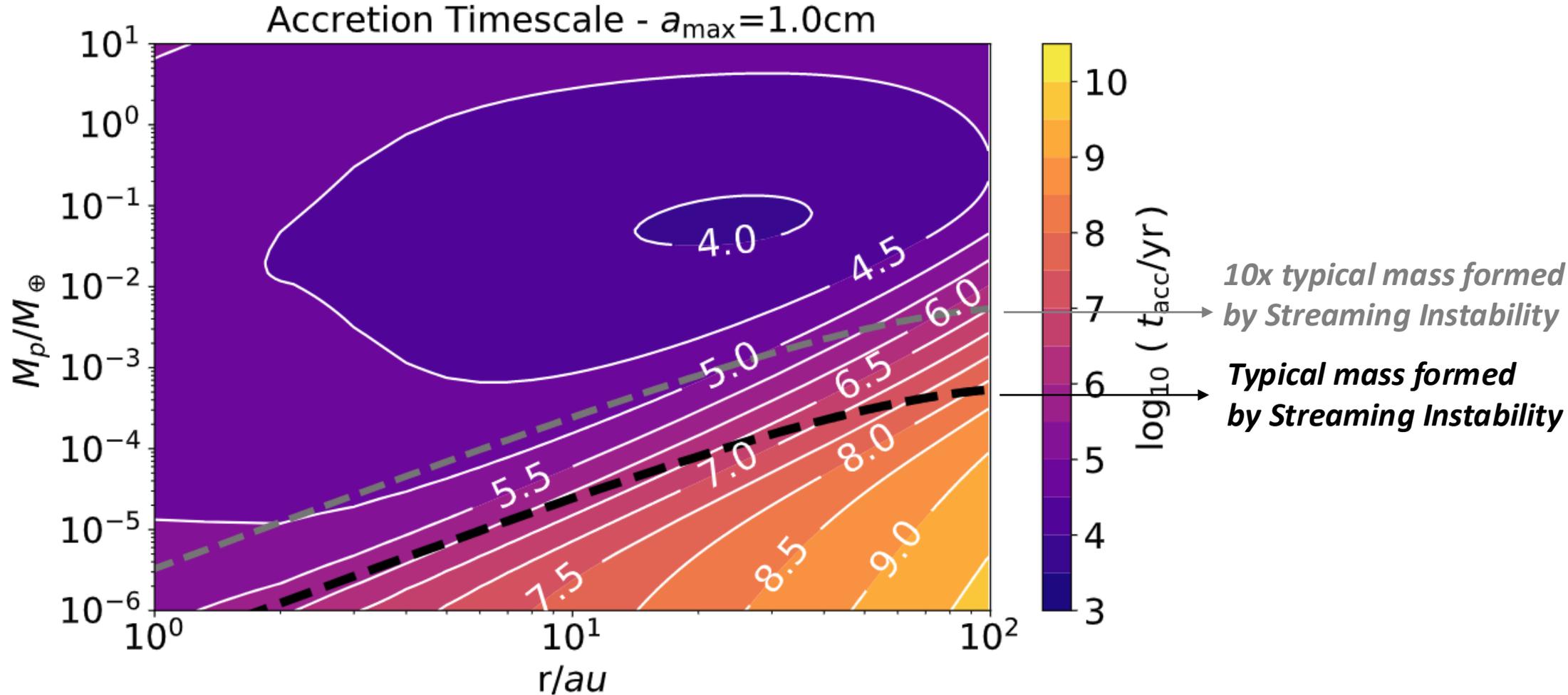


Differential Accretion Rates

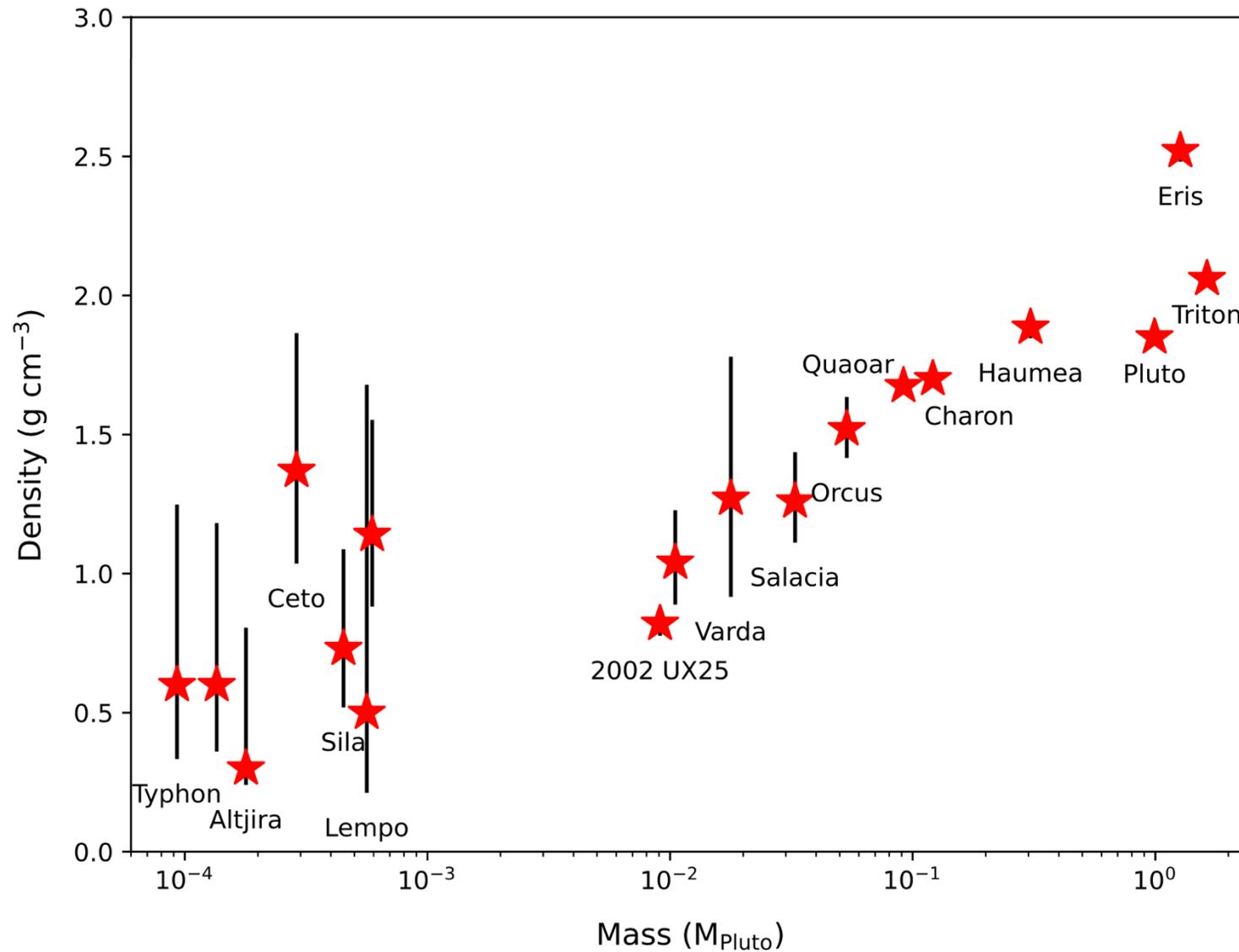


Accretion Timescales

Myr accretion timescales possible on top of planetesimals produced by Streaming Instability



The size-density relationship of Kuiper Belt objects

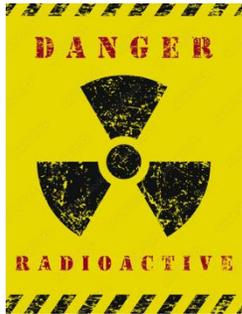


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Porosity removal by gravitational compaction

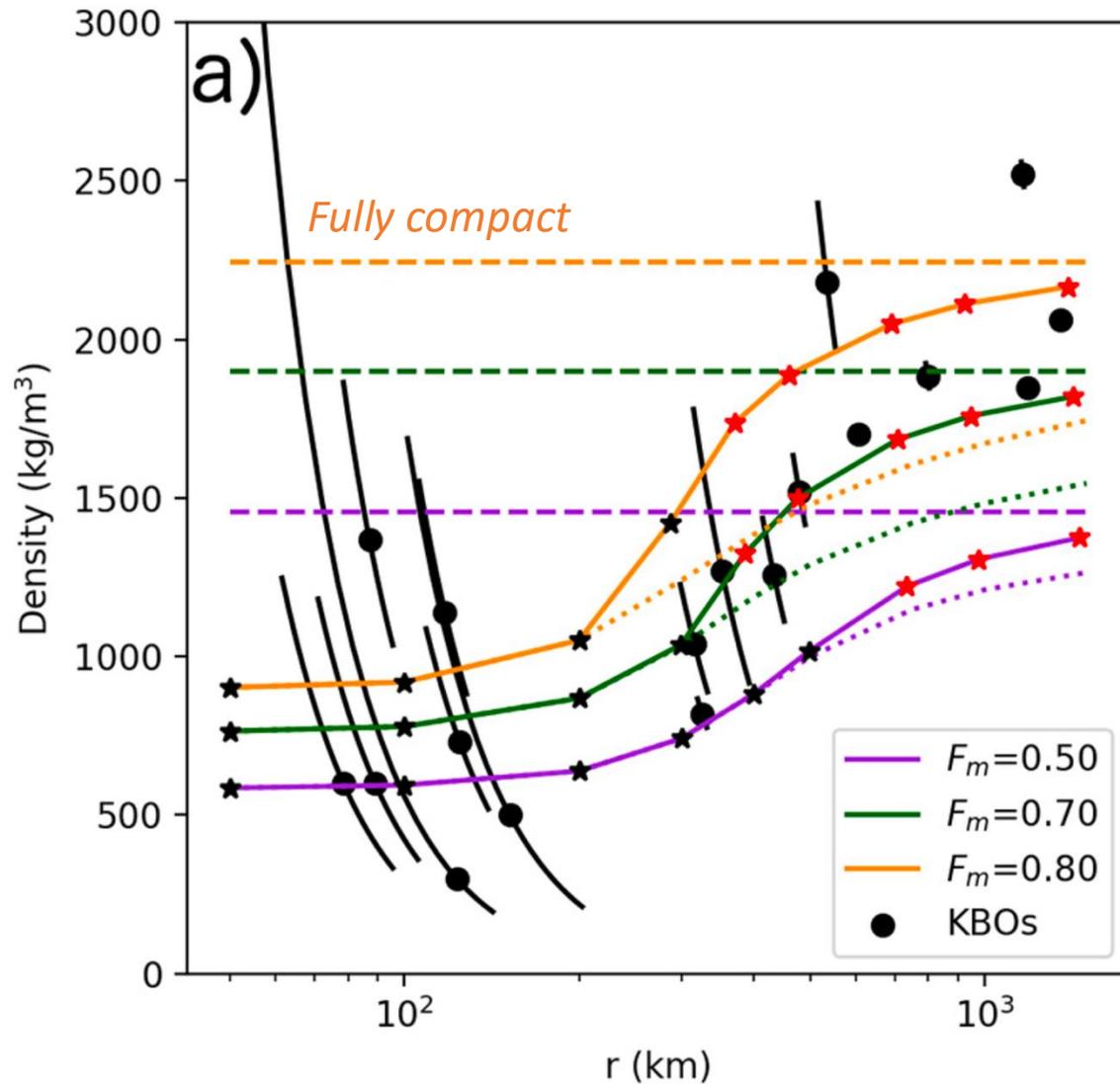
Problem

- Timing! ^{26}Al would melt if formed within 4 Myr



Assumption

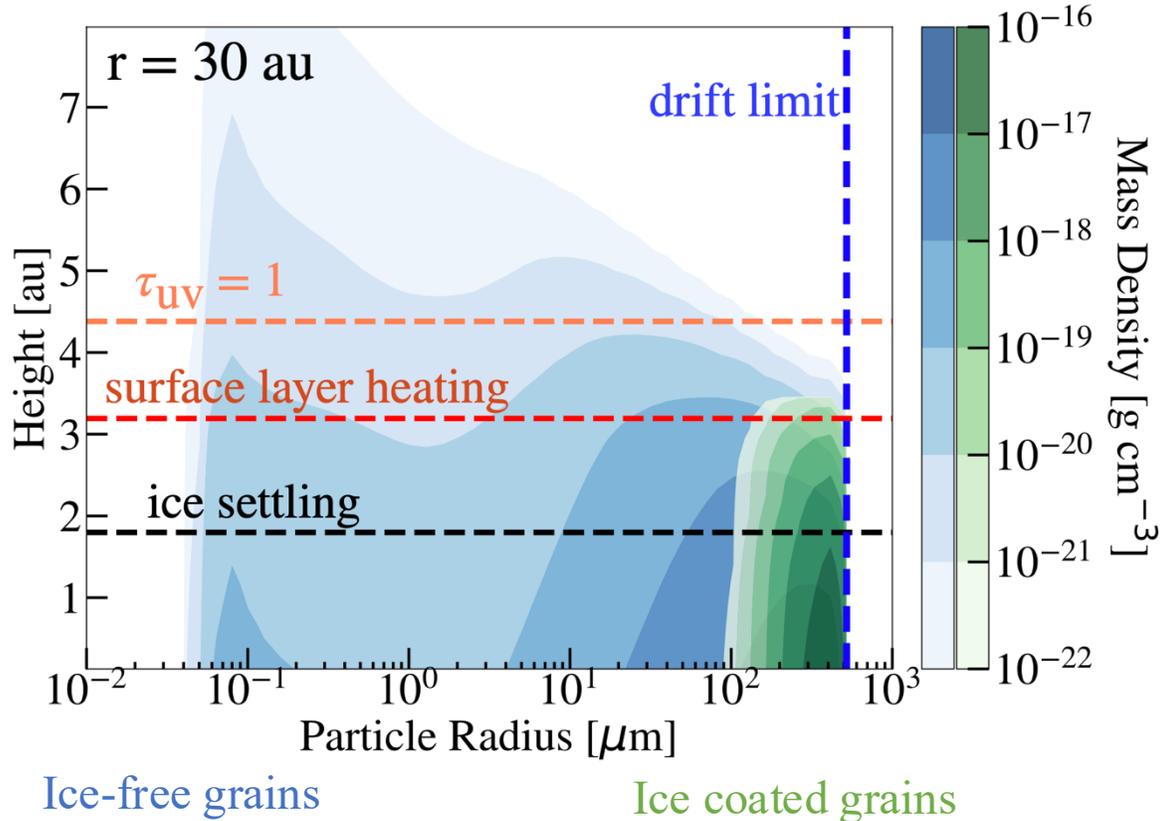
- Constant composition at birth and growth



Thermal and photo-desorption

Ice Removal

Heating and UV irradiation remove ice on Myr timescales (Harrison & Schoen 1967).

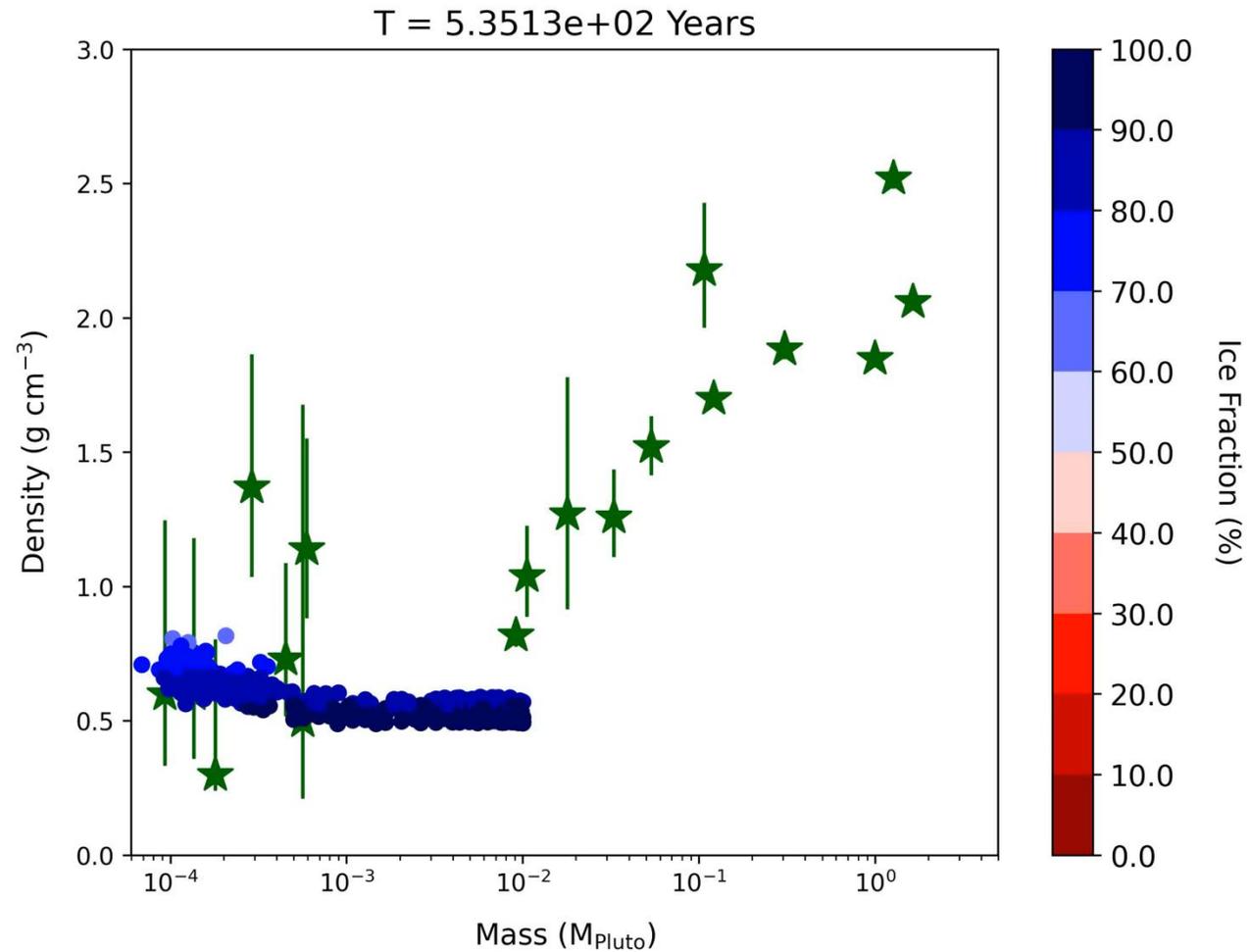
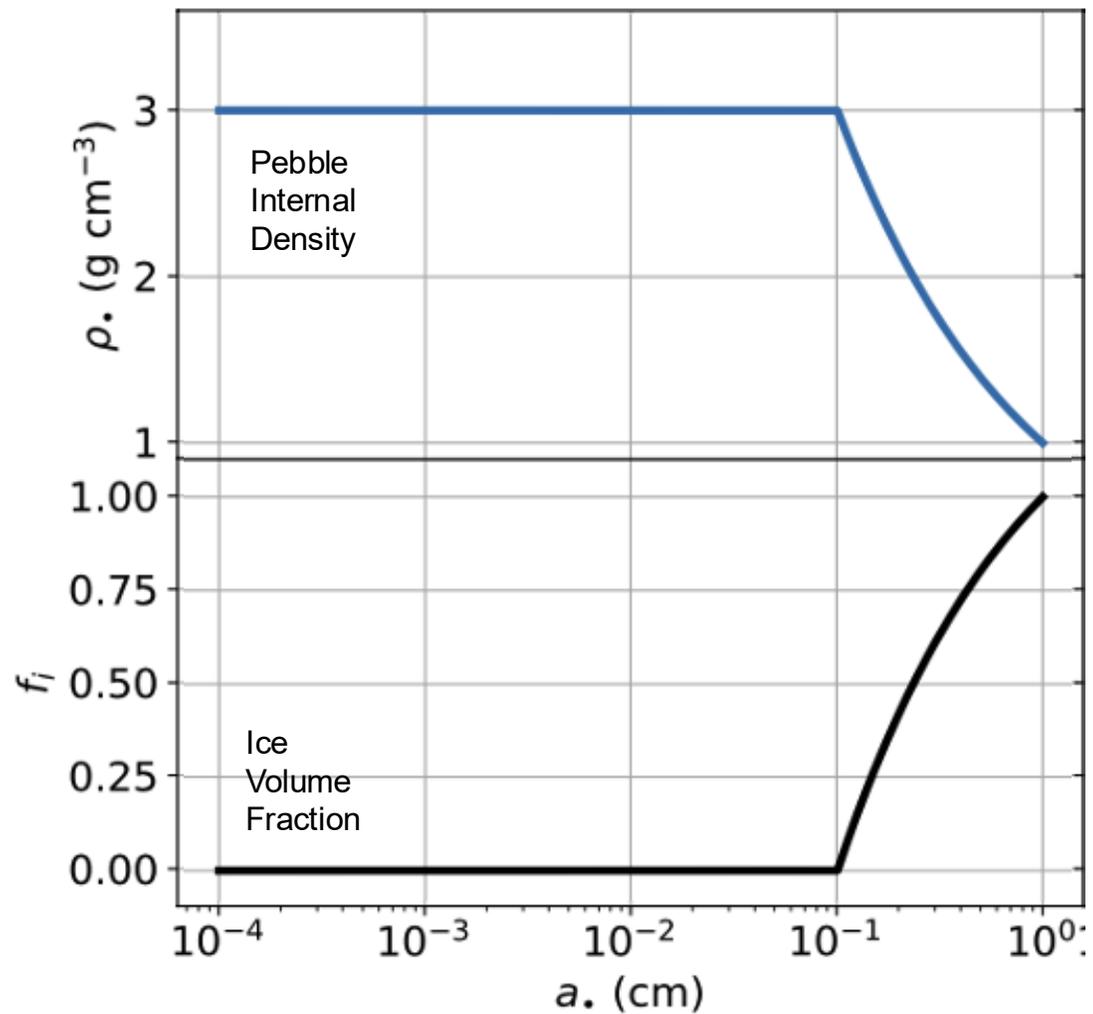


Proof-of-concept model

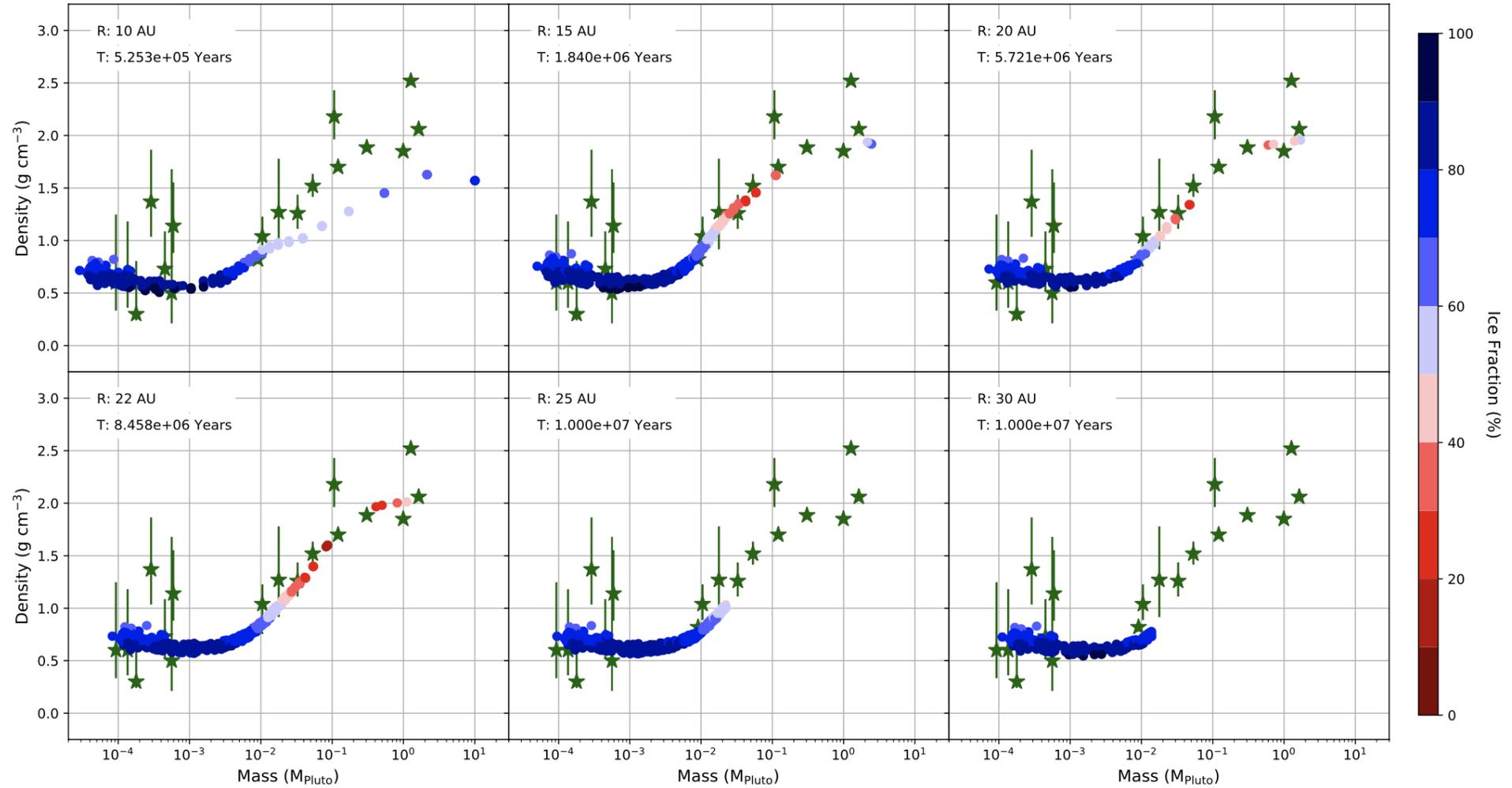
Small grains lofted in the atmosphere lose ice
Big grains are shielded and remain icy.



Growing Pluto by silicate pebble accretion



Distance Range 15 - 25AU



Conclusions

- Polydisperse Bondi accretion 1-2 orders of magnitude more efficient than monodisperse
 - Best accreted pebbles are those of drag time \sim Bondi time, not the largest ones
 - The largest ones dominate the mass budget, but accrete poorly
- Onset of Bondi accretion 1-2 orders of magnitude lower in mass compared to monodisperse
 - Bondi accretion possible on top of Streaming Instability planetary embryos within disk lifetime
 - Reaches 100-350km objects within Myr timescales
- Analytical solution to
 - Monodisperse general case
 - Polydisperse 2D Hill and 3D Bondi
- KBO density problem:
 - Two different pebble populations, maintained by ice desorption off small grains
 - Streaming instability: icy-rich small objects; nearly uniform composition
 - Polydisperse pebble accretion: silicate-rich larger objects; varied composition
 - Melting avoided by
 - ice-rich formation
 - ^{26}Al incorporated mostly in long ($>$ Myr) phase of silicate accretion
 - KBOs best reproduced between 15-25 AU

