

Consequences of multi-species streaming instability

*(or... The size-density relationship of Kuiper belt objects:
Evidence for streaming instability and pebble accretion)*

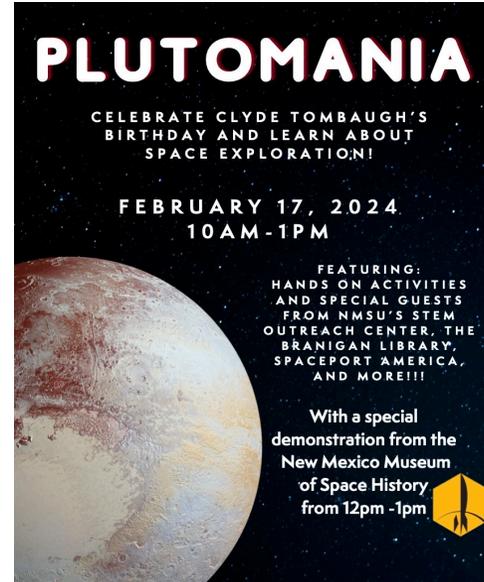
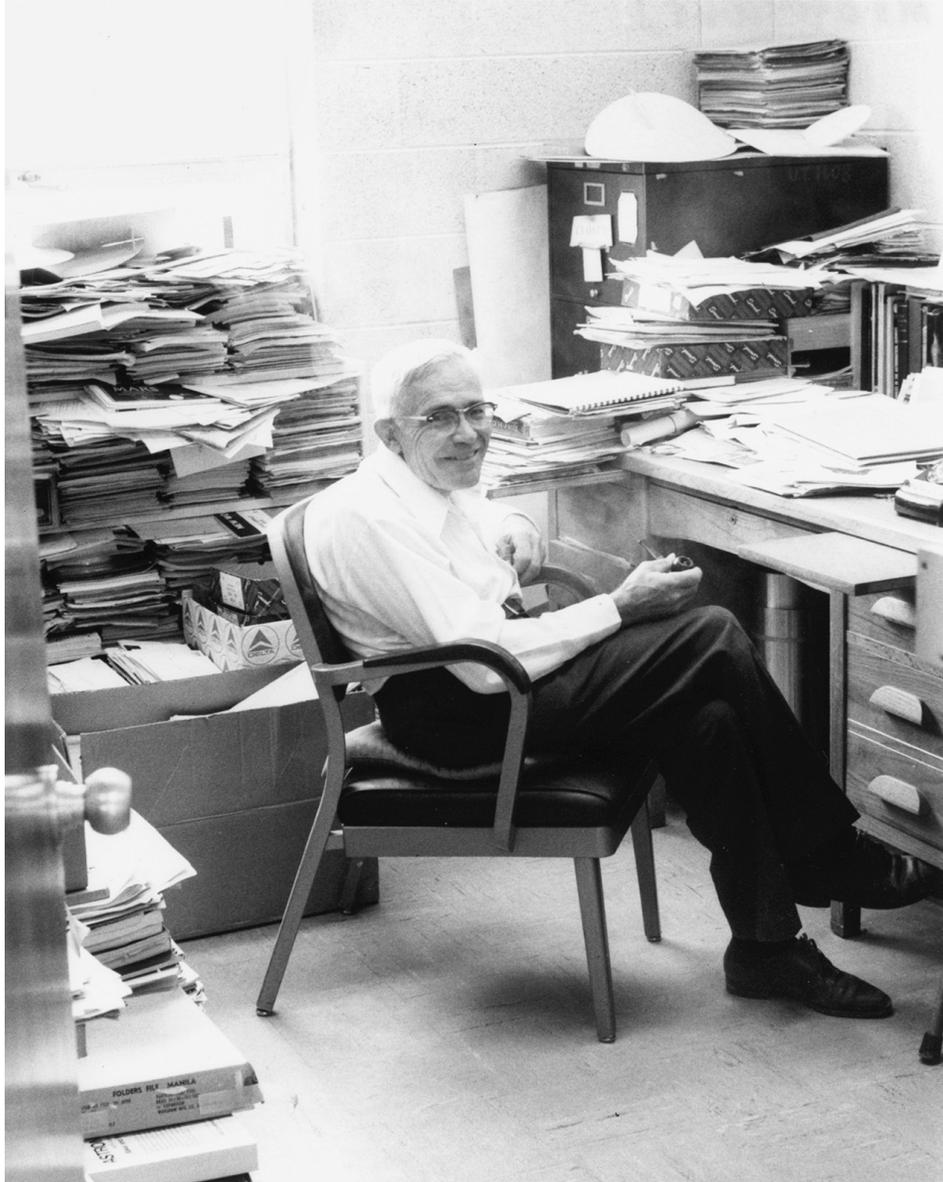


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Chao-Chin Yang (University of Alabama), **Andrew Youdin** (University of Arizona).

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NMSU professor awarded NASA grant, research may show how Pluto was formed

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This talk is based on these two papers

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A Solution for the Density Dichotomy Problem of Kuiper Belt Objects with Multispecies Streaming Instability and Pebble Accretion

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Abstract

Kuiper Belt objects (KBOs) show an unexpected trend, whereby large bodies have increasingly higher densities, up to five times greater than their smaller counterparts. Current explanations for this trend assume formation at constant composition, with the increasing density resulting from gravitational compaction. However, this scenario poses a timing problem to avoid early melting by decay of ²⁶Al. We aim to explain the density trend in the context of streaming instability and pebble accretion. Small pebbles experience lofting into the atmosphere of the disk, being exposed to UV and partially losing their ice via desorption. Conversely, larger pebbles are shielded and remain icier. We use a shearing box model including gas and solids, the latter split into ices and silicate pebbles. Self-gravity is included, allowing dense clumps to collapse into planetesimals. We find that the streaming instability leads to the formation of mostly icy planetesimals, albeit with an unexpected trend that the lighter ones are more silicate-rich than the heavier ones. We feed the resulting planetesimals into a pebble accretion integrator with a continuous size distribution, finding that they undergo drastic changes in composition as they preferentially accrete silicate pebbles. The density and masses of large KBOs are best reproduced if they form between 15 and 22 au. Our solution avoids the timing problem because the first planetesimals are primarily icy and ²⁶Al is mostly incorporated in the slow phase of silicate pebble accretion. Our results lend further credibility to the streaming instability and pebble accretion as formation and growth mechanisms.

Unified Astronomy Thesaurus concepts: Dwarf planets (419); Kuiper Belt (893); Pluto (1267); Hydrodynamics (1963); Planet formation (1241)



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An Analytical Theory for the Growth from Planetesimals to Planets by Polydisperse Pebble Accretion

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Abstract

Pebble accretion is recognized as a significant accelerator of planet formation. Yet only formulae for single-sized (monodisperse) distribution have been derived in the literature. These can lead to significant underestimates for Bondi accretion, for which the best accreted pebble size may not be the one that dominates the mass distribution. We derive in this paper the polydisperse theory of pebble accretion. We consider a power-law distribution in pebble radius, and we find the resulting surface and volume number density distribution functions. We derive also the exact monodisperse analytical pebble accretion rate for which 3D accretion and 2D accretion are limits. In addition, we find analytical solutions to the polydisperse 2D Hill and 3D Bondi limits. We integrate the polydisperse pebble accretion numerically for the MRN distribution, finding a slight decrease (by an exact factor 3/7) in the Hill regime compared to the monodisperse case. In contrast, in the Bondi regime, we find accretion rates 1–2 orders of magnitude higher compared to monodisperse, also extending the onset of pebble accretion to 1–2 orders of magnitude lower in mass. We find megayear timescales, within the disk lifetime, for Bondi accretion on top of planetary seeds of masses 10^{-6} to $10^{-4} M_{\oplus}$, over a significant range of the parameter space. This mass range overlaps with the high-mass end of the planetesimal initial mass function, and thus pebble accretion is possible directly following formation by streaming instability. This alleviates the need for mutual planetesimal collisions as a major contribution to planetary growth.

Unified Astronomy Thesaurus concepts: Planet formation (1241); Planetary system formation (1257)



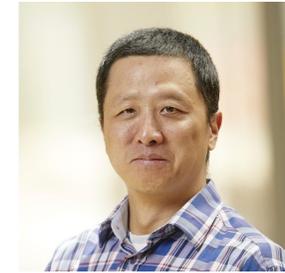
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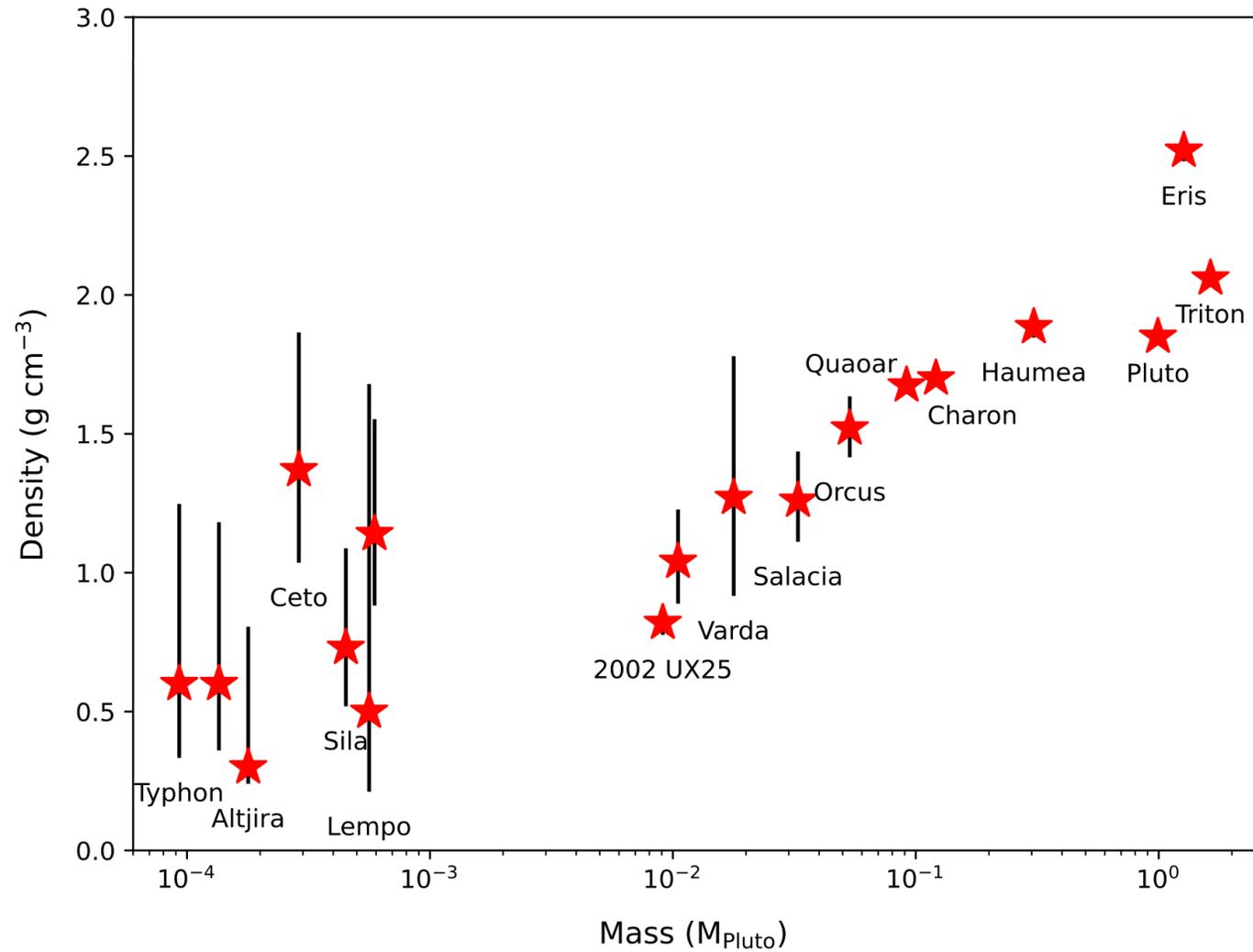


Chao-Chin Yang
U Alabama

Led by
Manny Cañas
(grad student).

The size-density relationship of Kuiper Belt objects

- Problem:
 - No high-density low-mass objects
 - No low-density high-mass objects
 - *How to form the high-mass objects from the low-mass ones via usual planetesimal accretion? Their compositions seem to be different (ice-rich for small objects, rock-rich for large objects)*
- Solutions:
 - Extremely low porosity?
 - *Even with the additional compression, would still have a density close to 1 g/cm³, not the 2.5 g/cm³ of Eris.*
 - Biased sample?
 - *Are we observationally missing populations of high-density low-mass and low-density high-mass objects?*
 - Compaction through giant impacts?
 - *Lots of giant impacts needed...*
- All alternatives unlikely. Different formation mechanism?
- *(Side question: is the gap between 10⁻³ and 10⁻² Pluto masses real?)*



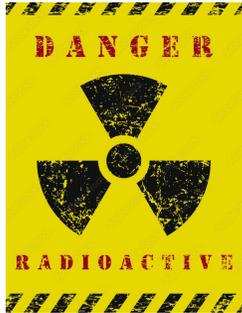
Cañas+Lyra et al. (2024)

Data; Thomas (2000), Stansberry et al. (2006), Grundy et al. (2007), Brown et al. (2011), Stansberry et al. (2012), Brown (2013), Fornasier et al. (2013), Vilenius, et al. (2014), Nimmo et al. (2016), Ortiz et al. (2017), Brown and Butler (2017), Grundy et al. (2019), Morgado et al. (2023), Pereira et al. (2023).

Current best bet: Porosity removal by gravitational compaction

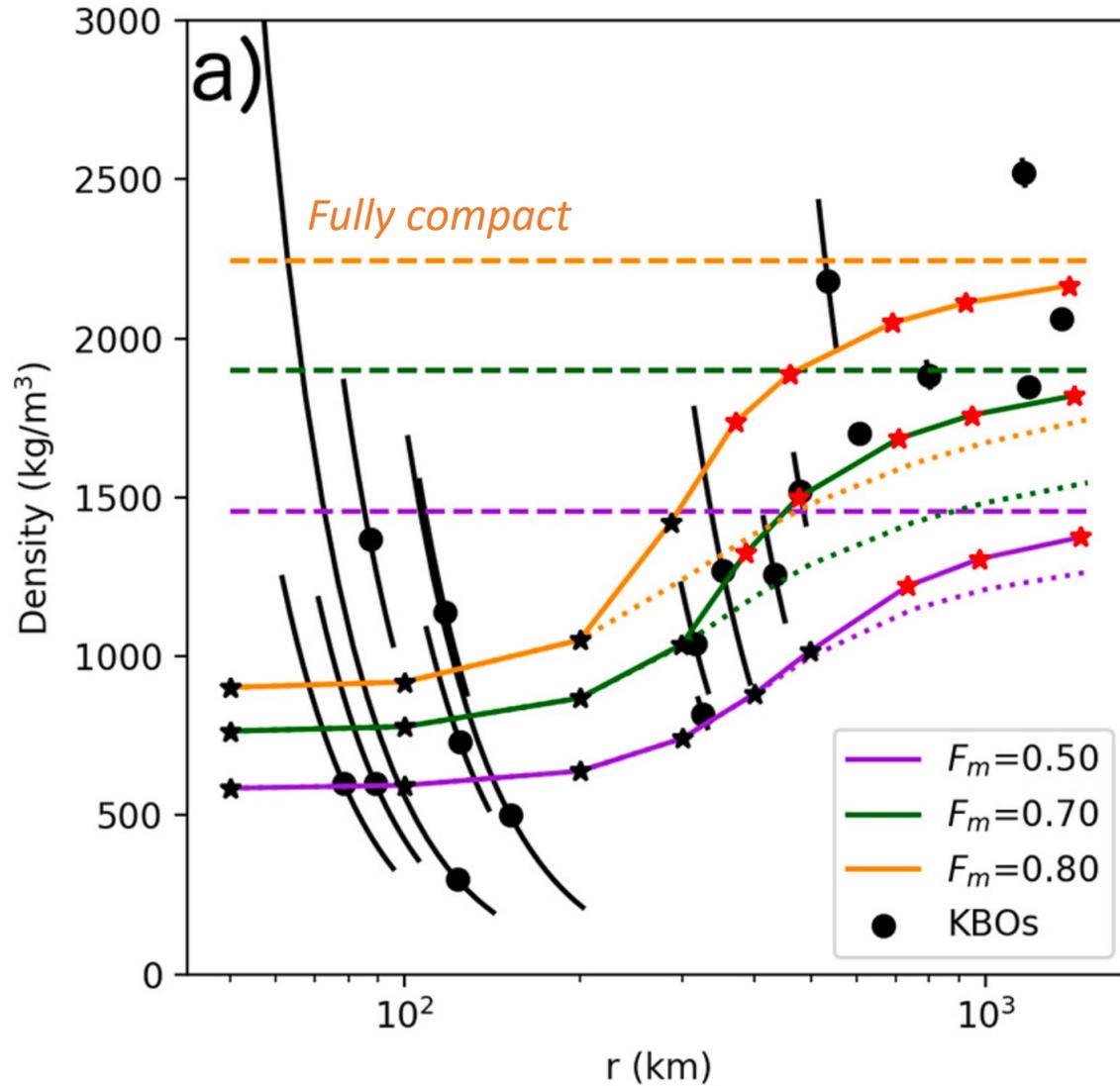
Problem

- Timing! ^{26}Al would melt if formed within 4 Myr



Assumptions

- ~~• Constant composition at birth and growth~~
- ~~• Growth by planetesimal accretion~~

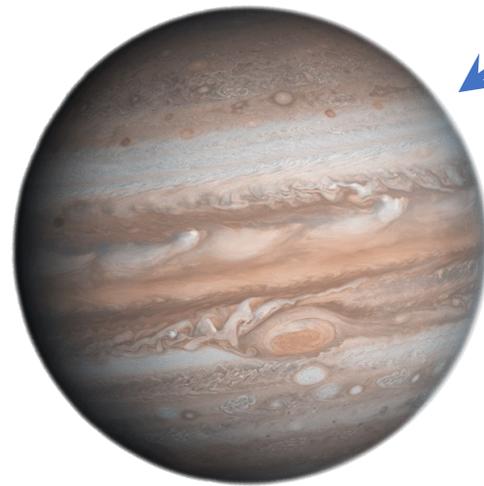
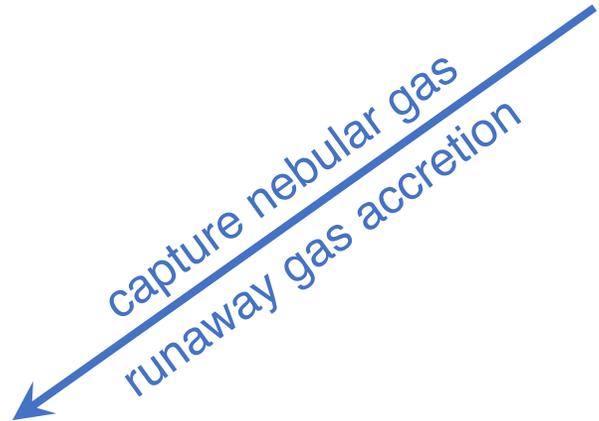
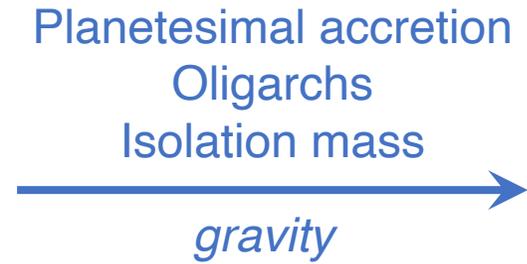
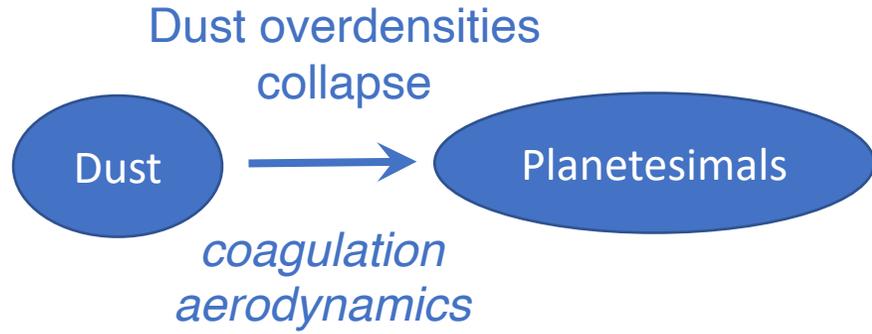


F_m = rock mass fraction

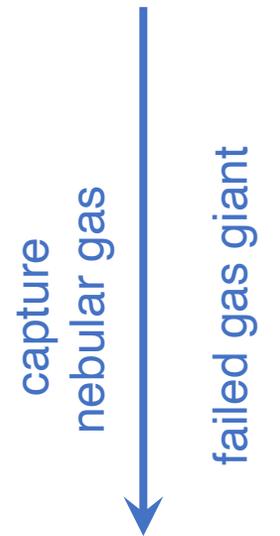
Core Accretion (... 15 years ago)

Rocky planets

**Core
Accretion**



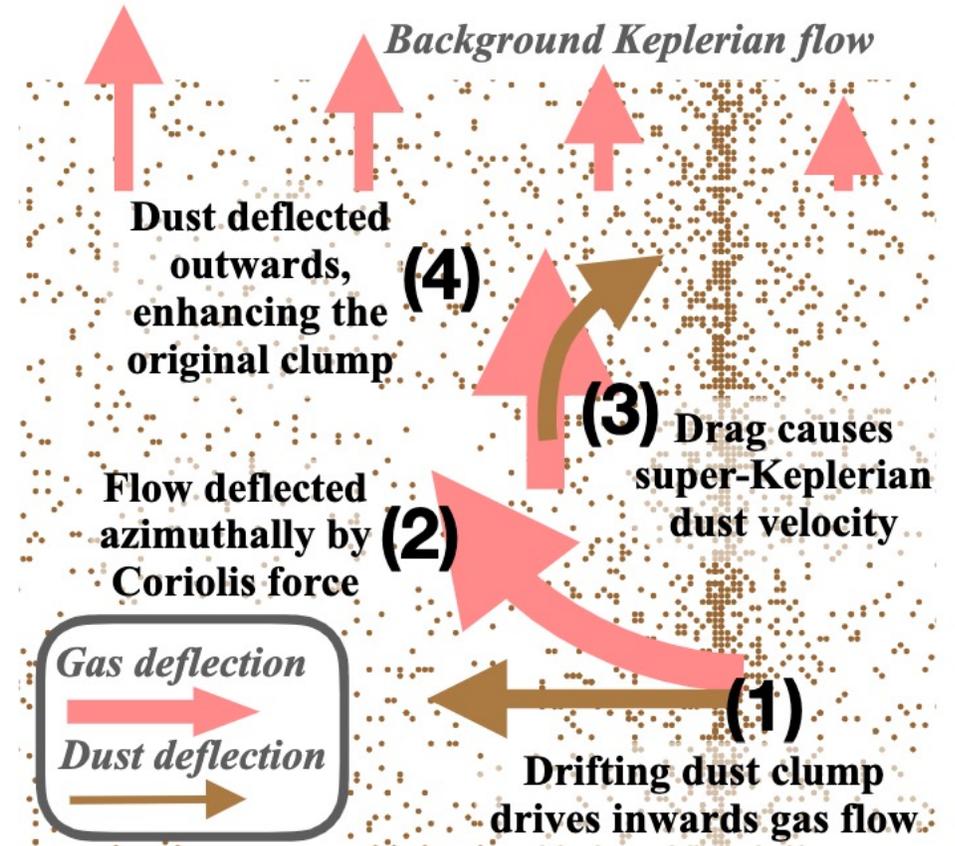
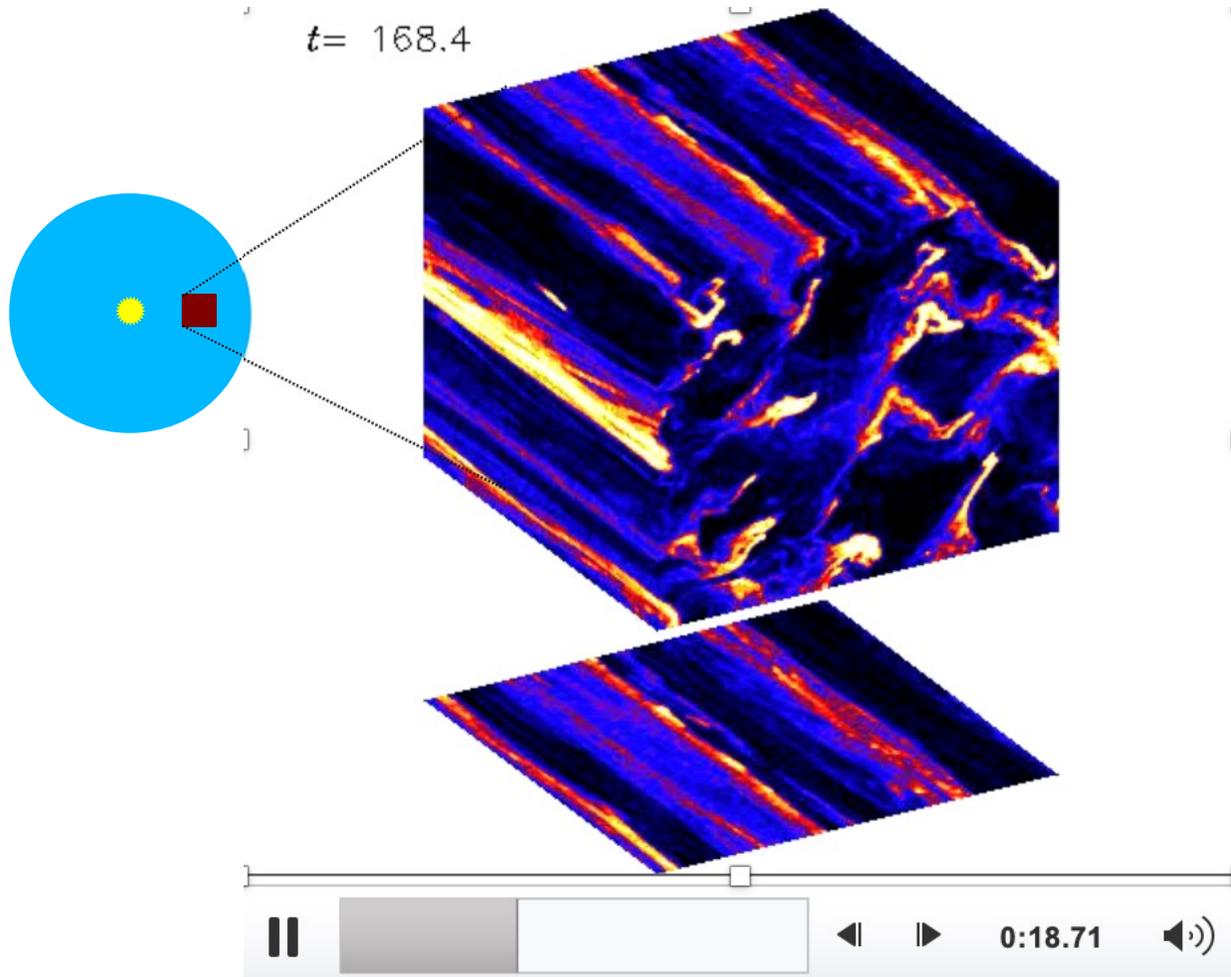
Gas giants



Ice giants

Streaming Instability

The dust drift is hydrodynamically unstable

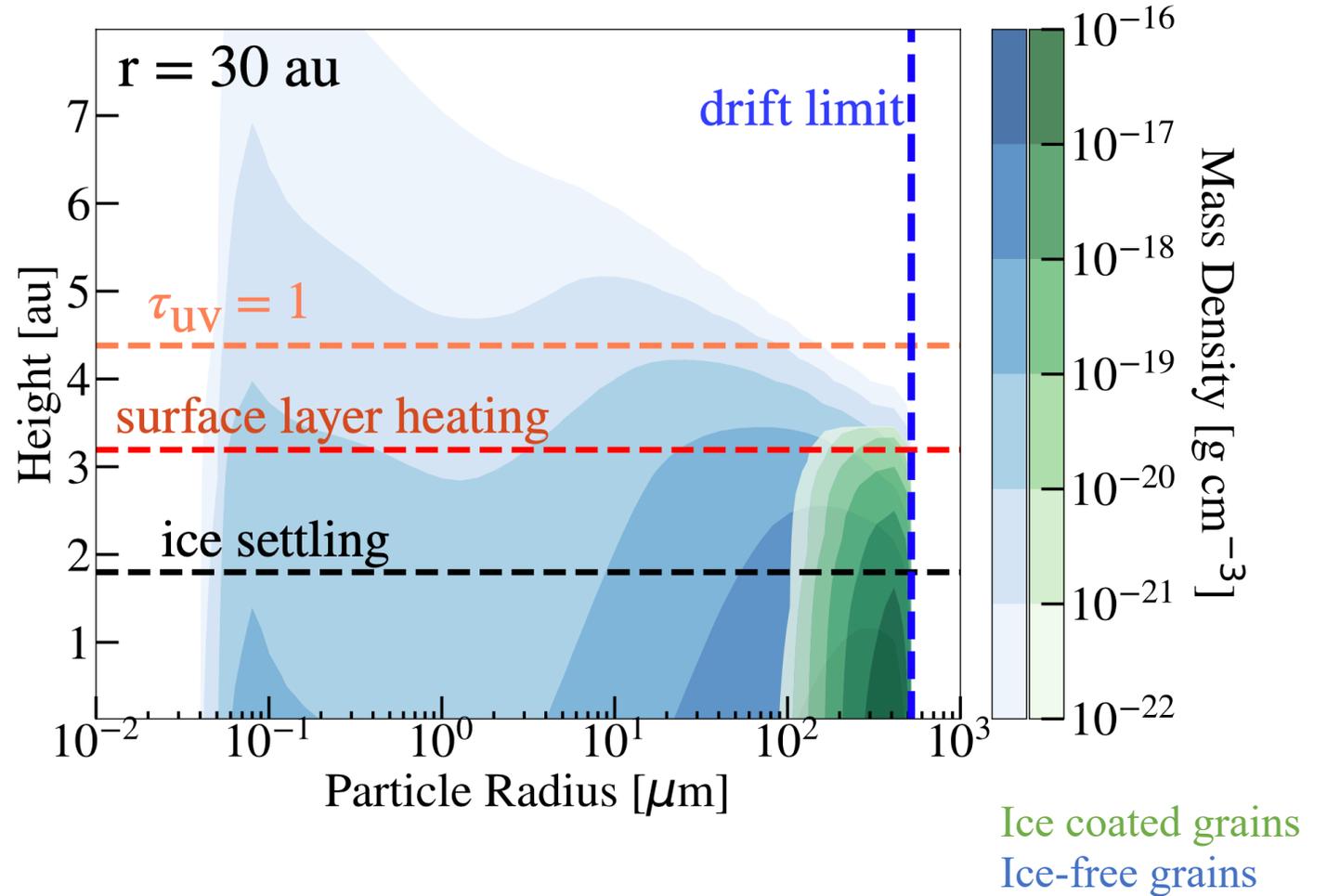
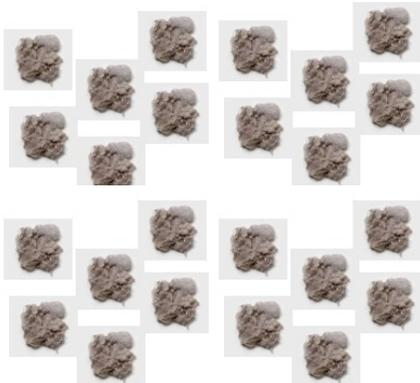


Lesur et al. (2022)

Abandoning Constant Composition

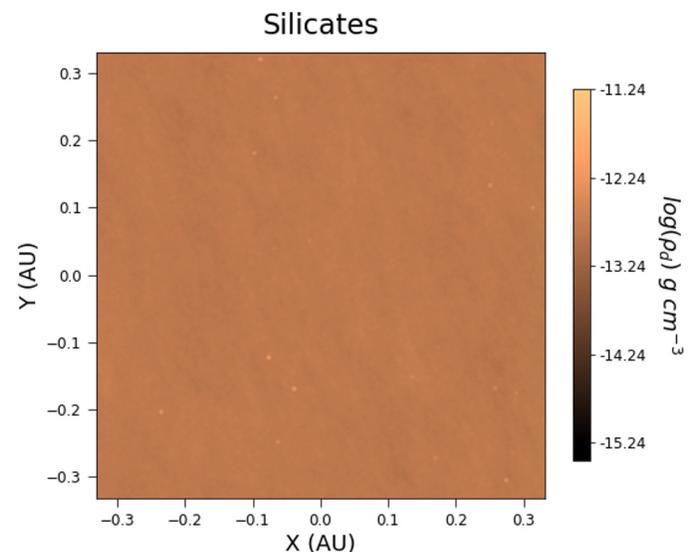
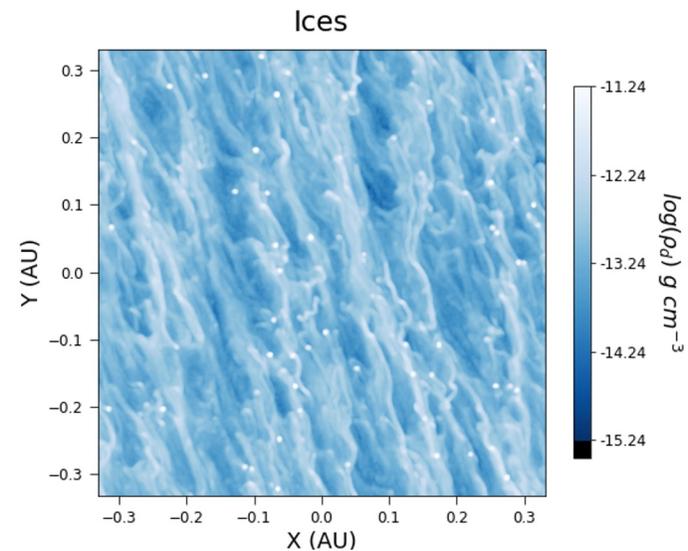
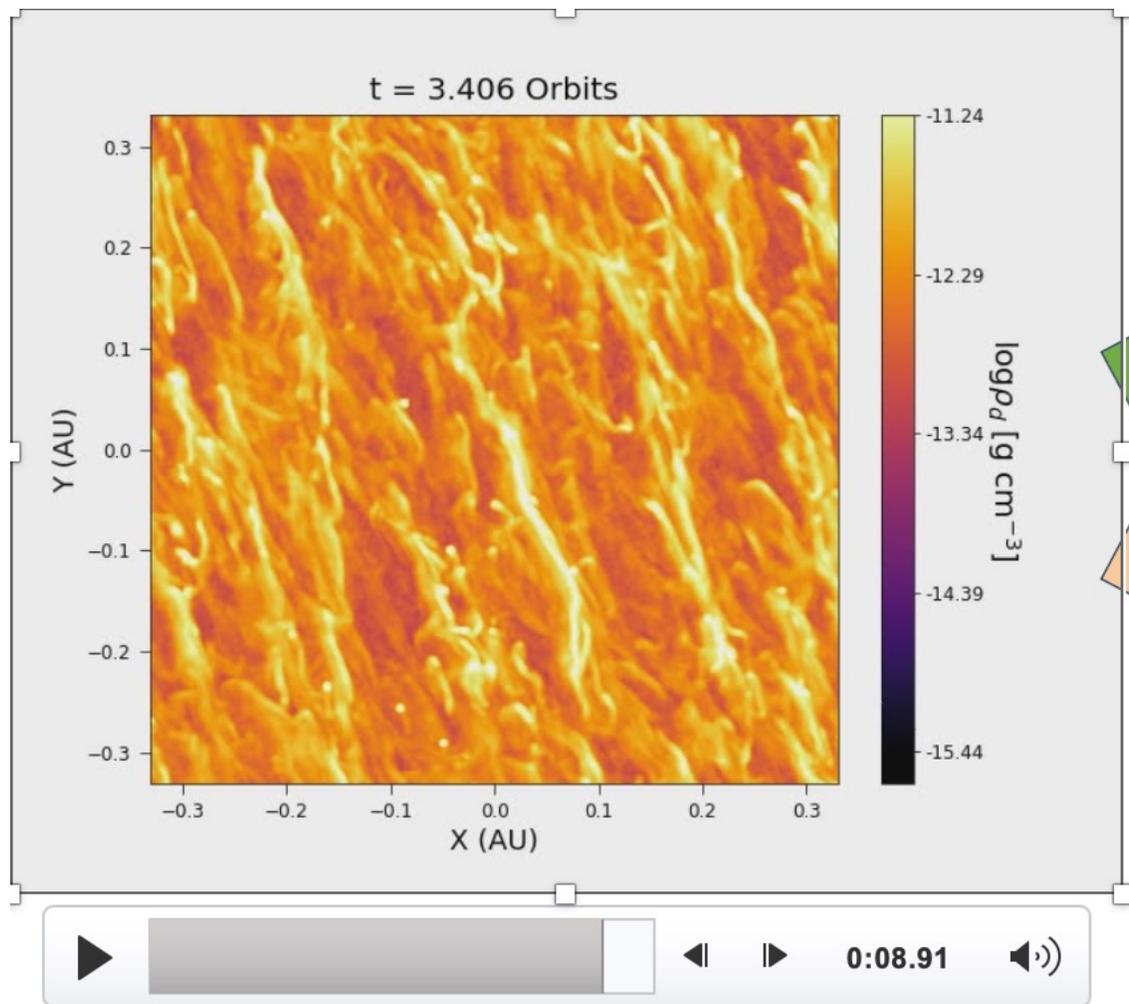
Heating and UV irradiation remove ice on Myr timescales (Harrison & Schoen 1967)

- Small grains lofted in the atmosphere lose ice
- Big grains are shielded and remain icy.
- *A bimodal population of pebbles is established*

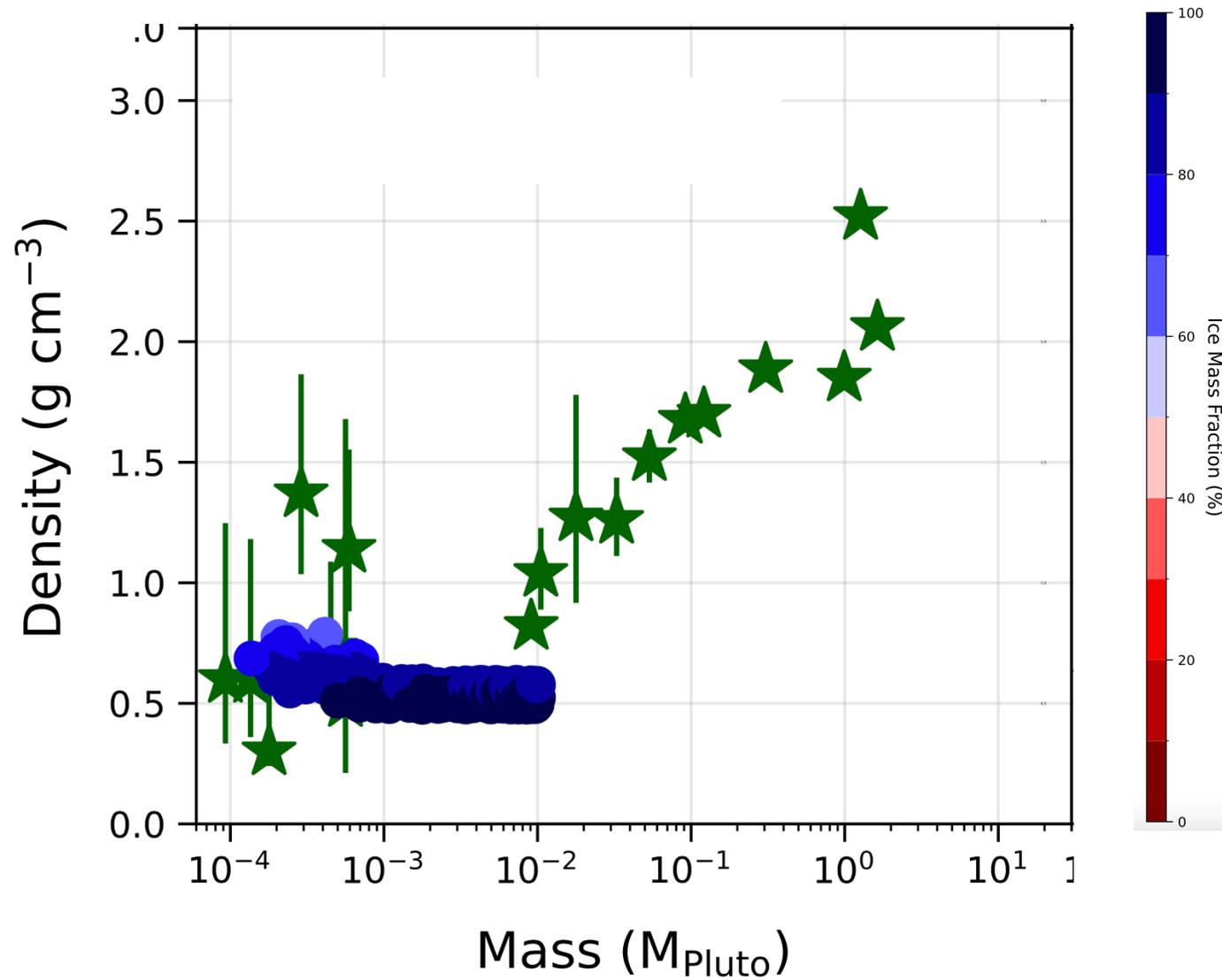


Split into icy and silicate pebbles

Streaming instability operates better on the large (icy) pebbles. The first planetesimals will thus be icy. That explains their low-density and avoids incorporation of ^{26}Al .



The first planetesimals are icy



The first planetesimals won't melt

Volumetric heating rate:

$$\mathcal{H} = \rho F_s [^{26}\text{Al}]_0 \mathcal{H}_0 e^{-\lambda t}$$

Integrate to find total heat:

$$Q(t) = V \int_0^t \mathcal{H}(t') dt'$$

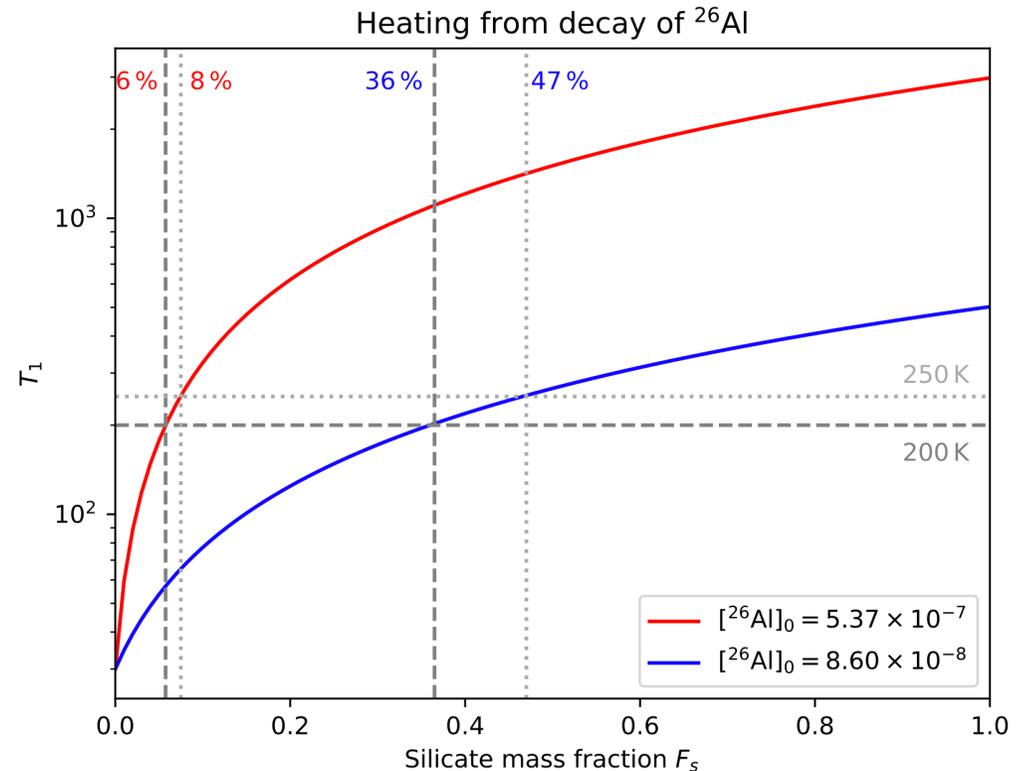
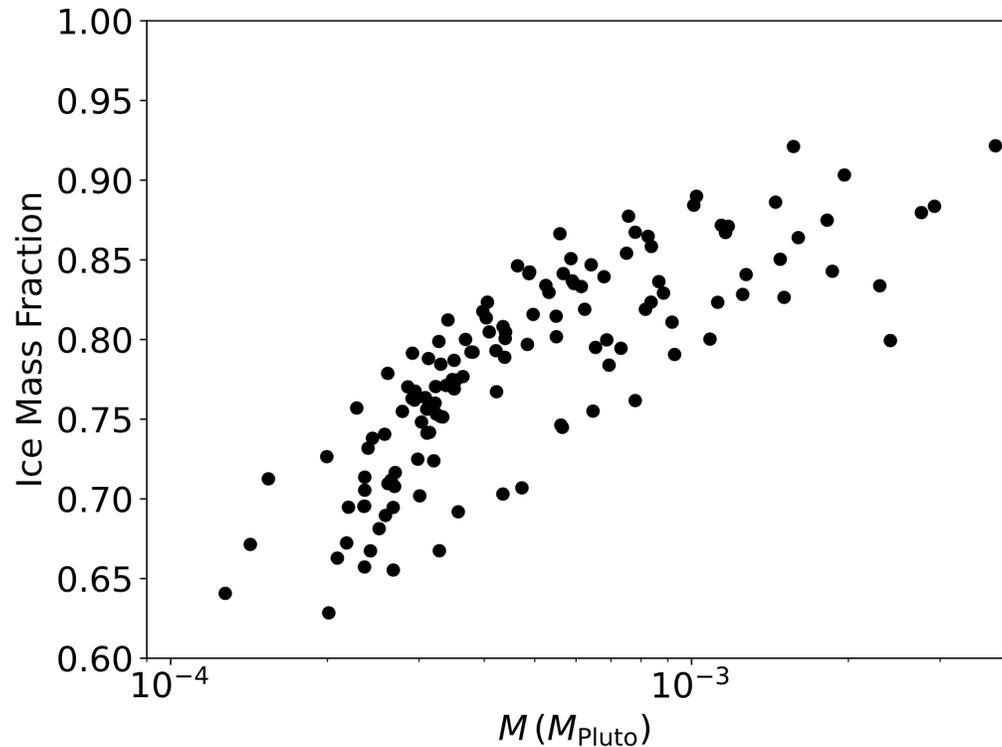
$$= M_p F_s [^{26}\text{Al}]_0 \mathcal{H}_0 \lambda^{-1} (1 - e^{-\lambda t})$$

Assume all the heat is used to raise the temperature of the body:

$$Q = M_p c_p \Delta T$$

$$\Delta T = F_s [^{26}\text{Al}]_0 \mathcal{H}_0 \lambda^{-1} c_p^{-1}$$

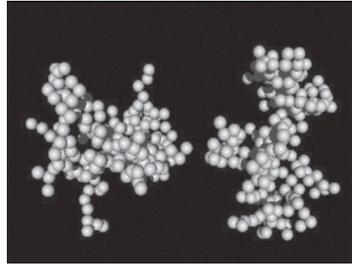
A high abundance of ^{26}Al would not allow for porosity retention, but a lower abundance accomodates the $\geq 65\%$ ice mass fraction seen for the products of streaming instability.



Core Accretion

**Core
Accretion**

Dust grains



0.1 – 1 μm

coagulation



Pebbles

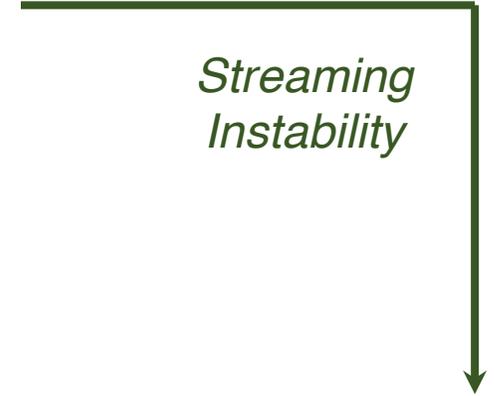


mm – cm

*Pebble
Accretion*



*Streaming
Instability*



Planetesimals

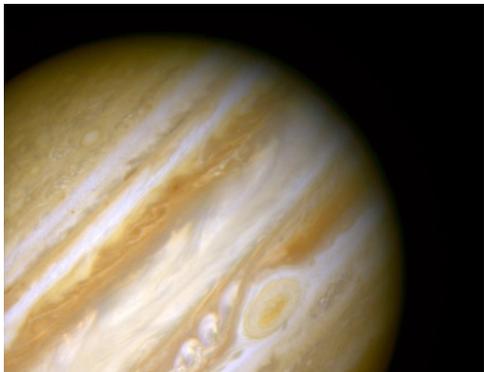


1-100km

Protoplanets
Rocky Planets
Planetary Cores

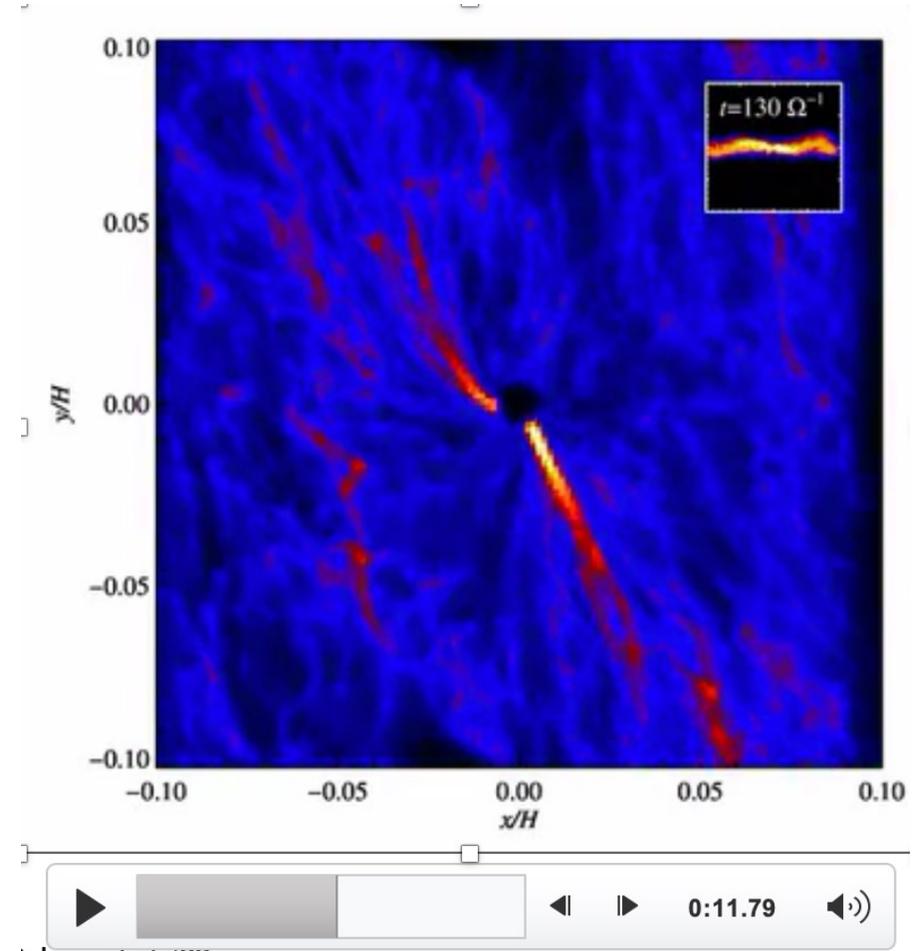
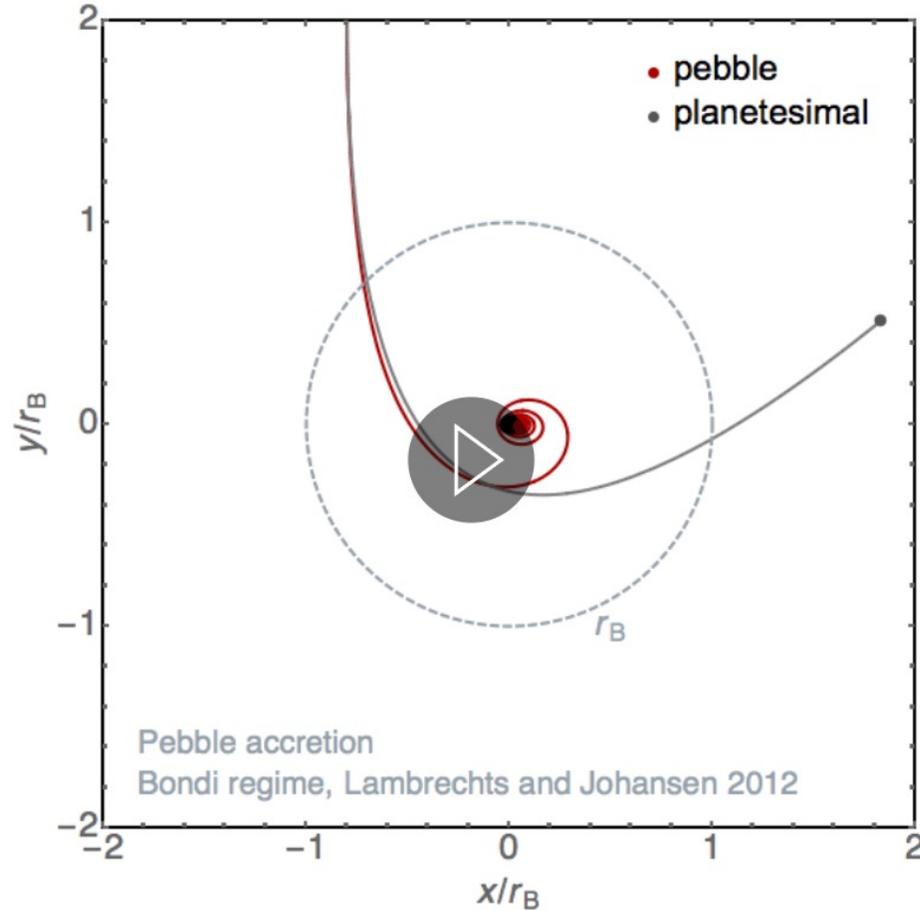


Gas Accretion



The capture radius for planetesimals by gravitational focusing is very small as the motion is conservative. Pebbles, however, are subject to gas drag, so the motion is dissipative, making capture much easier.

Pebble Accretion



Ormel & Klahr '10, Lambrechts & Johansen '12, Lyra et al. '23

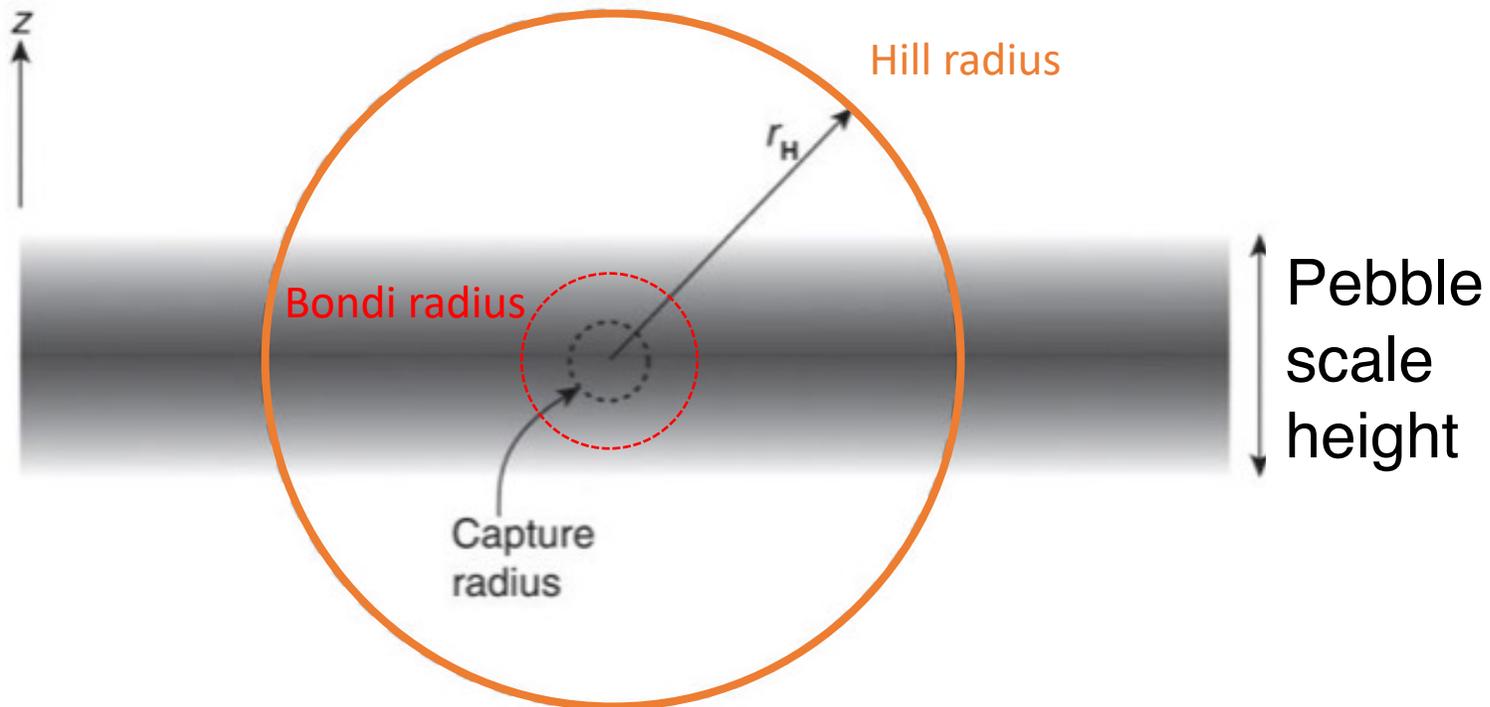
(see also Lyra et al. 2008b, Lyra et al. 2009a and Lyra et al. 2009b for stumbling upon pebble accretion and not recognizing it! *D'oh!*)

See Johansen & Lambrechts '17 for a review

Pebble Accretion: Geometric, Bondi, and Hill regime

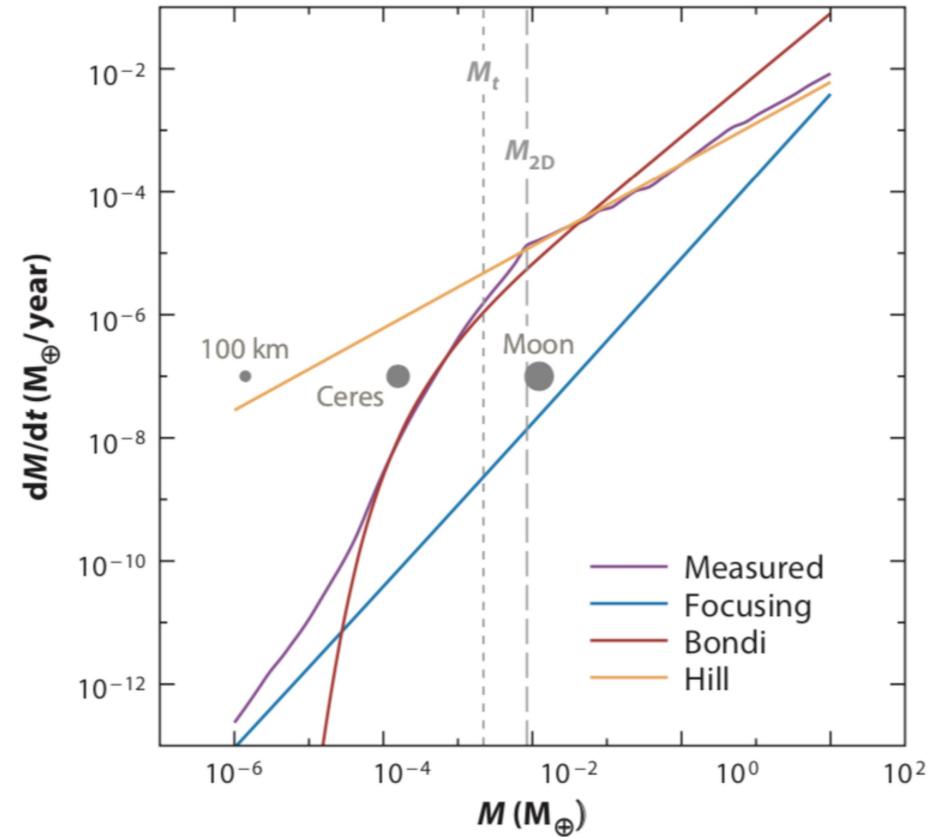
Bondi accretion - Bound against **headwind**

Hill accretion - Bound against **stellar tide**



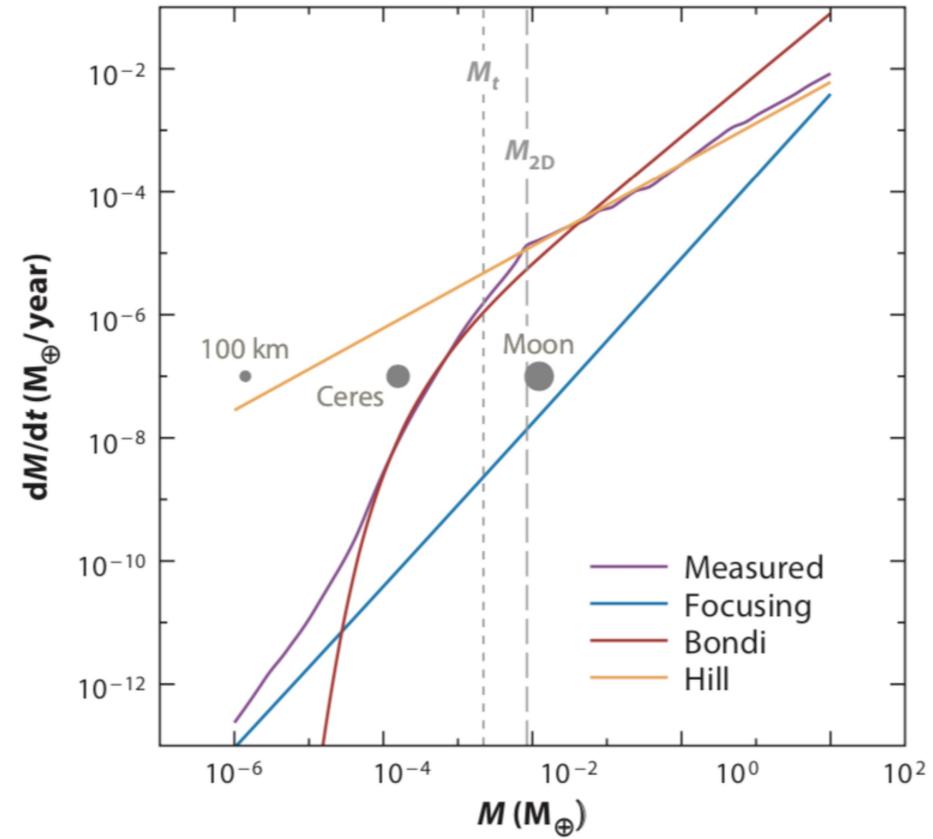
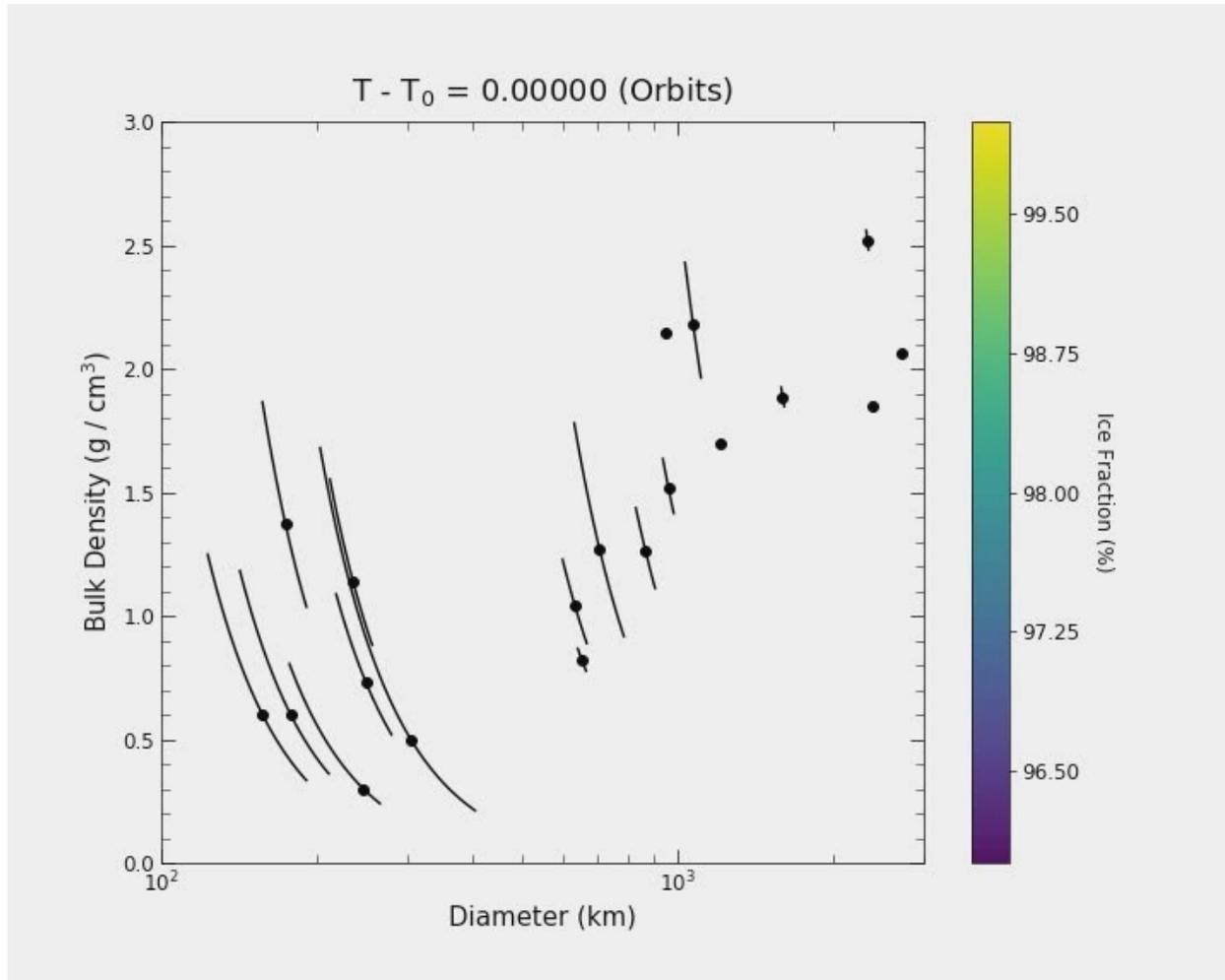
$$\xi \equiv \left(\frac{R_{\text{acc}}}{2H_d} \right)^2 \quad \begin{aligned} \dot{M}_{3D} &= \lim_{\xi \rightarrow 0} \dot{M} = \pi R_{\text{acc}}^2 \rho_{d0} \delta v, \\ \dot{M}_{2D} &= \lim_{\xi \rightarrow \infty} \dot{M} = 2R_{\text{acc}} \Sigma_d \delta v, \end{aligned}$$

Mass accretion rates



Starting pebble accretion, nothing happens.
 The seeds produced by streaming instability are still much too small, at the regime of gravitational focusing (Bondi radius smaller than gravitational capture radius). So what gives?

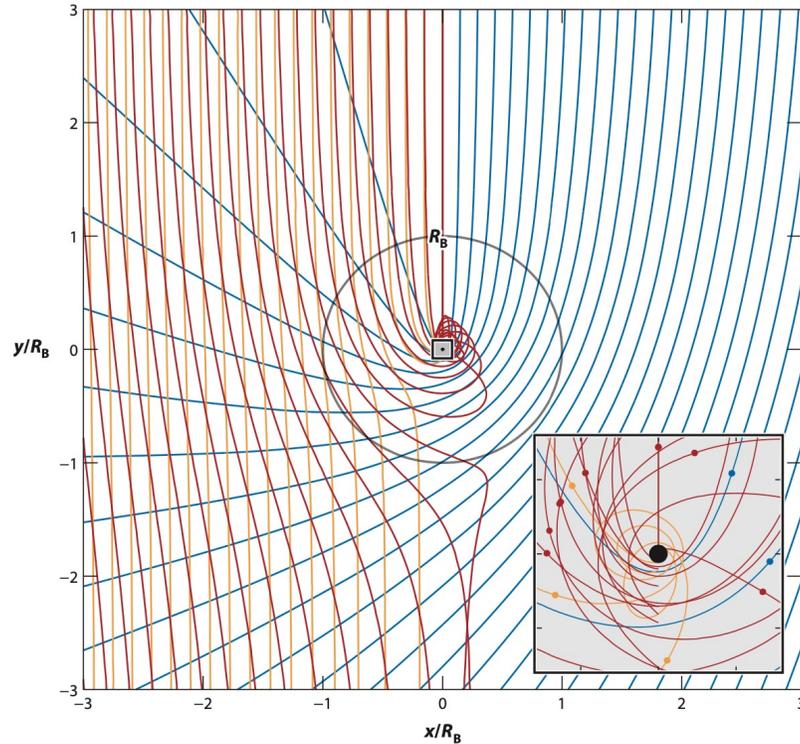
Integrate pebble accretion



1. The problem is that the mass accretion rates for pebble accretion considered only one pebble size: the biggest pebbles in the disk. This is reasonable, as the pebble distribution is top-heavy: most of the mass is in the large pebbles.

Pebble Accretion: Pebbles of different size accrete differently

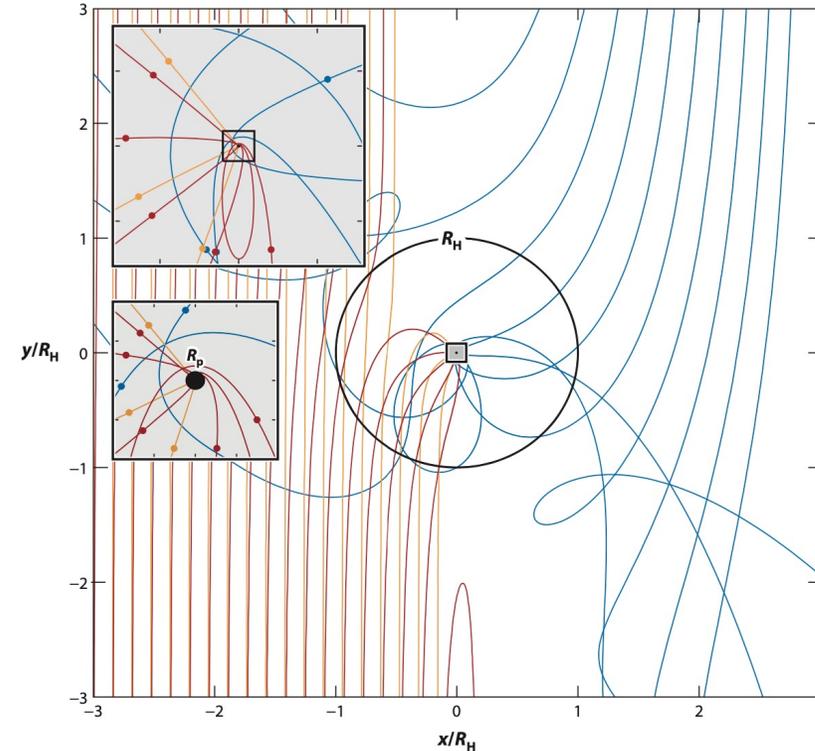
Bondi Regime



Best accreted pebble

Drag time ~ Bondi Time

Hill Regime



Best accreted pebble

Drag time ~ Orbital Time

2. That is ok for Hill accretion, as the pebbles that best accrete are the largest ones.

3. However, for Bondi accretion, the best accreting pebbles are those whose drag time is the time it takes to cross the Bondi sphere. These can be quite small. These small pebbles don't dominate the mass, but they may dominate the mass accretion rate.

In this case, the mass accretion rates in the Bondi regime will be severely underestimated if we don't consider the smaller pebbles.

Polydisperse (Multi-Species) Pebble Accretion

$$\rho_d(a, z) = \int_0^a m(a') F(a', z) da'.$$

$$F(a, z) \equiv f(a) e^{-z^2/2H_d^2},$$

$$f(a) = \frac{3(1-p)Z\Sigma_g}{2^{5/2}\pi^{3/2}H_g\rho_\bullet^{(0)}a_{\max}^{4-k}} \sqrt{1 + a \frac{\pi \rho_\bullet(a)}{2 \Sigma_g \alpha}} a^{-k}.$$

$$S \equiv \frac{1}{\pi R_{\text{acc}}^2} \int_{-R_{\text{acc}}}^{R_{\text{acc}}} 2\sqrt{R_{\text{acc}}^2 - z^2} \exp\left(-\frac{z^2}{2H_d^2}\right) dz,$$

$$W(a) = \frac{3(1-p)Z\Sigma_g}{4\pi\rho_\bullet^{(0)}a_{\max}^{4-k}} a^{-k},$$

$$\delta v \equiv \Delta v + \Omega R_{\text{acc}},$$

$$R_{\text{acc}} \equiv \hat{R}_{\text{acc}} \exp[-\chi(\tau_f/t_p)^\gamma],$$

$$\hat{R}_{\text{acc}}^{(\text{Bondi})} = \left(\frac{4\tau_f}{t_B}\right)^{1/2} R_B,$$

$$\hat{R}_{\text{acc}}^{(\text{Hill})} = \left(\frac{\text{St}}{0.1}\right)^{1/3} R_H,$$

$$\frac{\partial \Sigma_d(a)}{\partial a} \propto a^{-p};$$

$$\rho_\bullet \propto a^{-q};$$

$$t_p \equiv \frac{GM_p}{(\Delta v + \Omega R_H)^3}$$

$$\dot{M}(a) = \int_0^a \frac{\partial \dot{M}(a')}{\partial a'} da',$$

$$\frac{\partial \dot{M}(a)}{\partial a} = \pi R_{\text{acc}}^2(a) \delta v(a) S(a) m(a) f(a).$$

$$\dot{M}_{2\text{D, Hill}} = 2 \times 10^{2/3} \Omega R_H^2 \int_0^{a_{\max}} \text{St}(a)^{2/3} m(a) W(a) da.$$

$$\dot{M}_{3\text{D, Bondi}} = \frac{4\pi R_B \Delta v^2}{\Omega} \times \int_0^{a_{\max}} \text{St} e^{-2\psi} m(a) f(a) \left[1 + 2 \left(\text{St} \frac{\Omega R_B}{\Delta v} \right)^{1/2} e^{-\psi} \right] da,$$

$$\psi \equiv \chi [\text{St}/(\Omega t_p)]^\gamma.$$

Analytical theory of polydisperse (multi-species) pebble accretion

Monodisperse (single species)

$$\dot{M}_{3D} = \lim_{\xi \rightarrow 0} \dot{M} = \pi R_{\text{acc}}^2 \rho_{d0} \delta v,$$

$$\dot{M}_{2D} = \lim_{\xi \rightarrow \infty} \dot{M} = 2R_{\text{acc}} \Sigma_d \delta v,$$

$$\xi \equiv \left(\frac{R_{\text{acc}}}{2H_d} \right)^2$$

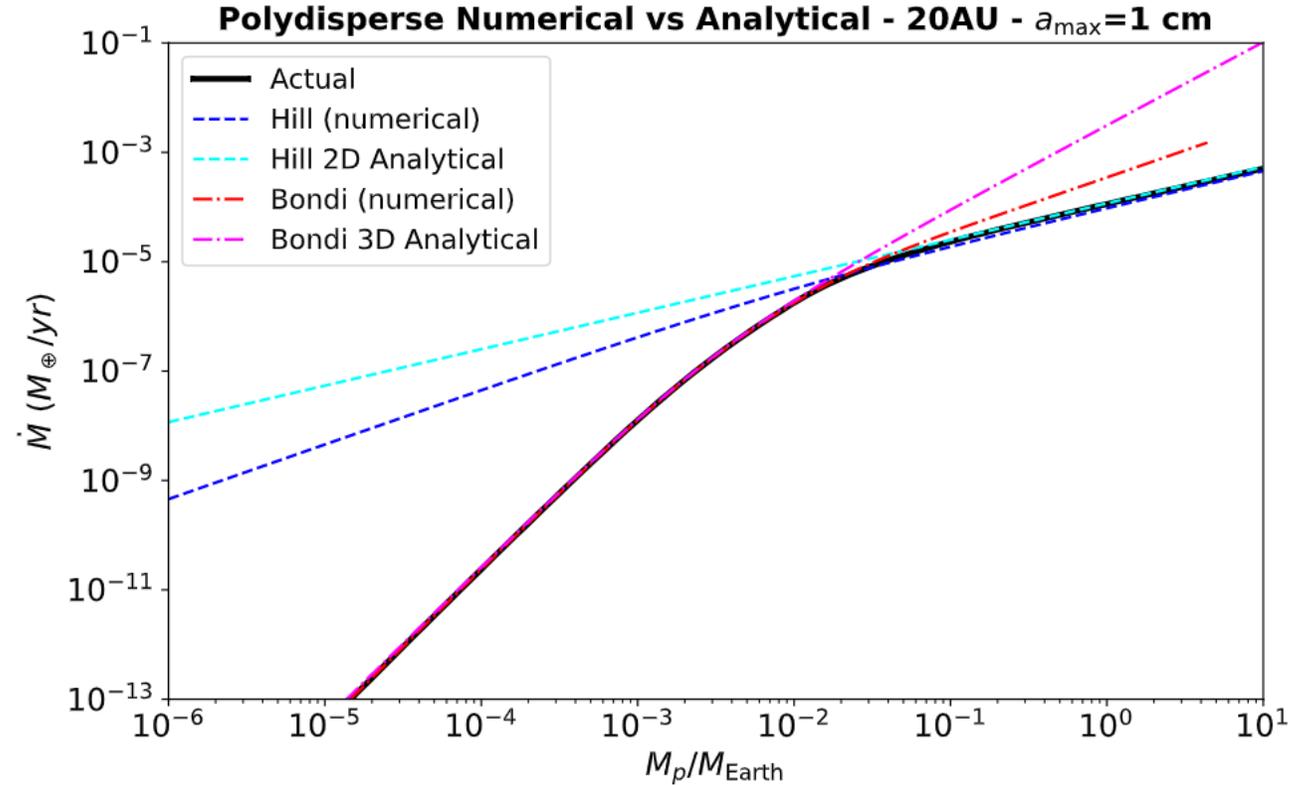
Lambrechts & Johansen (2012)

Polydisperse (multiple species)

$$\dot{M}_{2D,\text{Hill}} = \frac{6(1-p)}{14-5q-3k} \left(\frac{\text{St}_{\text{max}}}{0.1} \right)^{2/3} \Omega R_H^2 Z \Sigma_g.$$

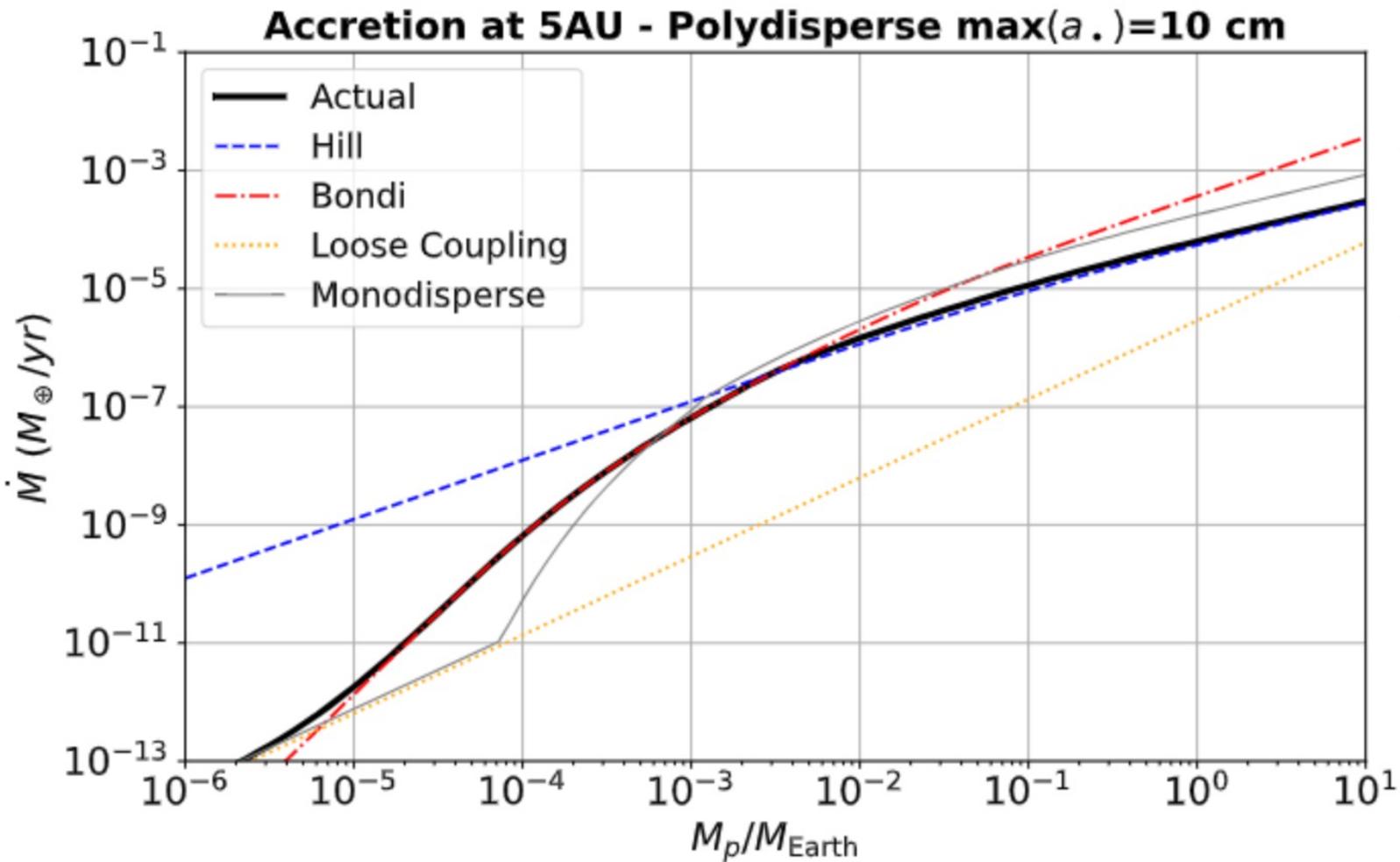
$$\dot{M}_{3D,\text{Bondi}} \approx C_1 \frac{\gamma_l \left(\frac{b_1+1}{s}, j_1 a_{\text{max}}^s \right)}{s J_1^{(b_1+1)/s}} + C_2 \frac{\gamma_l \left(\frac{b_2+1}{s}, j_2 a_{\text{max}}^s \right)}{s J_2^{(b_2+1)/s}} + C_3 \frac{\gamma_l \left(\frac{b_3+1}{s}, j_3 a_{\text{max}}^s \right)}{s J_3^{(b_3+1)/s}} + C_4 \frac{\gamma_l \left(\frac{b_4+1}{s}, j_4 a_{\text{max}}^s \right)}{s J_4^{(b_4+1)/s}},$$

Lyra et al. (2023)



Lyra et al. (2023)

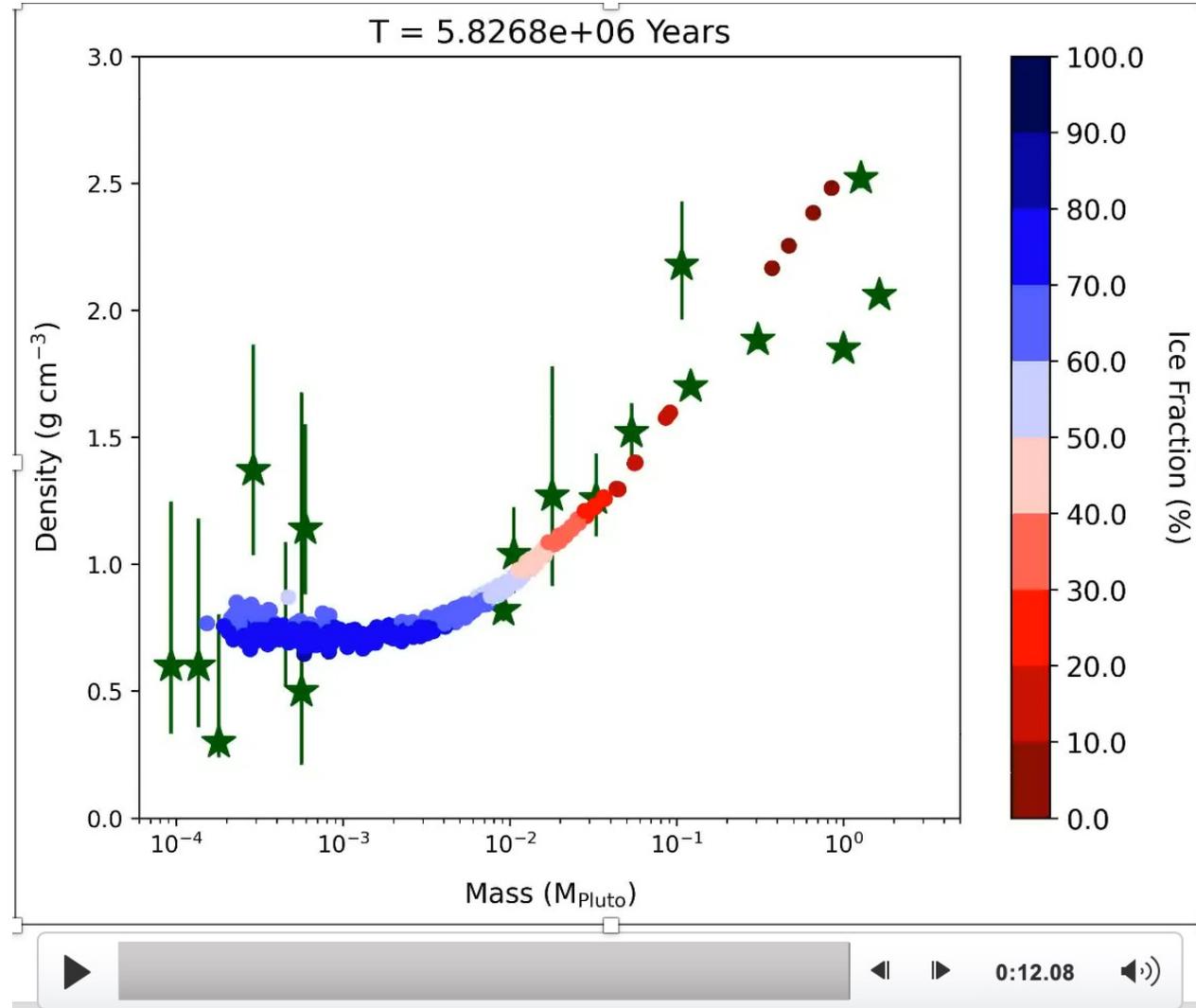
Accretion Rates



Bondi accretion: Indeed the monodisperse severely underestimates the mass accretion. Polydisperse is 2 orders of magnitude more efficient, and brings the onset of Bondi accretion to 2 orders of magnitude lower in mass. This is because monodisperse is considering only the largest grains. For small accreting mass, the Bondi sphere is small, so the largest grains accrete like planetesimals (poorly).

Hill accretion: the monodisperse is slightly overestimated. This is because monodisperse assumes all grains are in the largest pebble size. Naturally it will overestimate the pebble accretion rate – but not my much.

Growing Pluto by silicate pebble accretion

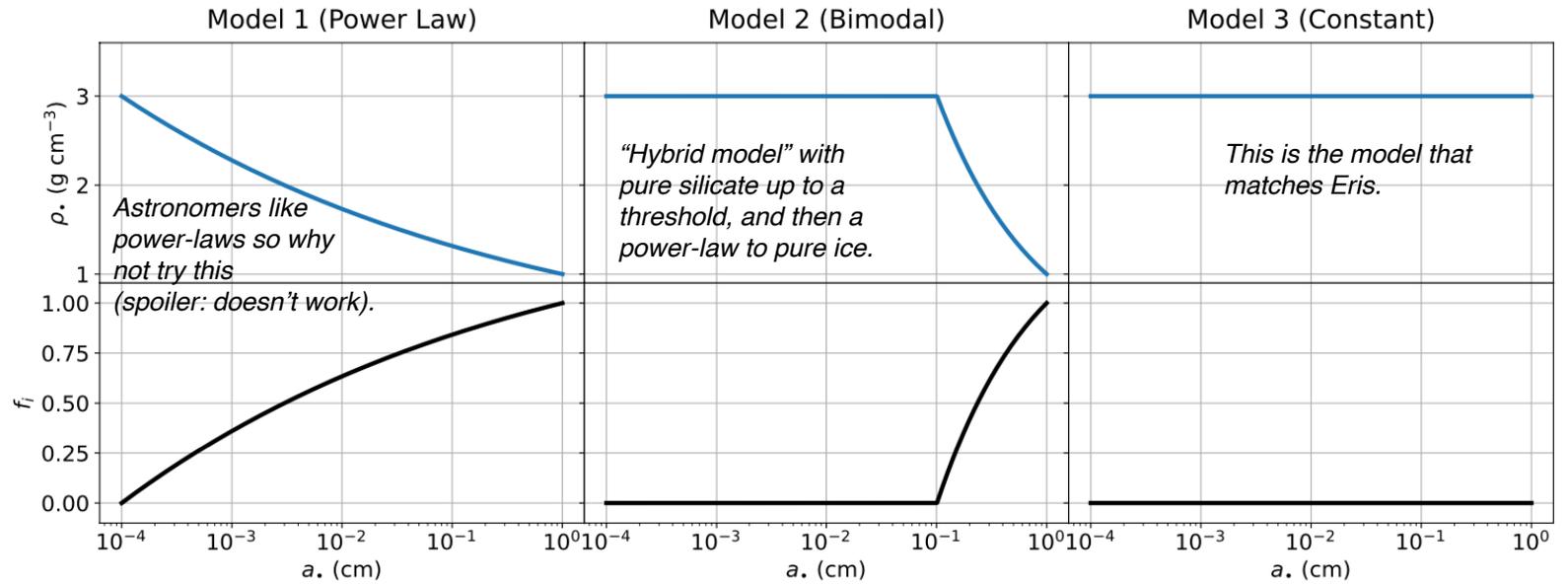


Pure silicate pebble accretion. Overshoots Pluto and Triton but matches Eris.

Try different models!

Pebble Internal Density

Ice Volume Fraction

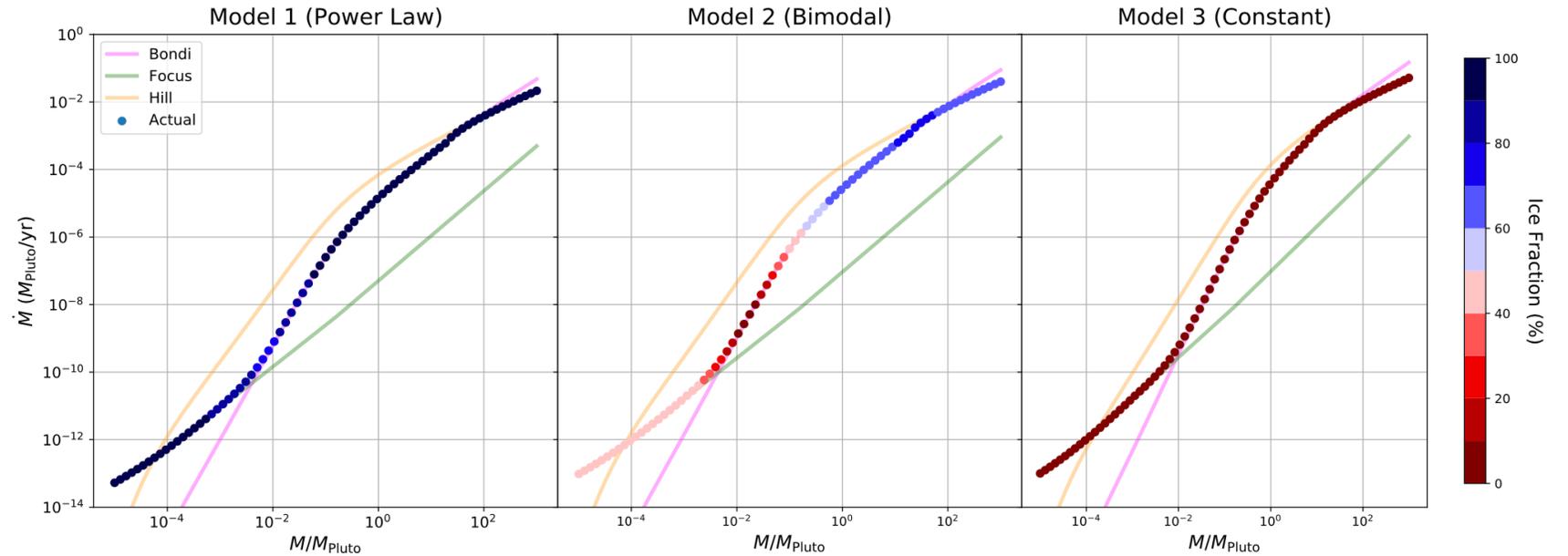


Mass Accretion rate

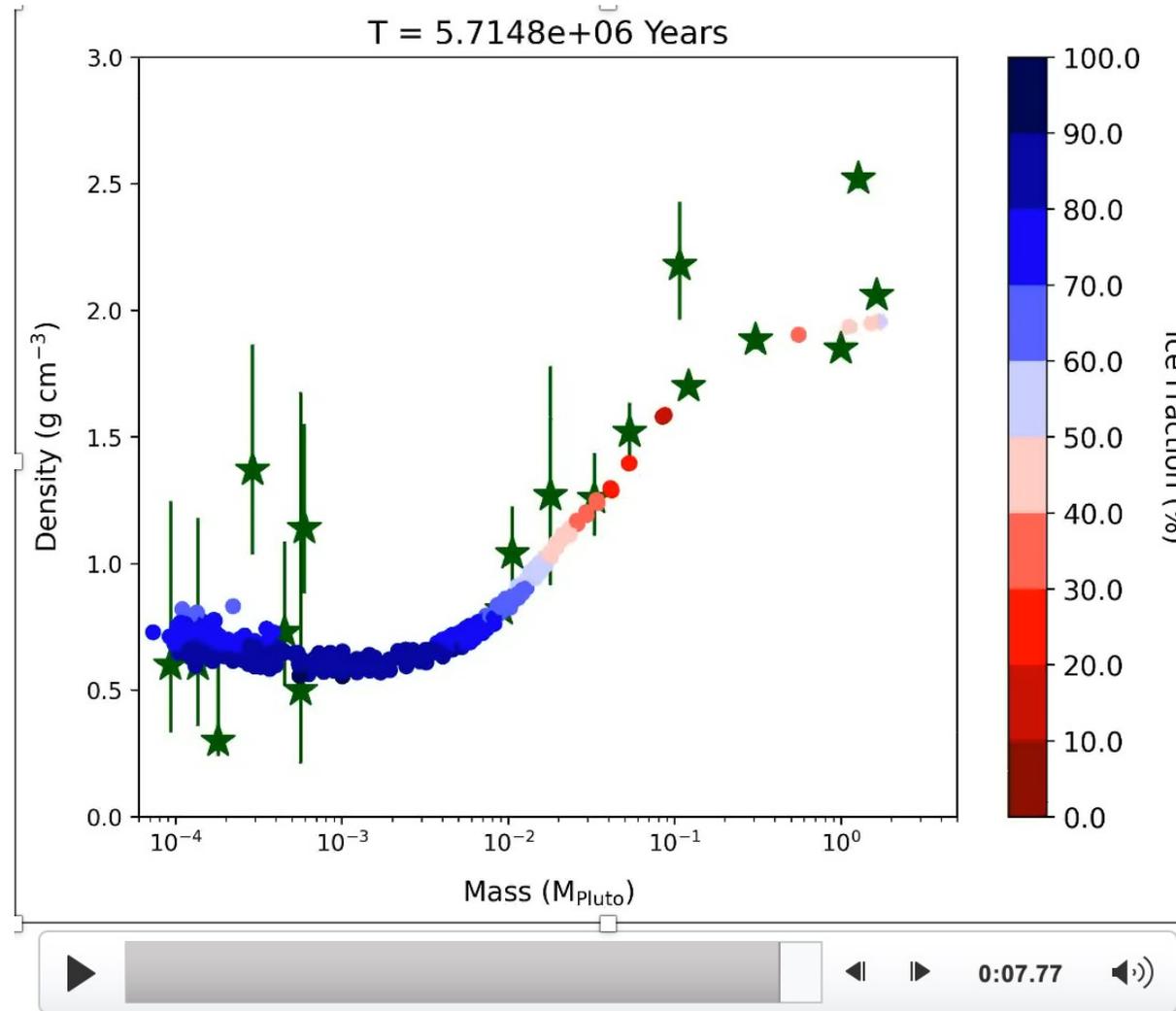
Color-coded the ice fraction of the best-accreting pebble size at each accretor mass.

Model 1 accretes mostly ice pebbles.

Model 2 has a window of silicate accretion.



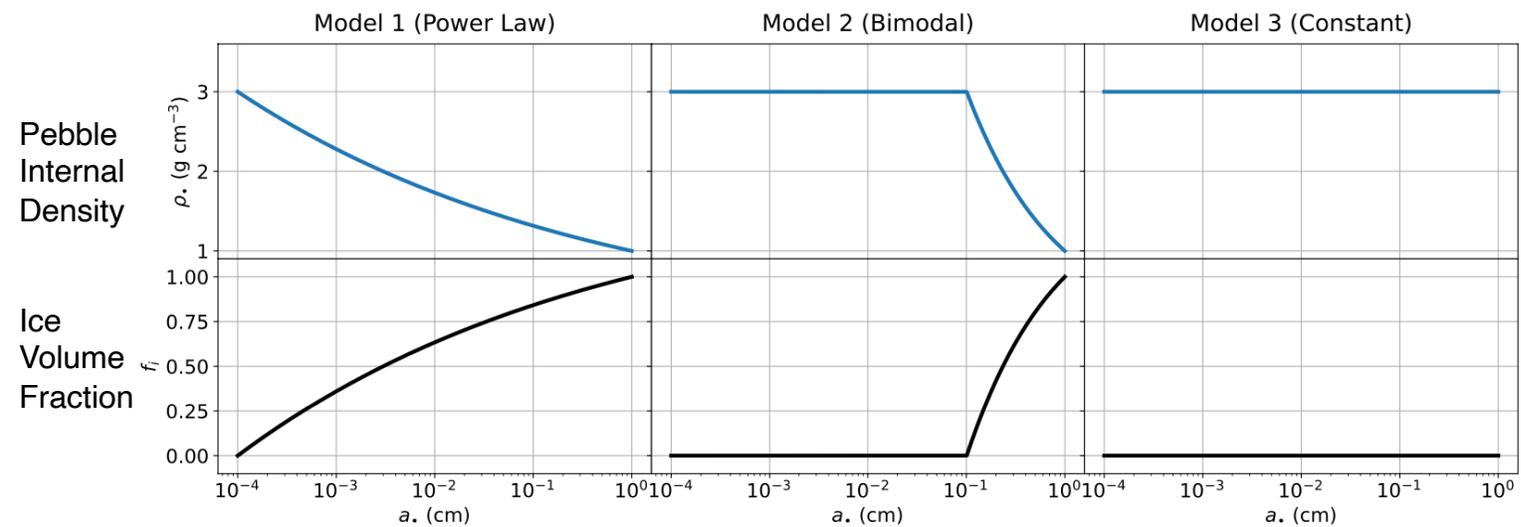
Growing Pluto by silicate pebble accretion



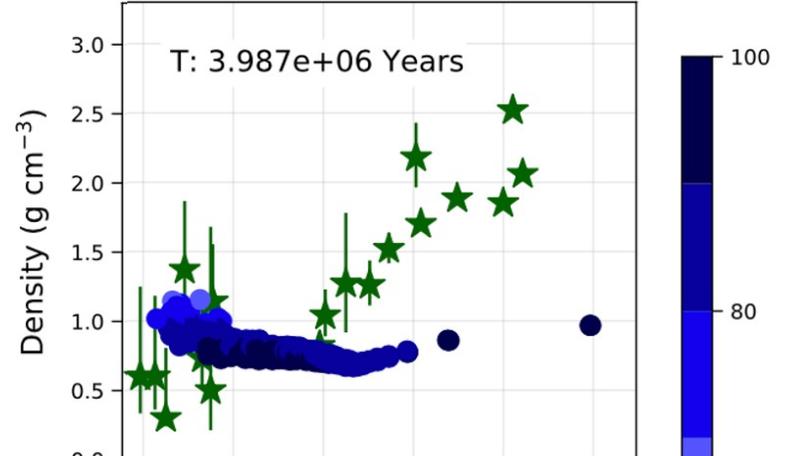
Model 2 (hybrid) fits Pluto and Triton.

Resulting Densities vs Mass relations

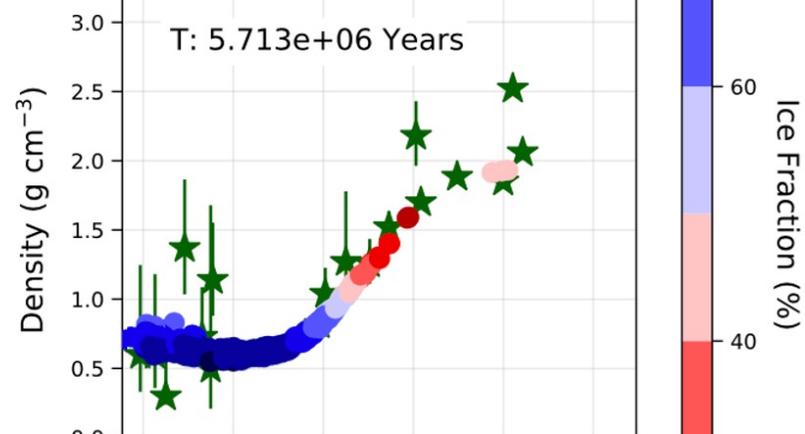
*Model 1 accretes mostly ice pebbles.
Doesn't reproduce densities of high mass objects.*



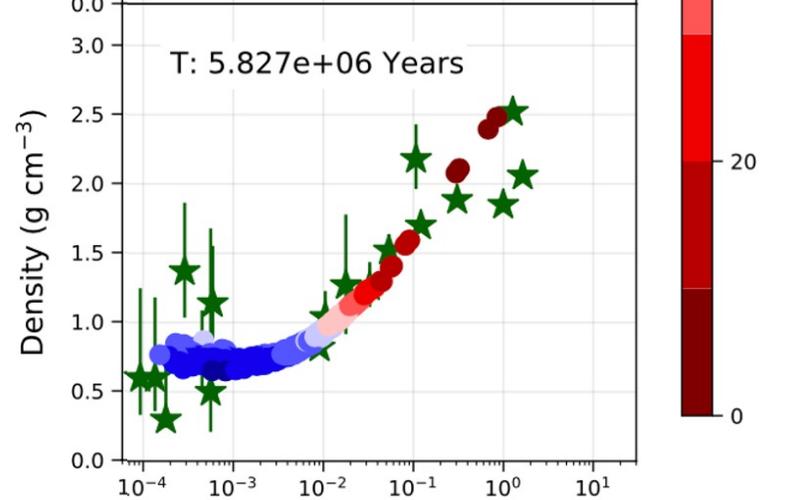
Model 1 (Power Law)



Model 2 (Bimodal)



Model 3 (Constant)

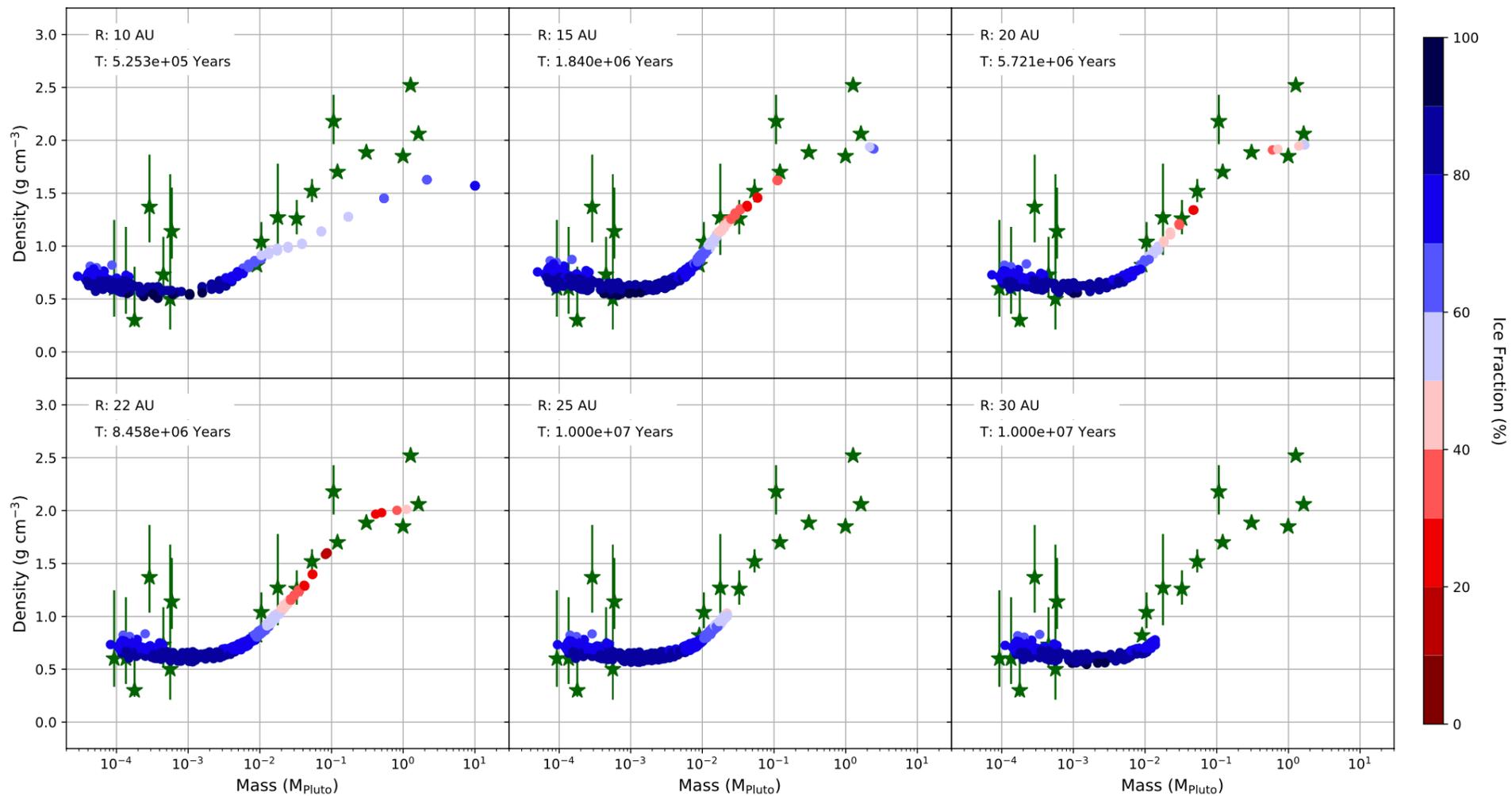


Distance Range

The model works best between 15 and 25AU

At 10 AU the mass accretion rates are too large, in half Myr the objects easily grow to 10 Pluto masses. Also the densities are mostly icy.

At 25AU the mass accretion rates are too low to significantly grow within the lifetime of the disk. The planetesimals formed by streaming instability grow by a factor unity, remaining planetesimals.

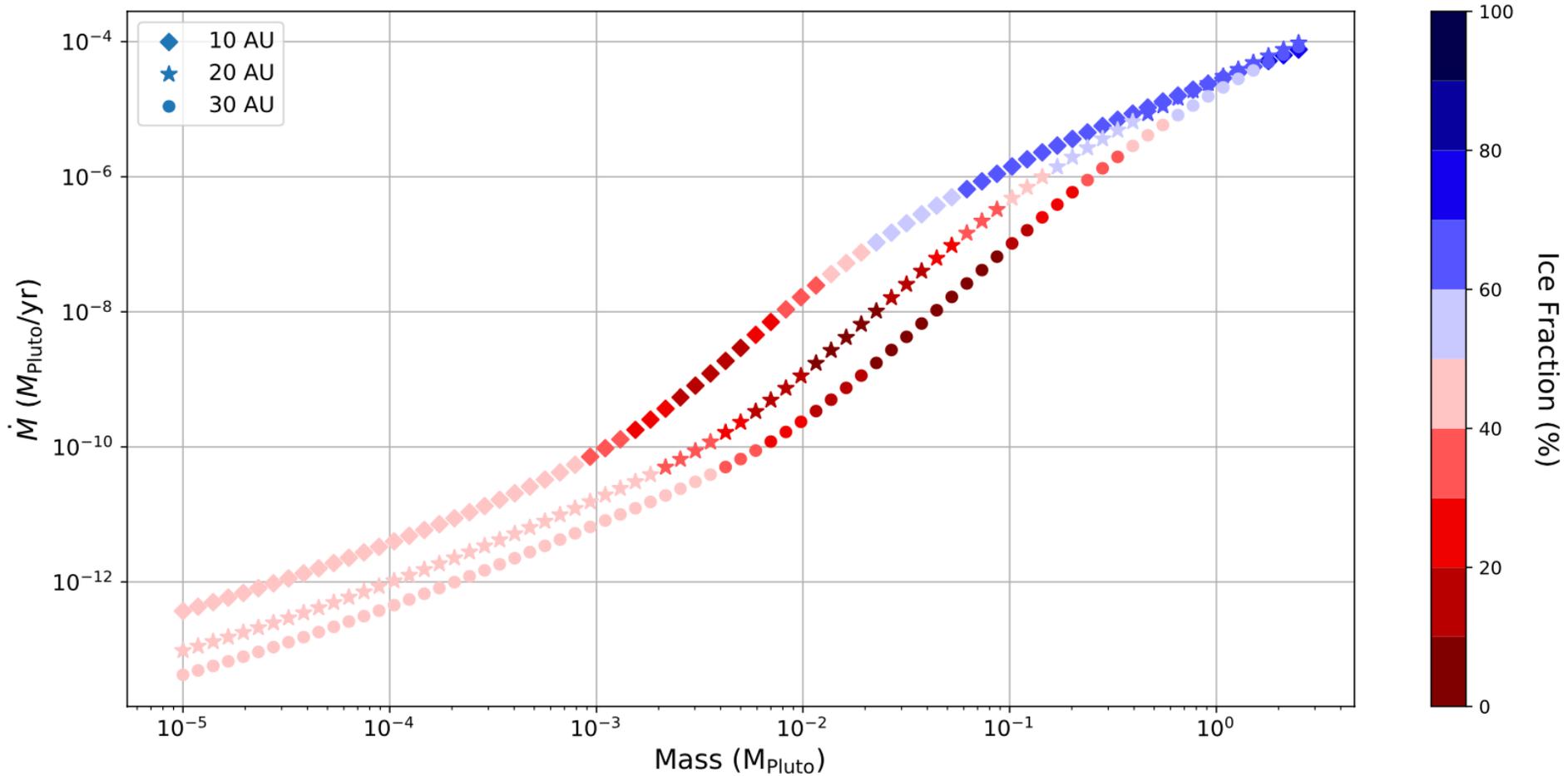


The window of silicate accretion

To understand the distance relation, we plot the mass accretion rates and the ice fraction of the best accreted pebble at 10, 20, and 30 AU.

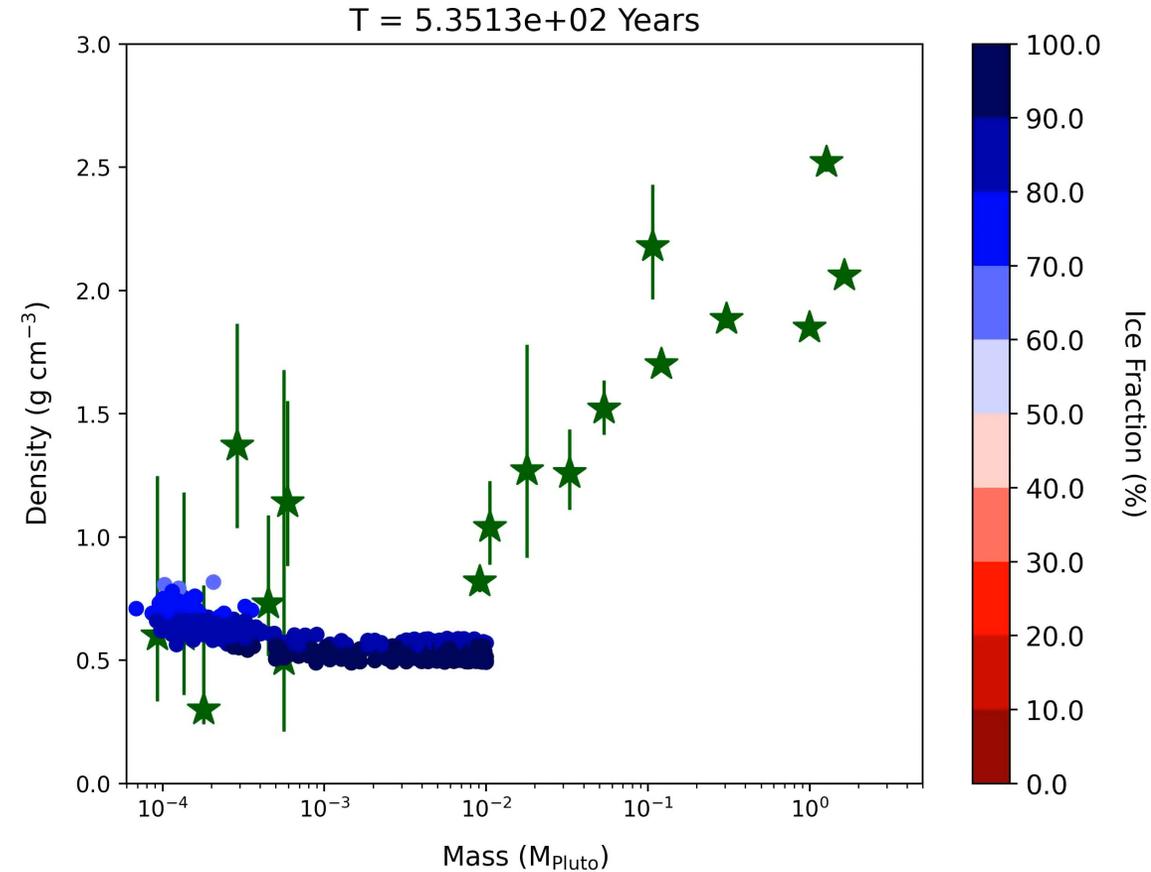
Consider a seed mass of 10^{-2} Pluto mass. At 10AU the accretion rate is high but it will soon accrete icy pebbles. It misses the window of silicate accretion, which was around a few $\times 10^{-3}$ Pluto masses.

At 30AU it will accrete silicates but the mass accretion rates are low (10^{-10} $M_{\text{Pluto}}/\text{yr}$). It would take Gyrs to accrete Pluto.



Conclusions

- Polydisperse Bondi accretion 1-2 orders of magnitude more efficient than monodisperse
 - Best accreted pebbles are those of drag time \sim Bondi time, not the largest ones
 - The largest ones dominate the mass budget, but accrete poorly
- Onset of Bondi accretion 1-2 orders of magnitude lower in mass compared to monodisperse
 - Bondi accretion possible on top of Streaming Instability planetary embryos within disk lifetime
 - Reaches 100-350km objects within Myr timescales
- Analytical solution to
 - Monodisperse general case
 - Polydisperse 2D Hill and 3D Bondi
- KBO density dichotomy problem:
 - Two different pebble populations, maintained by ice desorption off small grains
 - Streaming instability: icy-rich small objects; nearly uniform composition
 - Polydisperse pebble accretion: silicate-rich larger objects; varied composition
 - Melting avoided by
 - ice-rich formation
 - ^{26}Al incorporated mostly in long ($>$ Myr) phase of silicate accretion
 - KBOs best reproduced between 15-25 AU



Analytical Solution for General Monodisperse (single species) Pebble Accretion

$$\dot{M} = \pi R_{\text{acc}}^2 \rho_{d0} S \delta v.$$

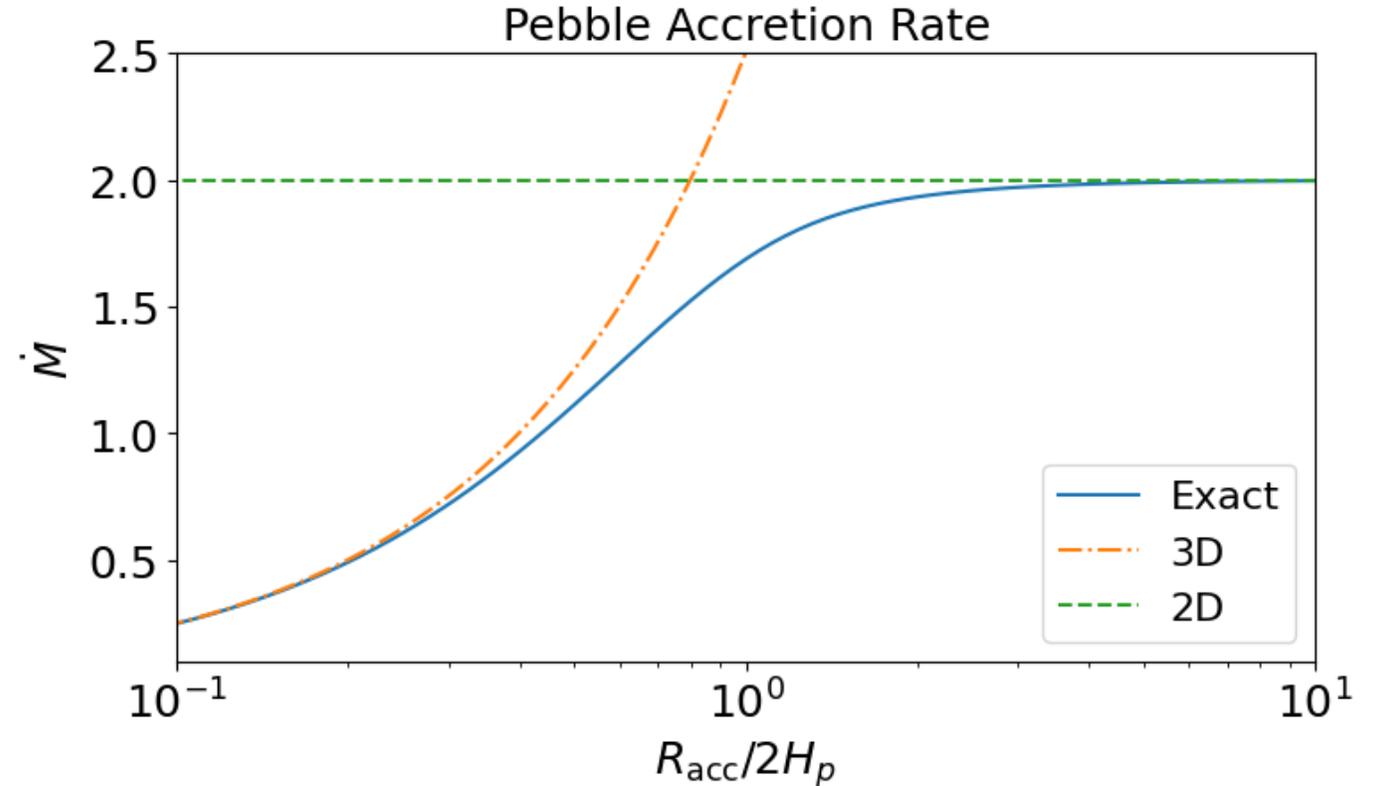
$$S \equiv \frac{1}{\pi R_{\text{acc}}^2} \int_{-R_{\text{acc}}}^{R_{\text{acc}}} 2\sqrt{R_{\text{acc}}^2 - z^2} \exp\left(-\frac{z^2}{2H_d^2}\right) dz,$$

$$S = e^{-\xi} [I_0(\xi) + I_1(\xi)], \quad \xi \equiv \left(\frac{R_{\text{acc}}}{2H_d}\right)^2$$

```

y = (x/2)**2
# Modified Bessel function of the first kind of real order.
I0 = sp.special.iv(0, y)
I1 = sp.special.iv(1, y)

Sint = np.exp(-y) * (I0 + I1)
rho_int = rhop * Sint
Mdot = pi*r**2 * rho_int * deltav
    
```



Analytical Solutions for 2D and 3D Polydisperse (multi-species) Pebble Accretion

$$\dot{M}_{2D,Hill} = \frac{6(1-p)}{14-5q-3k} \left(\frac{St_{max}}{0.1} \right)^{2/3} \Omega R_H^2 Z \Sigma_g$$

$$\dot{M}_{3D,Bondi} \approx C_1 \frac{\gamma_l \left(\frac{b_1+1}{s}, j_1 a_{max}^s \right)}{s j_1^{(b_1+1)/s}} + C_2 \frac{\gamma_l \left(\frac{b_2+1}{s}, j_2 a_{max}^s \right)}{s j_2^{(b_2+1)/s}} + C_3 \frac{\gamma_l \left(\frac{b_3+1}{s}, j_3 a_{max}^s \right)}{s j_3^{(b_3+1)/s}} + C_4 \frac{\gamma_l \left(\frac{b_4+1}{s}, j_4 a_{max}^s \right)}{s j_4^{(b_4+1)/s}}$$

```

gamma1 = sp.special.gammaln((b1+1)/s, j1*a**s)*sp.special.gamma((b1+1)/s)
gamma2 = sp.special.gammaln((b2+1)/s, j2*a**s)*sp.special.gamma((b2+1)/s)
gamma3 = sp.special.gammaln((b3+1)/s, j3*a**s)*sp.special.gamma((b3+1)/s)
gamma4 = sp.special.gammaln((b4+1)/s, j4*a**s)*sp.special.gamma((b4+1)/s)

```

```

G1 = C1*gamma1/s/j1**((b1+1)/s)
G2 = C2*gamma2/s/j2**((b2+1)/s)
G3 = C3*gamma3/s/j3**((b3+1)/s)
G4 = C4*gamma4/s/j4**((b4+1)/s)

```

```

mbondi3d = G1 + G2 + G3 + G4

```

