Polydisperse Pebble Accretion
Doing away with planetesimal accretion

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The size-density relationship of Kuiper Belt objects

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THE DENSITY OF MID-SIZED KUIPER BELT OBJECT 2002 UX25
AND THE FORMATION OF THE DWARF PLANETS

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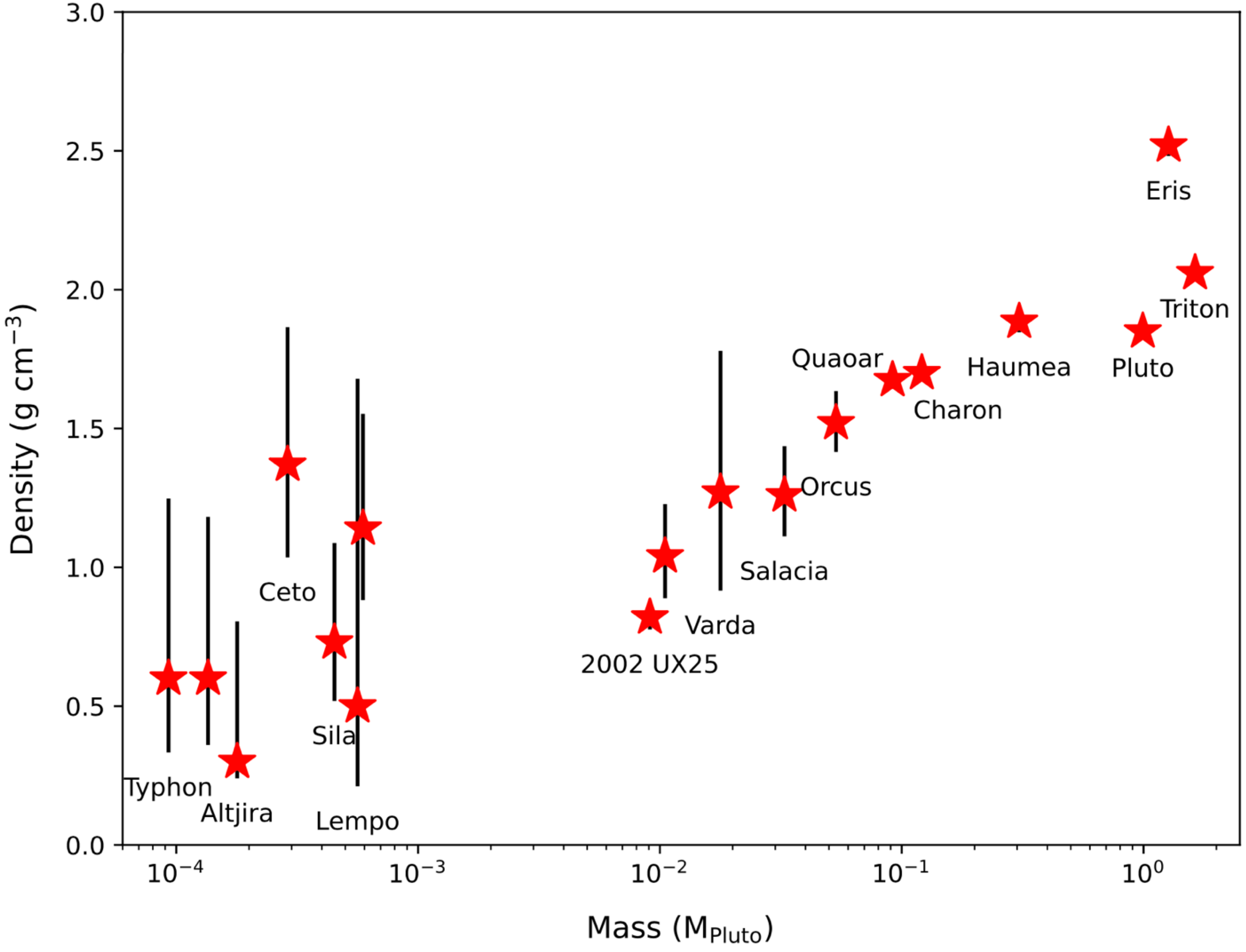
ABSTRACT

higher, from / hout invoking / in to increase / on occurs for / een ~350 km / could provide / racterization, / a diameter of / solar system / unlikely to be / epresentation / l higher from / es: formation

The inferred low rock fraction of the 2002 UX25 system makes the formation of rock-rich larger objects difficult to explain in any standard coagulation scenario. For example, to create an object with the volume of Eris would require assembling ~40 objects of the size of 2002 UX25. Yet the assembled object, even with the additional compression, would still have a density close to 1 g cm^{-3} rather than the 2.5 g cm^{-3} density of Eris (Sicardy et al. 2011).

- Extremely low porosity;
- Biased sample;
- Compaction through giant impacts

None of these alternatives appears likely. We are left in the uncomfortable state of having no satisfying mechanism to explain the formation of the icy dwarf planets. While objects up to the size of 2002 UX25 can easily be formed through standard coagulation scenarios, the rock-rich larger bodies may require a formation mechanism separate from the rest of the Kuiper belt.



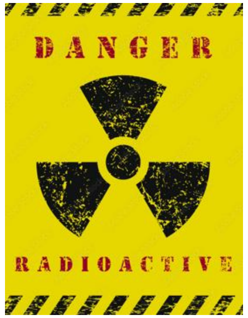
Cañas+Lyra et al. (2024)

Data; *Thomas (2000), Stansberry et al. (2006), Grundy et al. (2007), Brown et al. (2011), Stansberry et al. (2012), Brown (2013), Fornasier et al. (2013), Vilenius, et al. (2014), Nimmo et al. (2016), Ortiz et al. (2017), Brown and Butler (2017), Grundy et al. (2019), Morgado et al. (2023), Pereira et al. (2023).*

Previous best bet:
Porosity removal by gravitational compaction

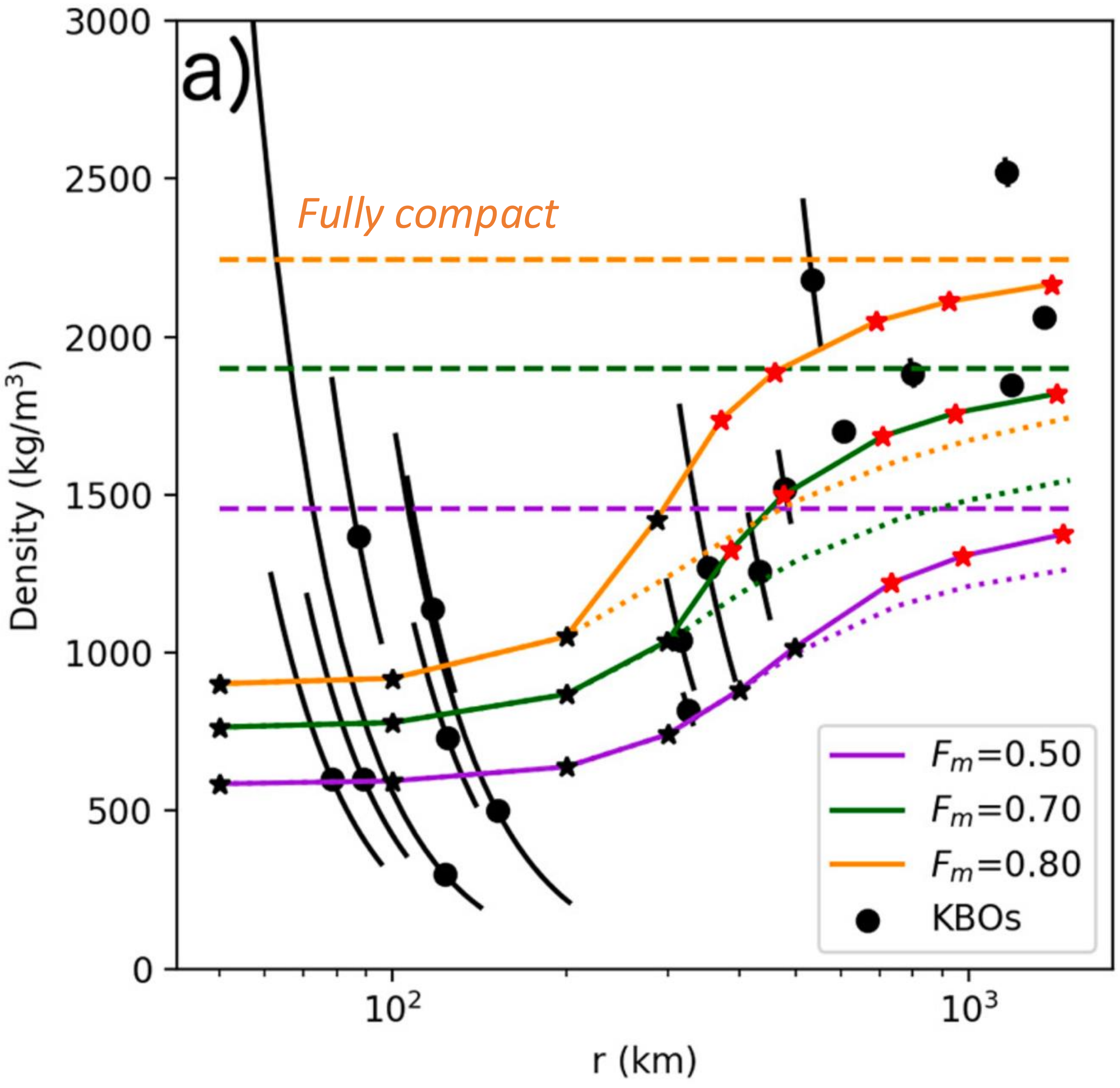
Problem

- Timing! ^{26}Al would melt if formed within 4 Myr



Assumptions

- ~~Constant composition at birth and growth~~
- ~~Growth by planetesimal accretion~~



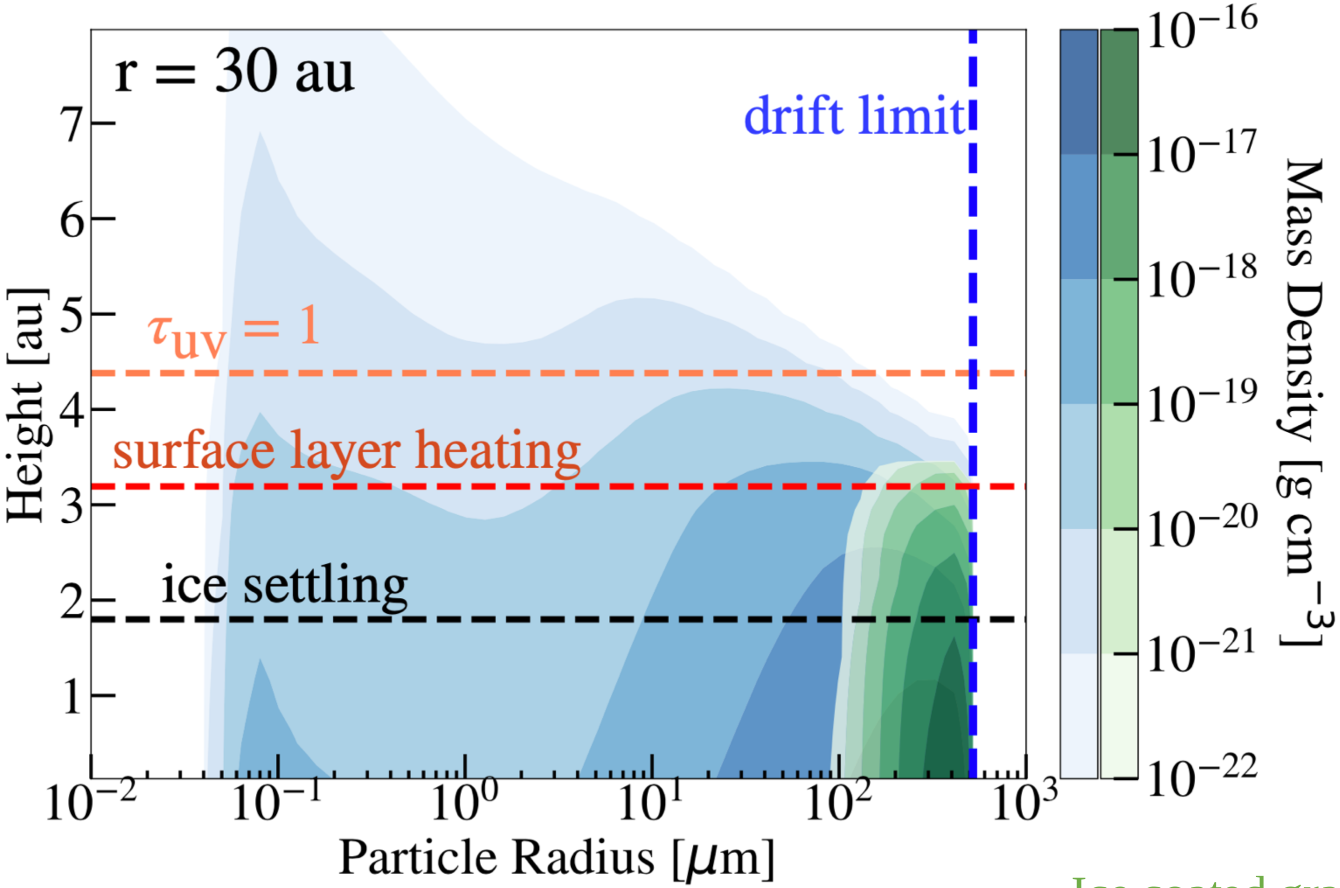
F_m = rock mass fraction

Bierson & Nimmo (2019)

Abandoning Constant Composition

Heating and UV irradiation remove ice on Myr timescales (Harrison & Schoen 1967)

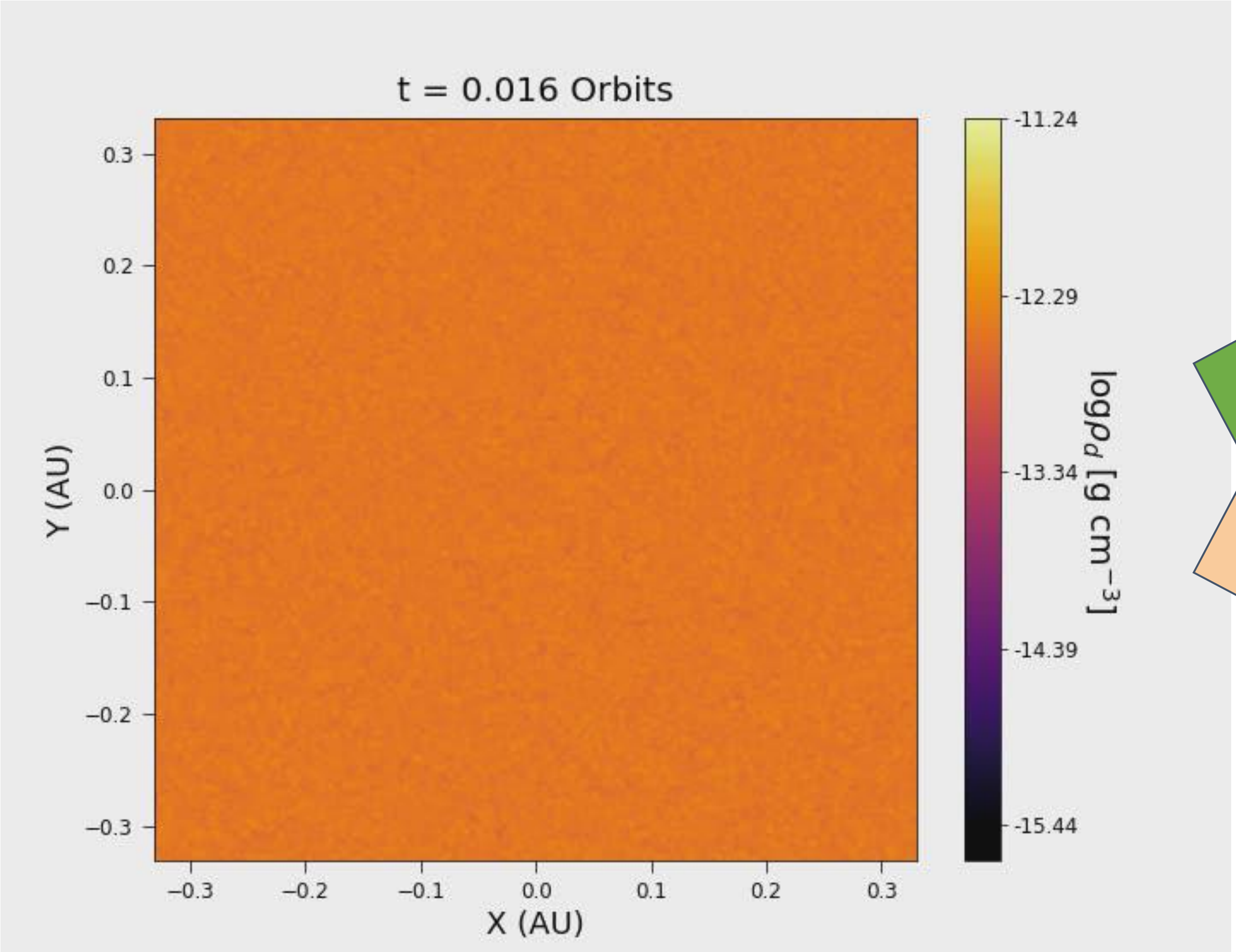
- Small grains lofted in the atmosphere lose ice
- Big grains are shielded and remain icy.

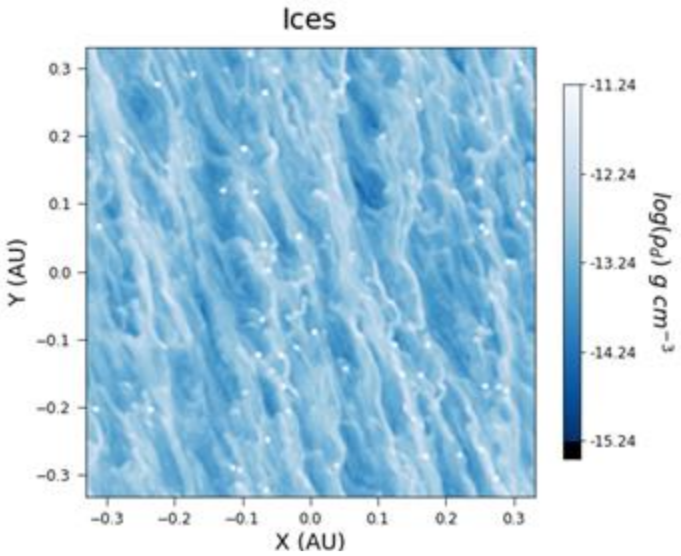


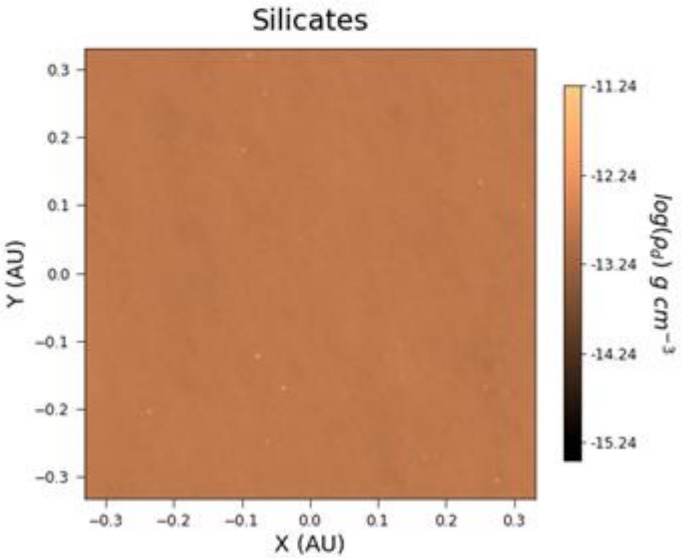
Ice coated grains
Ice-free grains

Powell et al. 2022

Split into icy and silicate pebbles

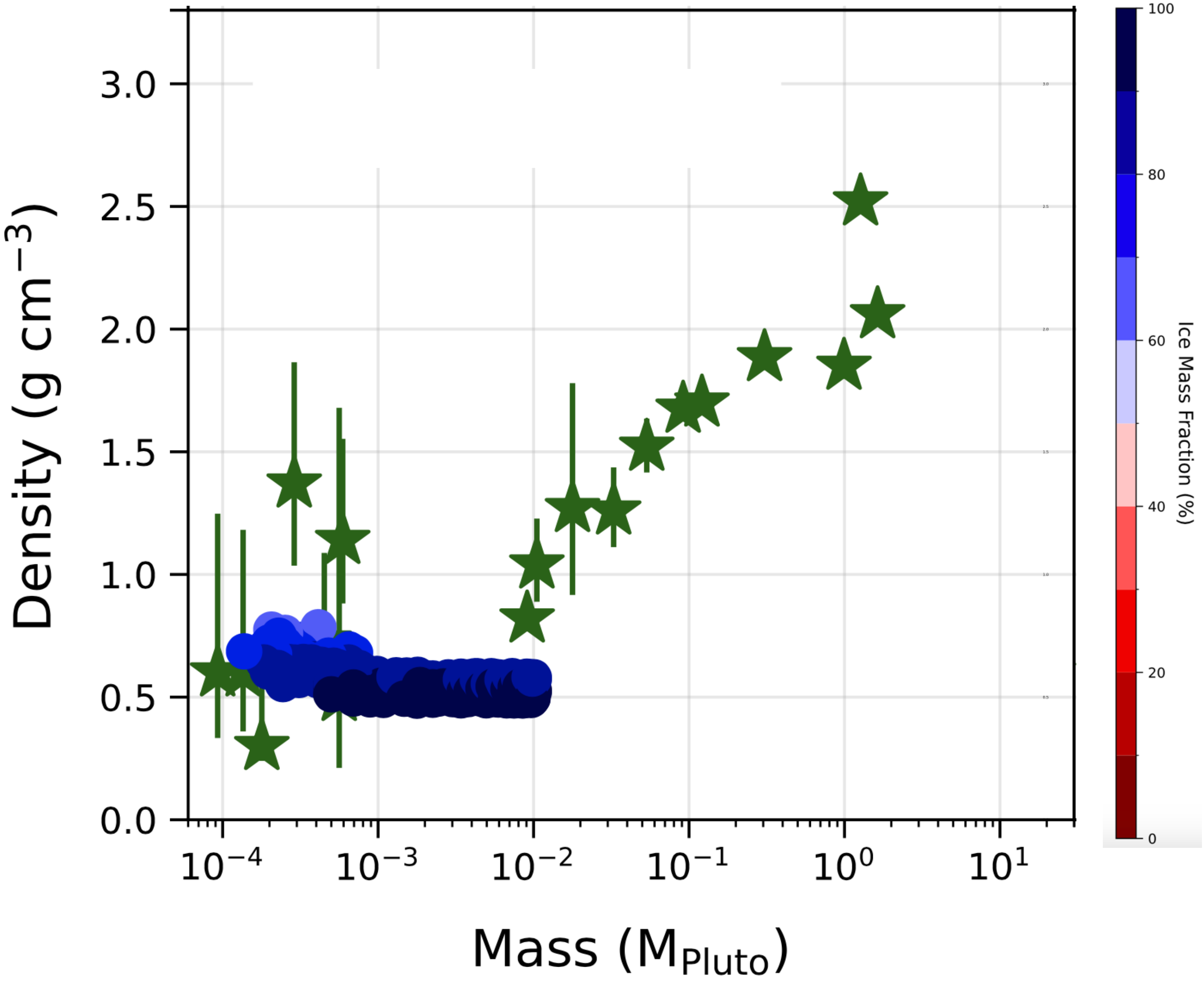






Canas+Lyra et al. 2024

The first planetesimals are icy



Canas+Lyra et al. 2024

The first planetesimals won't melt

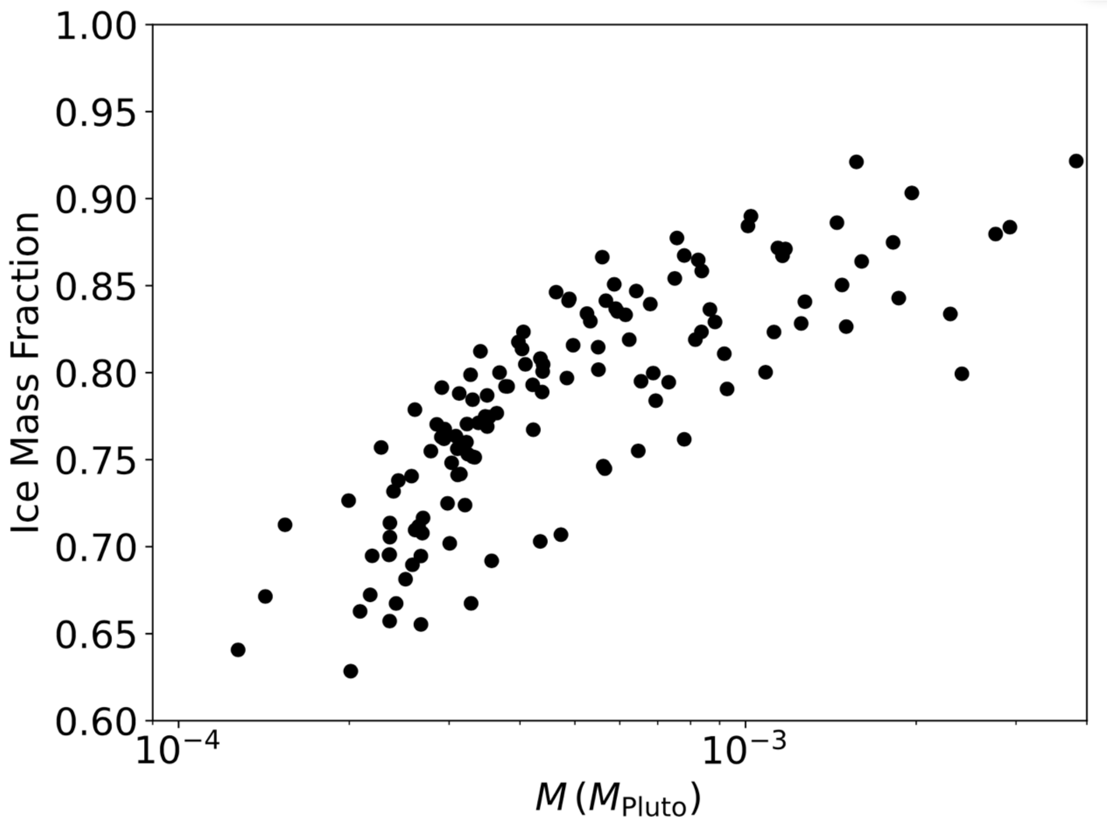
$$\mathcal{H} = \rho F_s [^{26}\mathrm{Al}]_0 \mathcal{H}_0 e^{-\lambda t}$$

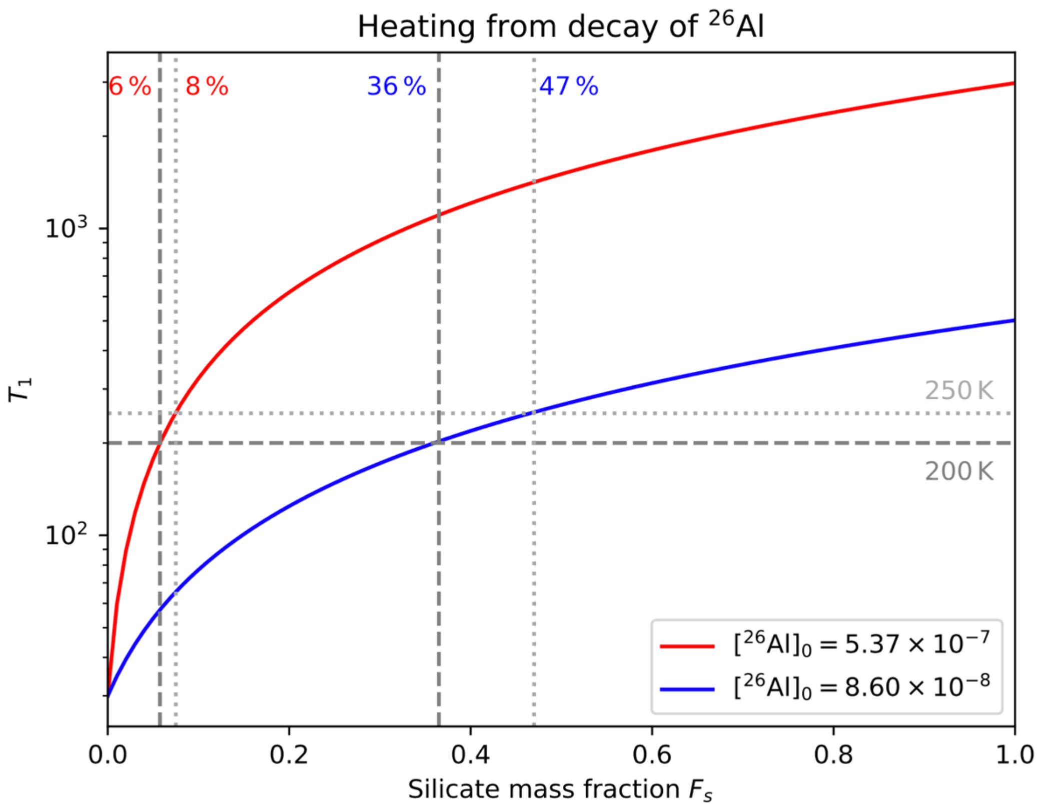
$$Q(t) = V \int_0^t \mathcal{H}(t')dt'$$

$$= M_p F_s [^{26}\mathrm{Al}]_0 \mathcal{H}_0 \lambda^{-1} \left(1 - e^{-\lambda t}\right)$$

$$Q = M_p c_p \Delta T$$

$$\Delta T = F_s [^{26}\mathrm{Al}]_0 \mathcal{H}_0 \lambda^{-1} c_p^{-1}$$



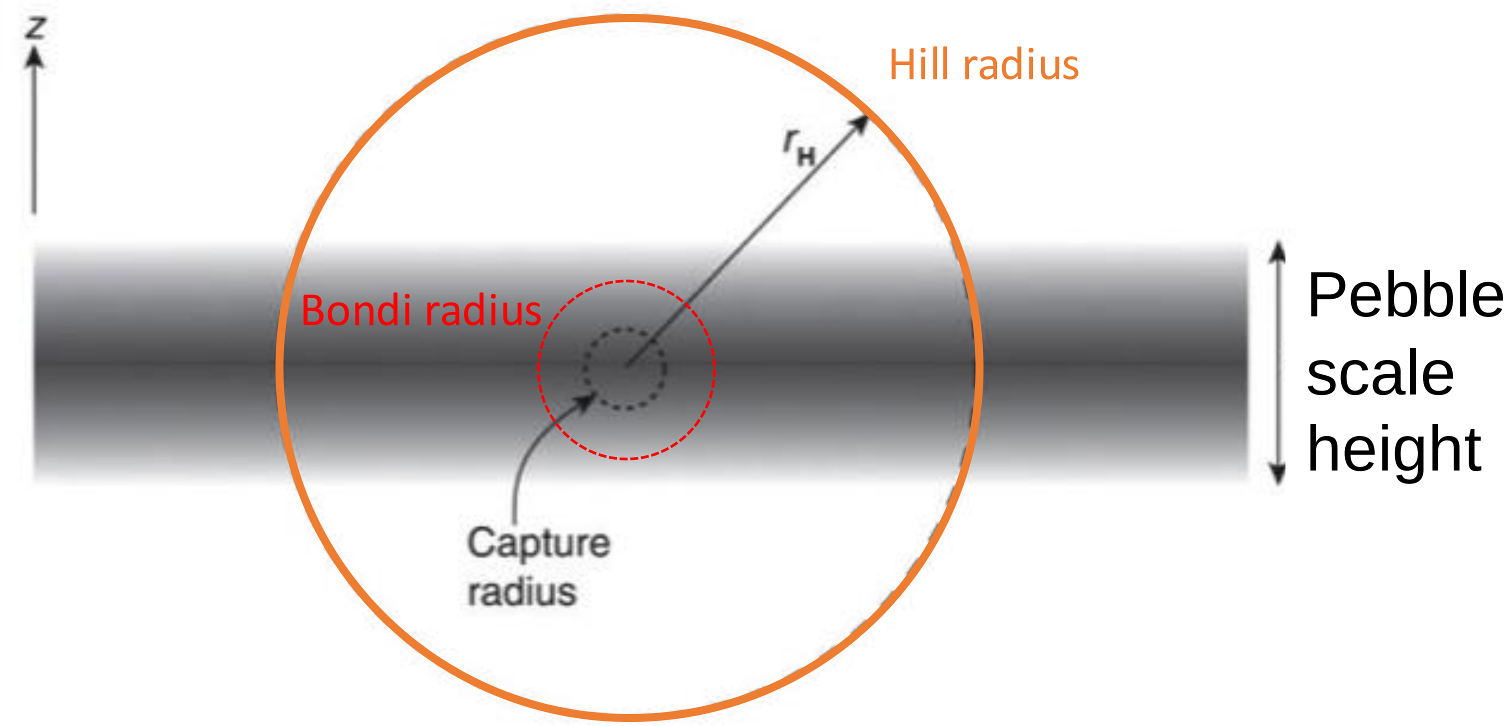


Pebble Accretion: Geometric, Bondi, and Hill regime

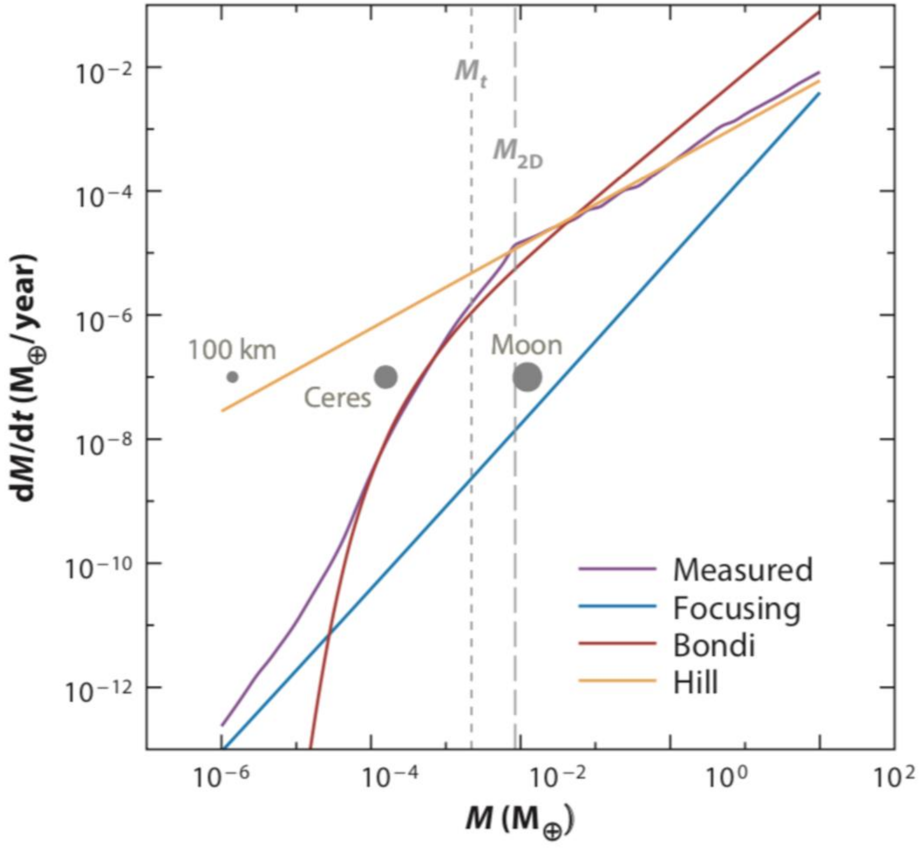
$$\xi \equiv \left(\frac{R_{acc}}{2H_d}\right)^2 \qquad \dot{M}_{3D} = \lim_{\xi\to 0} \dot{M} = \pi R_{acc}^2 \rho_{d0} \delta v, \qquad \dot{M}_{2D} = \lim_{\xi\to\infty} \dot{M} = 2R_{acc}\Sigma_d \delta v,$$

Bondi accretion - Bound against headwind

Hill accretion - Bound against stellar tide

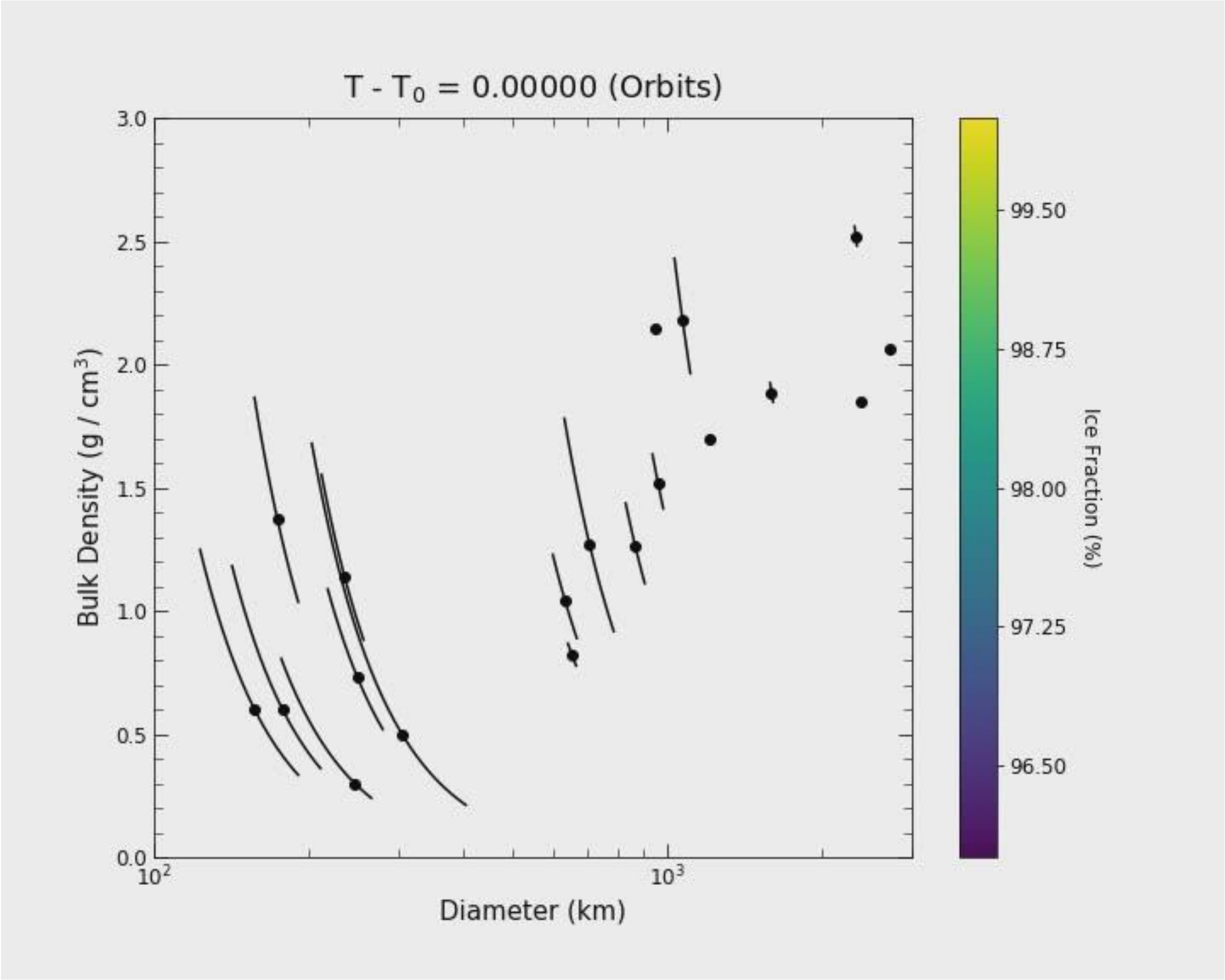


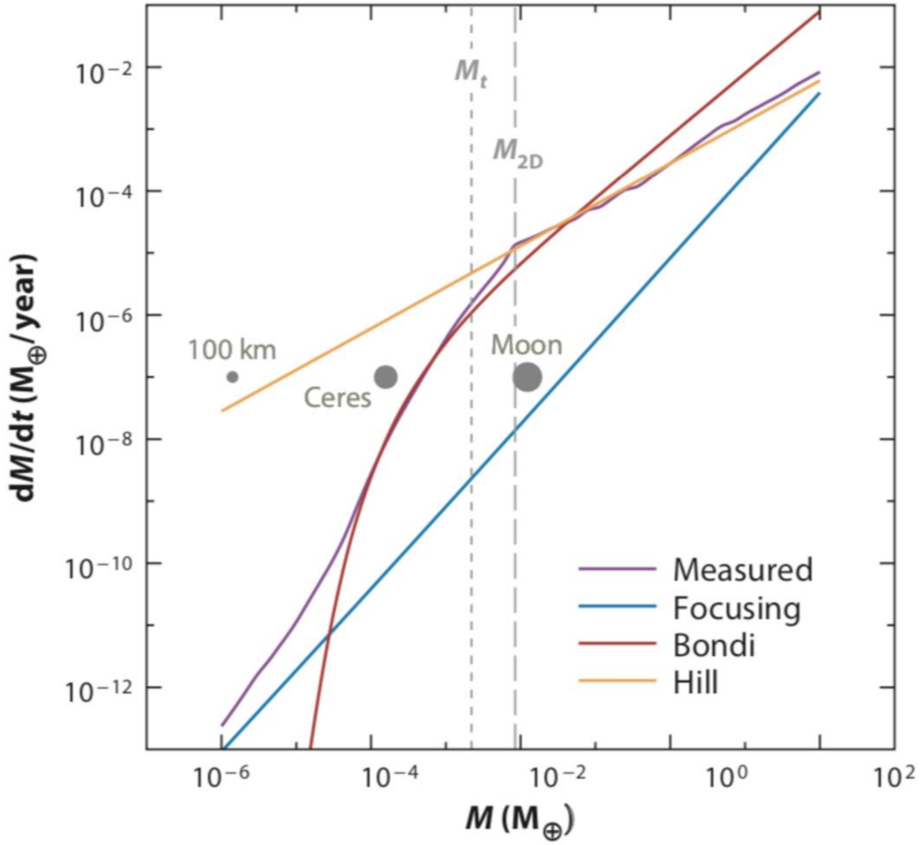
Mass Accretion rates



Johansen & Lambrechts '17

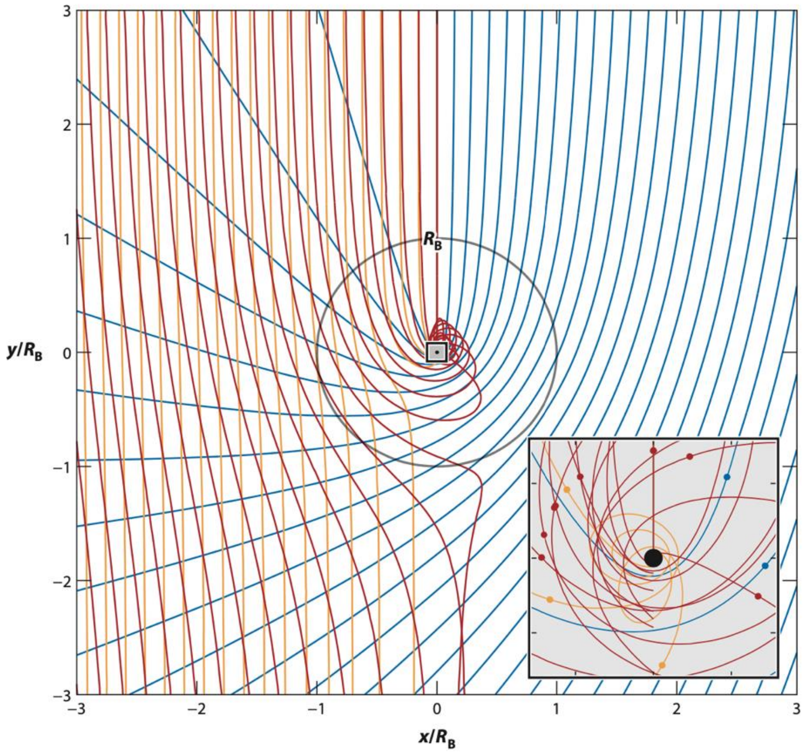
Integrate pebble accretion





Pebble Accretion: Pebbles of different size accrete differently

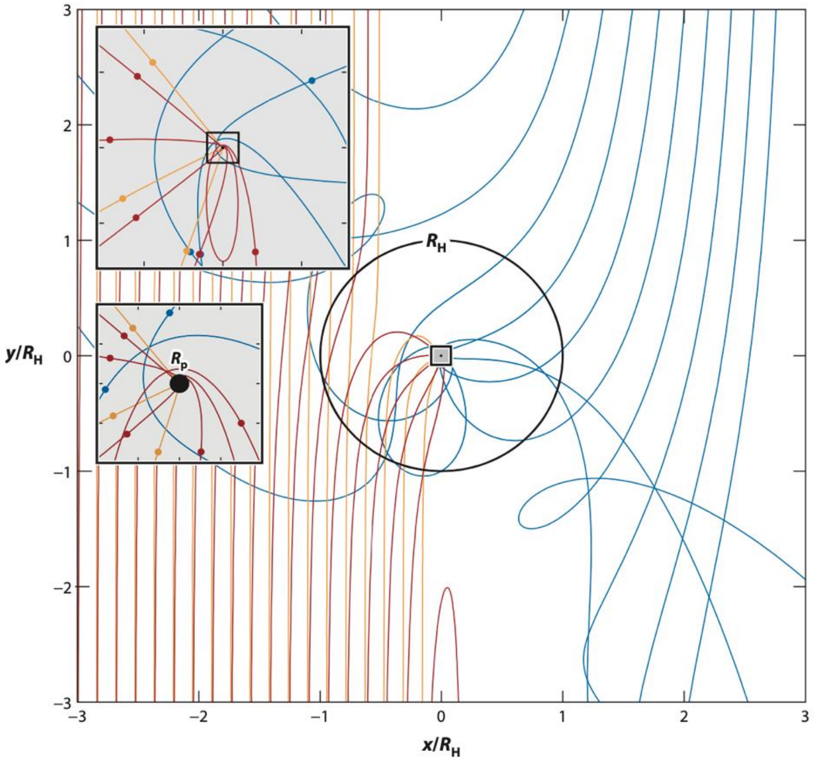
Bondi Regime



Best accreted pebble

Drag time ~ Bondi Time

Hill Regime



Best accreted pebble

Drag time ~ Orbital Time

Polydisperse (Multi-Species) Pebble Accretion

$$\rho_d(a, z) = \int_0^a m(a')\, F(a', z)\, da'.$$

$$F(a, z) \equiv f(a)\, e^{-z^2/2H_d^2},$$

$$f(a) = \frac{3(1-p)Z\Sigma_g}{2^{5/2}\pi^{3/2}H_g\rho_\bullet^{(0)}a_{\max}^{4-k}}\sqrt{1 + a\frac{\pi}{2}\frac{\rho_\bullet(a)}{\Sigma_g\alpha}}\; a^{-k}.$$

$$\dot{M}(a) = \int_0^a \frac{\partial \dot{M}(a')}{\partial a'}da',$$

$$\frac{\partial \dot{M}(a)}{\partial a} = \pi R_{\rm acc}^2(a)\delta v(a) S(a) m(a) f(a).$$

$$S \equiv \frac{1}{\pi R_{\rm acc}^2}\int_{-R_{\rm acc}}^{R_{\rm acc}} 2\sqrt{R_{\rm acc}^2 - z^2}\,\exp\left(-\frac{z^2}{2H_d^2}\right)dz,$$

$$W(a) = \frac{3(1-p)Z\Sigma_g}{4\pi\rho_\bullet^{(0)}a_{\max}^{4-k}}\,a^{-k},$$

$$\hat{R}_{\rm acc}^{\rm (Bondi)} = \left(\frac{4\tau_f}{t_{\rm B}}\right)^{1/2}R_{\rm B},$$

$$\delta v \equiv \Delta v + \Omega R_{\rm acc},$$

$$\hat{R}_{\rm acc}^{\rm (Hill)} = \left(\frac{\rm St}{0.1}\right)^{1/3}R_{\rm H},$$

$$R_{\rm acc} \equiv \hat{R}_{\rm acc}\exp[-\chi(\tau_f/t_p)^\gamma],$$

$$\frac{\partial \Sigma_d(a)}{\partial a} \propto a^{-p};$$

$$\rho_\bullet \propto a^{-q};$$

$$t_p \equiv \frac{GM_p}{(\Delta v + \Omega R_{\rm H})^3}$$

$$\dot{M}_{\rm 2D,\,Hill} = 2\times 10^{2/3}\Omega R_H^2\int_0^{a_{\max}} {\rm St}(a)^{2/3}\, m(a)\, W(a)\, da.$$

$$\dot{M}_{\rm 3D,\,Bondi} = \frac{4\pi R_{\rm B}\Delta v^2}{\Omega} \times \int_0^{a_{\max}} {\rm St}\; e^{-2\psi}m(a)f(a) \times \left[1 + 2\left({\rm St}\frac{\Omega R_{\rm B}}{\Delta v}\right)^{1/2}e^{-\psi}\right]da,$$

$$\psi \equiv \chi[{\rm St}/(\Omega t_p)]^\gamma.$$

Lyra et al. 2023

Analytical theory of polydisperse (multi-species) pebble accretion

Monodisperse (single species)

$$\xi \equiv \left(\frac{R_{\rm acc}}{2H_d}\right)^2$$

$$\dot{M}_{\rm 3D} = \lim_{\xi \to 0} \dot{M} = \pi R_{\rm acc}^2 \rho_{d0} \delta v,$$

$$\dot{M}_{\rm 2D} = \lim_{\xi \to \infty} \dot{M} = 2 R_{\rm acc} \Sigma_d \delta v,$$

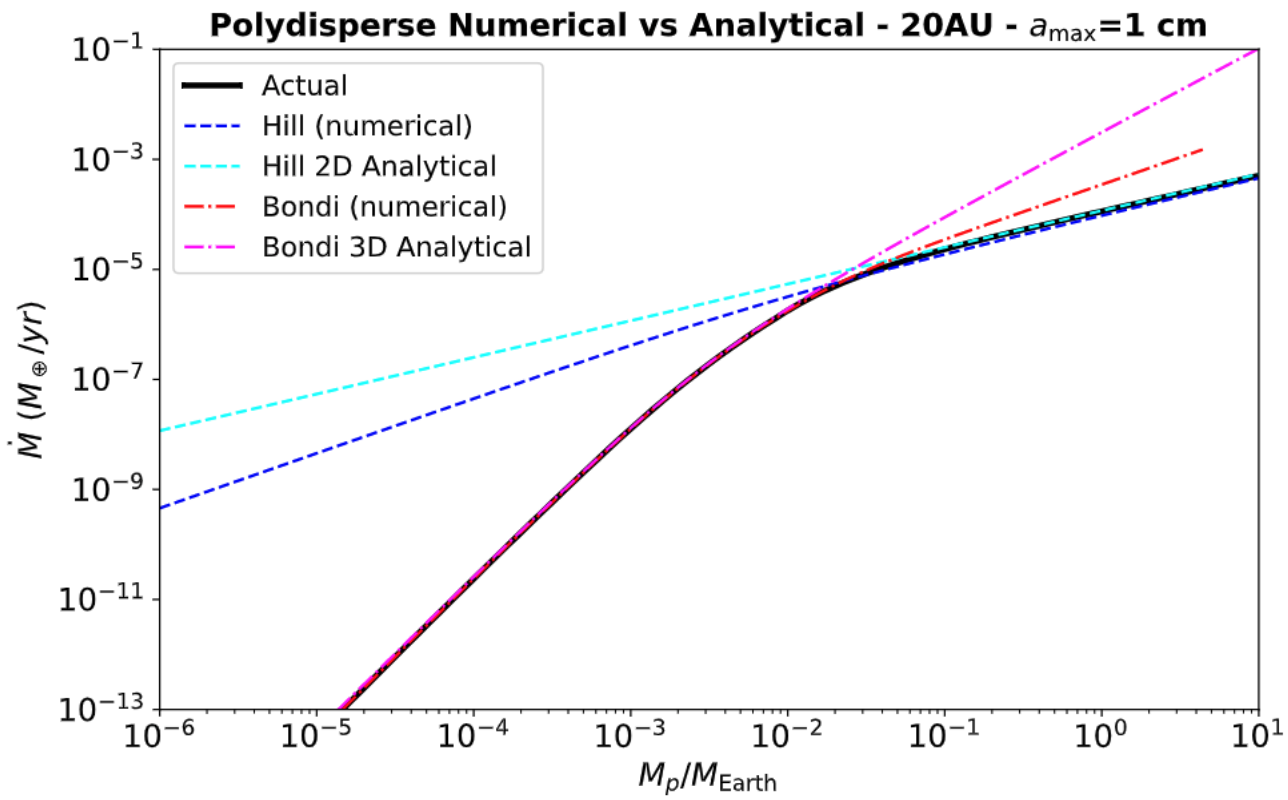
Lambrechts & Johansen (2012)

Polydisperse (multiple species)

$$\dot{M}_{\rm 2D,Hill} = \frac{6(1-p)}{14-5q-3k}\left(\frac{{\rm St}_{\rm max}}{0.1}\right)^{2/3} \Omega R_H^2 Z \Sigma_g.$$

$$\dot{M}_{\rm 3D,Bondi} \approx C_1 \frac{\gamma_l\left(\frac{b_1+1}{s}, j_1 a_{\rm max}^s\right)}{s\, j_1^{(b_1+1)/s}} + C_2 \frac{\gamma_l\left(\frac{b_2+1}{s}, j_2 a_{\rm max}^s\right)}{s\, j_2^{(b_2+1)/s}} + C_3 \frac{\gamma_l\left(\frac{b_3+1}{s}, j_3 a_{\rm max}^s\right)}{s\, j_3^{(b_3+1)/s}} + C_4 \frac{\gamma_l\left(\frac{b_4+1}{s}, j_4 a_{\rm max}^s\right)}{s\, j_4^{(b_4+1)/s}},$$

Lyra et al. (2023)



Lyra et al. 2023

Analytical Solution for
General Monodisperse (single species) Pebble Accretion

$$\dot{M} = \pi R_{\rm acc}^2 \rho_{d0}\, S\, \delta v.$$

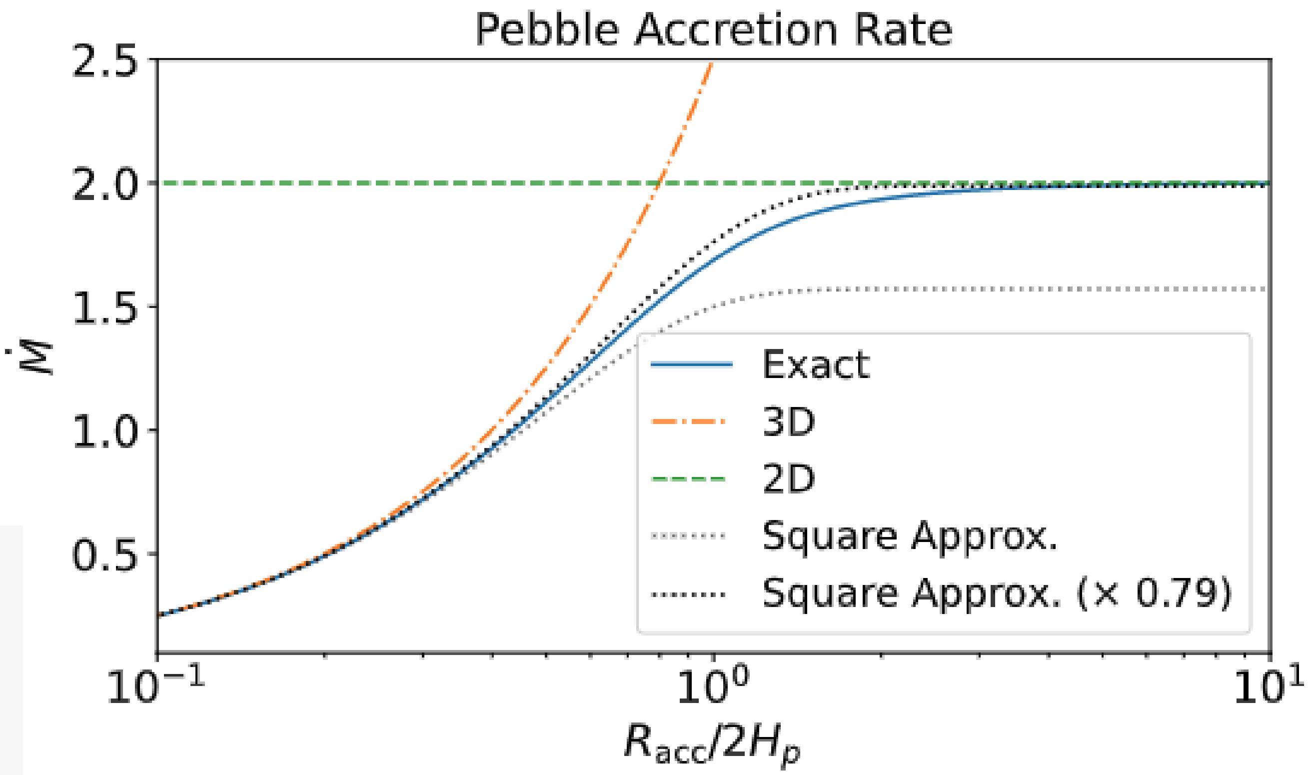
$$S \equiv \frac{1}{\pi R_{\rm acc}^2} \int_{-R_{\rm acc}}^{R_{\rm acc}} 2\sqrt{R_{\rm acc}^2 - z^2}\, \exp\left(-\frac{z^2}{2H_d^2}\right) dz,$$

$$S = e^{-\xi}\left[I_0(\xi) + I_1(\xi)\right], \qquad \xi \equiv \left(\frac{R_{\rm acc}}{2H_d}\right)^2$$

```
y = (x/2)**2

# Modified Bessel function of the first kind of real order.
I0 = sp.special.iv(0, y)
I1 = sp.special.iv(1, y)

Sint = np.exp(-y) * (I0 + I1)
rho_int = rhop * Sint
Mdot = pi*r**2 * rho_int * deltav
```



Lyra et al. 2023

Analytical Solutions for
2D and 3D Polydisperse (multi-species) Pebble Accretion

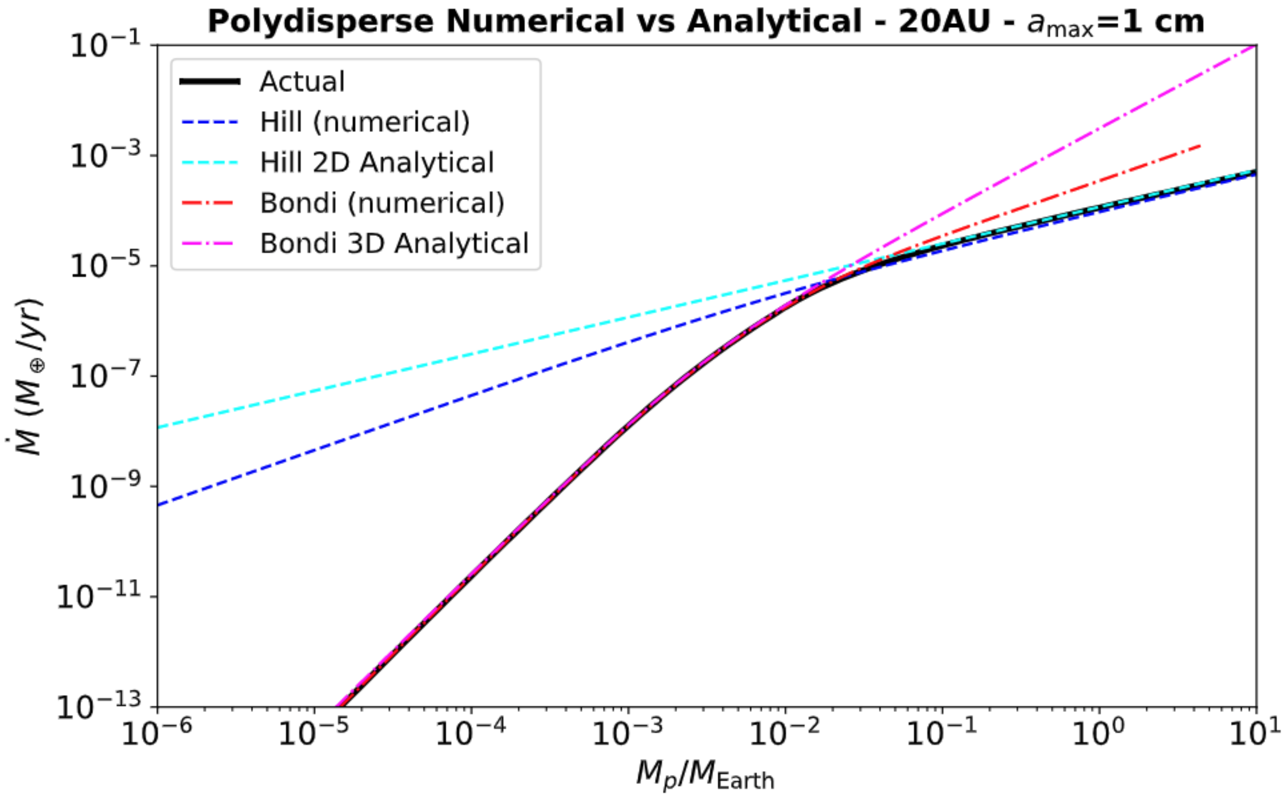
$$\dot{M}_{\rm 2D,Hill} = \frac{6(1-p)}{14-5q-3k}\left(\frac{\rm St_{max}}{0.1}\right)^{2/3}\Omega R_H^2 Z\Sigma_g.$$

$$\dot{M}_{\rm 3D,Bondi} \approx C_1 \frac{\gamma_l\left(\frac{b_1+1}{s}, j_1 a^s_{\rm max}\right)}{s j_1^{(b_1+1)/s}} + C_2 \frac{\gamma_l\left(\frac{b_2+1}{s}, j_2 a^s_{\rm max}\right)}{s j_2^{(b_2+1)/s}} + C_3 \frac{\gamma_l\left(\frac{b_3+1}{s}, j_3 a^s_{\rm max}\right)}{s j_3^{(b_3+1)/s}} + C_4 \frac{\gamma_l\left(\frac{b_4+1}{s}, j_4 a^s_{\rm max}\right)}{s j_4^{(b_4+1)/s}}$$

```
gamma11 = sp.special.gammainc((b1+1)/s,j1*a**s)*sp.special.gamma((b1+1)/s)
gamma12 = sp.special.gammainc((b2+1)/s,j2*a**s)*sp.special.gamma((b2+1)/s)
gamma13 = sp.special.gammainc((b3+1)/s,j3*a**s)*sp.special.gamma((b3+1)/s)
gamma14 = sp.special.gammainc((b4+1)/s,j4*a**s)*sp.special.gamma((b4+1)/s)

G1 = C1*gamma11/s/j1**((b1+1)/s)
G2 = C2*gamma12/s/j2**((b2+1)/s)
G3 = C3*gamma13/s/j3**((b3+1)/s)
G4 = C4*gamma14/s/j4**((b4+1)/s)

Mbondi3d = G1 + G2 + G3 + G4
```



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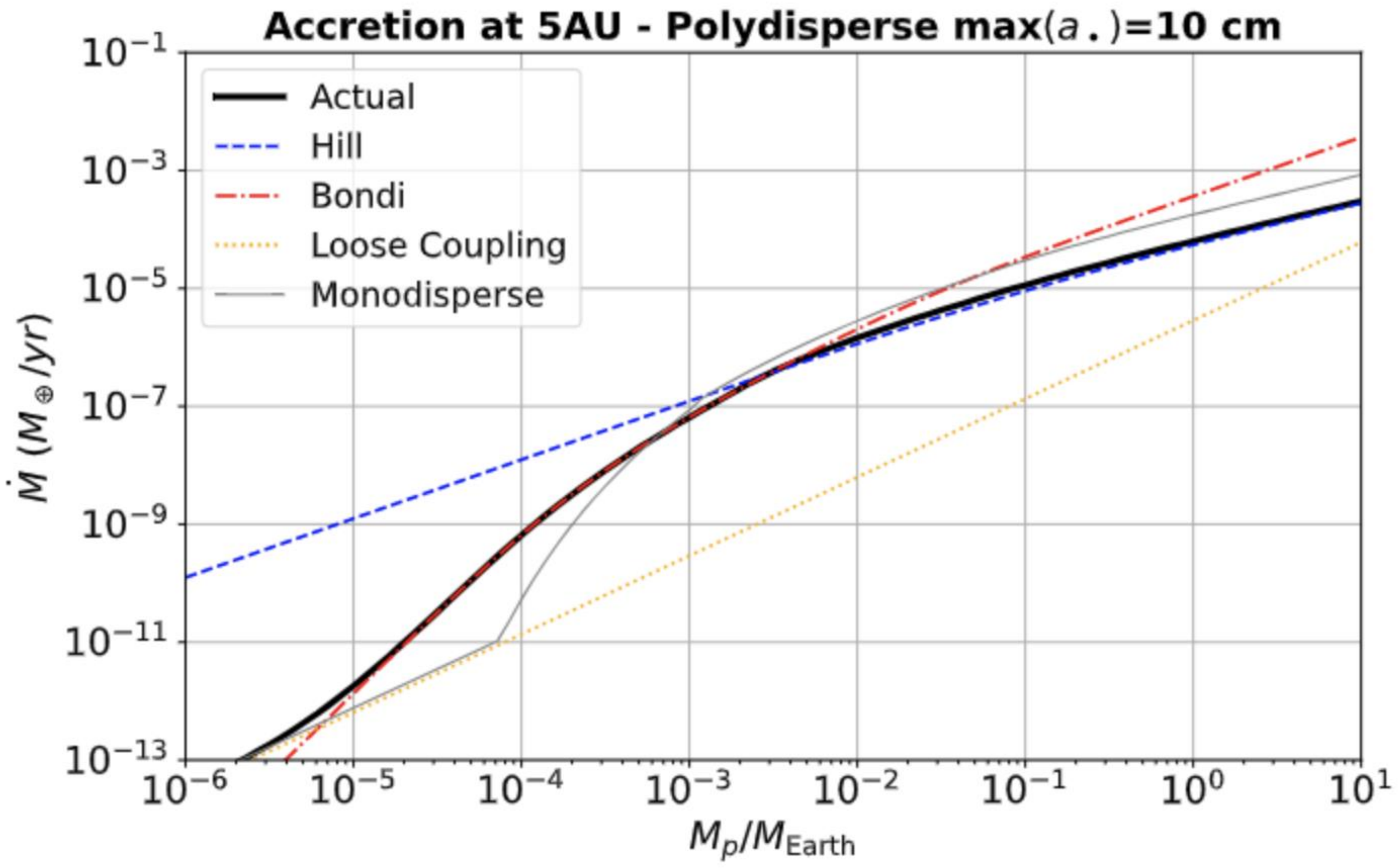
Accretion Rates



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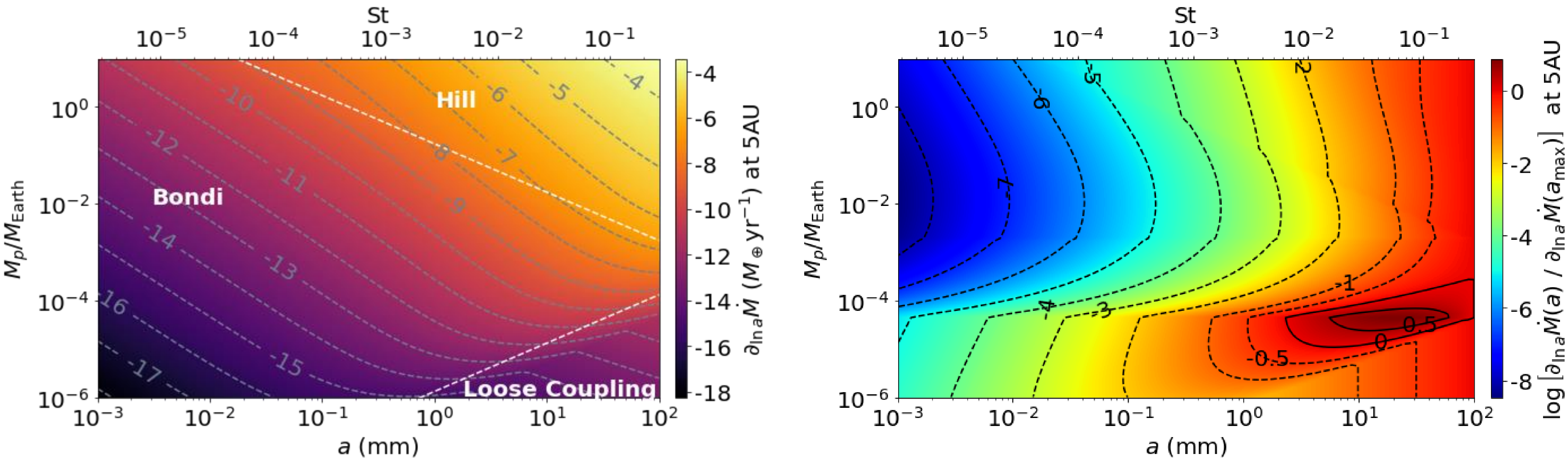
Accretion Rates



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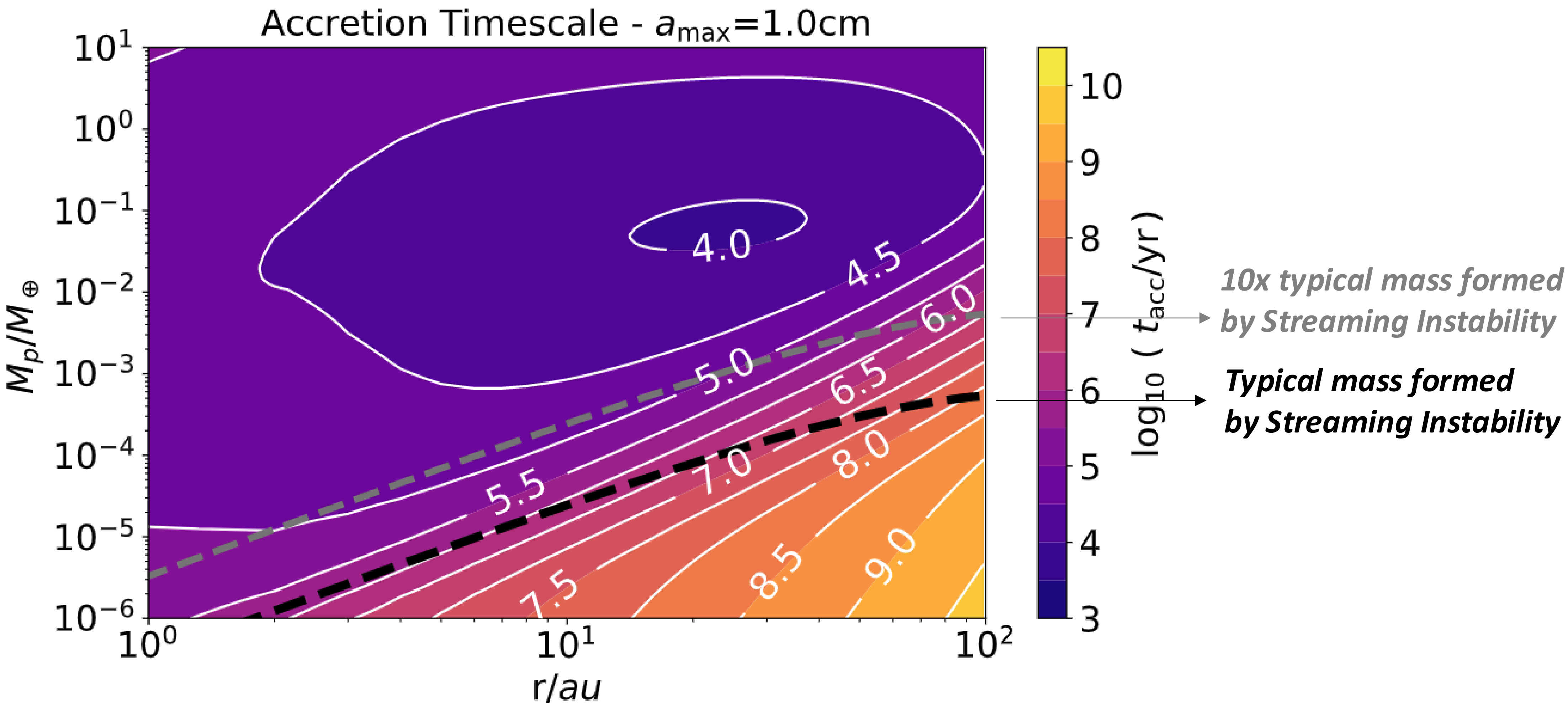
Accretion Timescales

Myr accretion timescales possible on top of planetesimals produces by Streaming Instability



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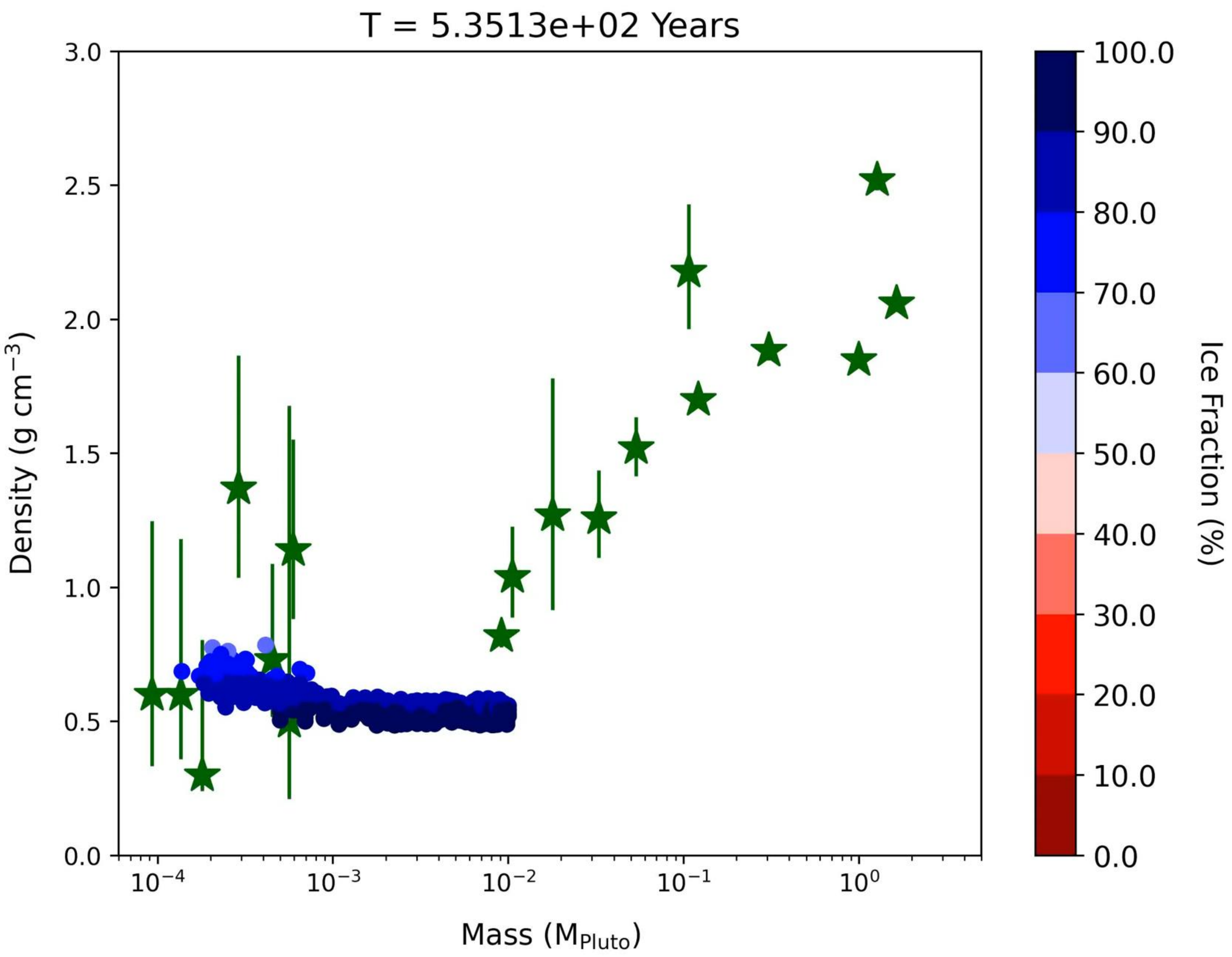
Growing Pluto by silicate pebble accretion



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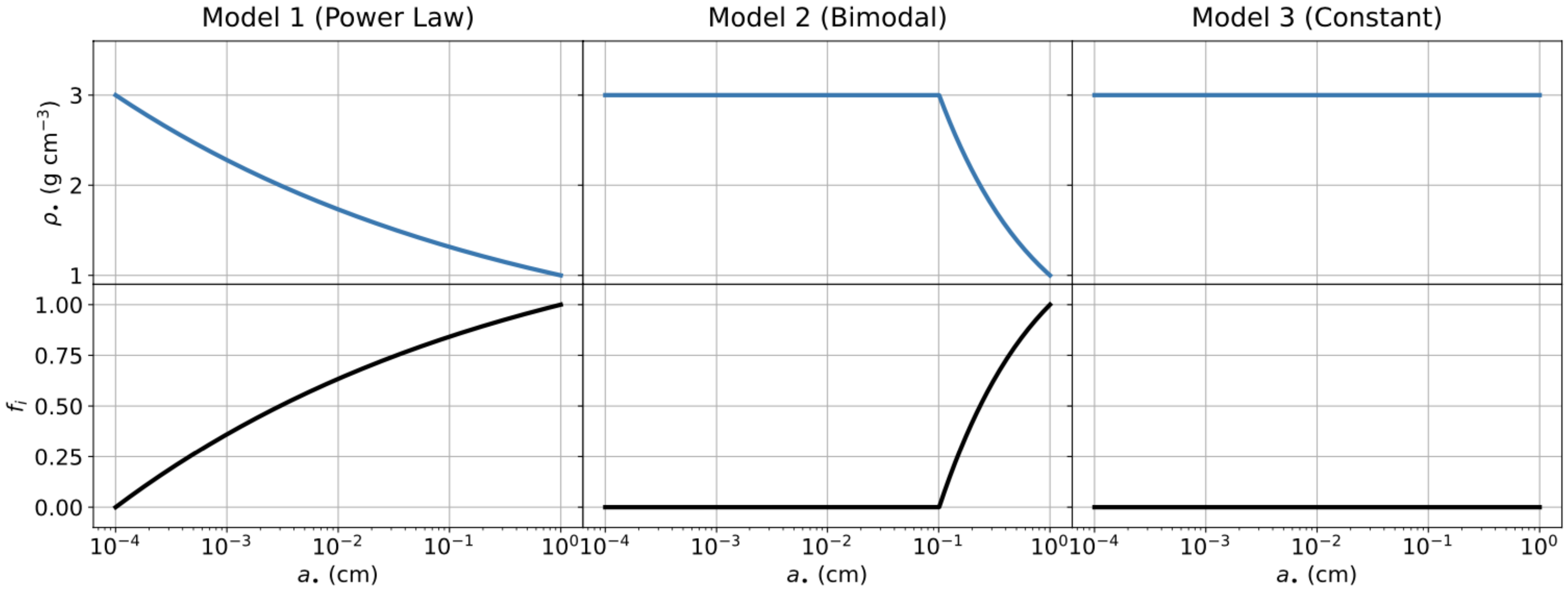
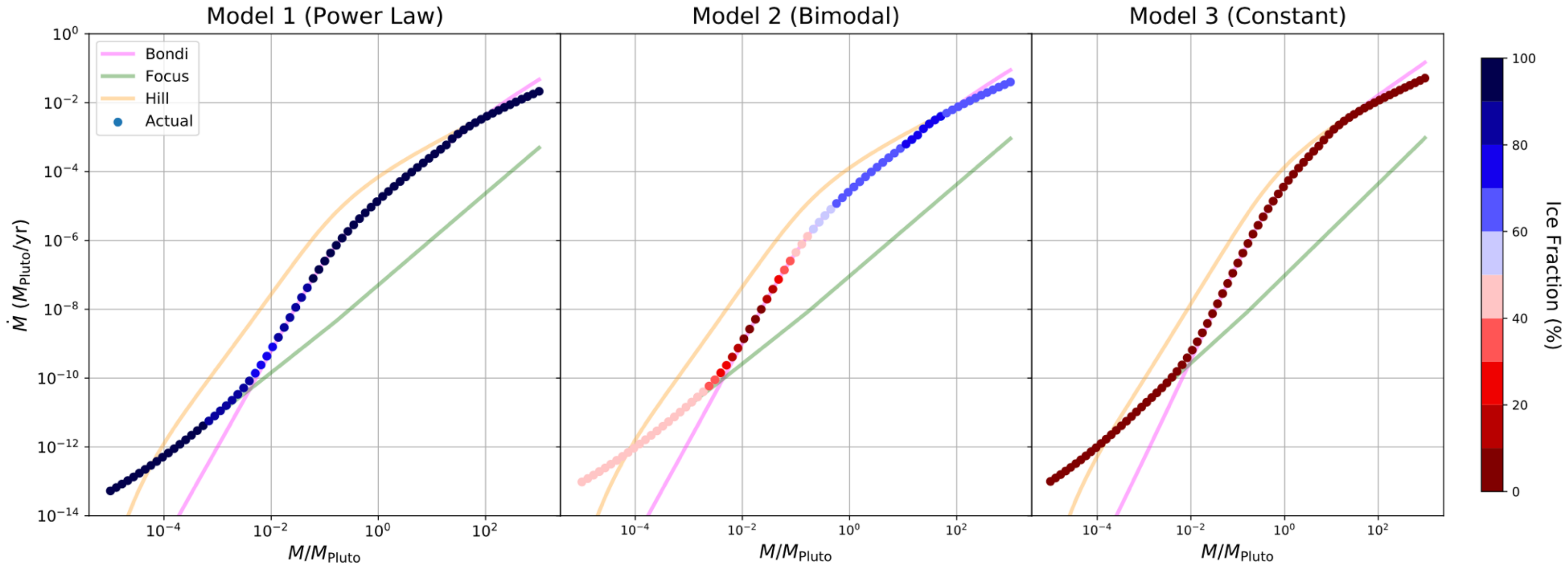
Pebble Internal Density

Ice Volume Fraction



Mass Accretion rate



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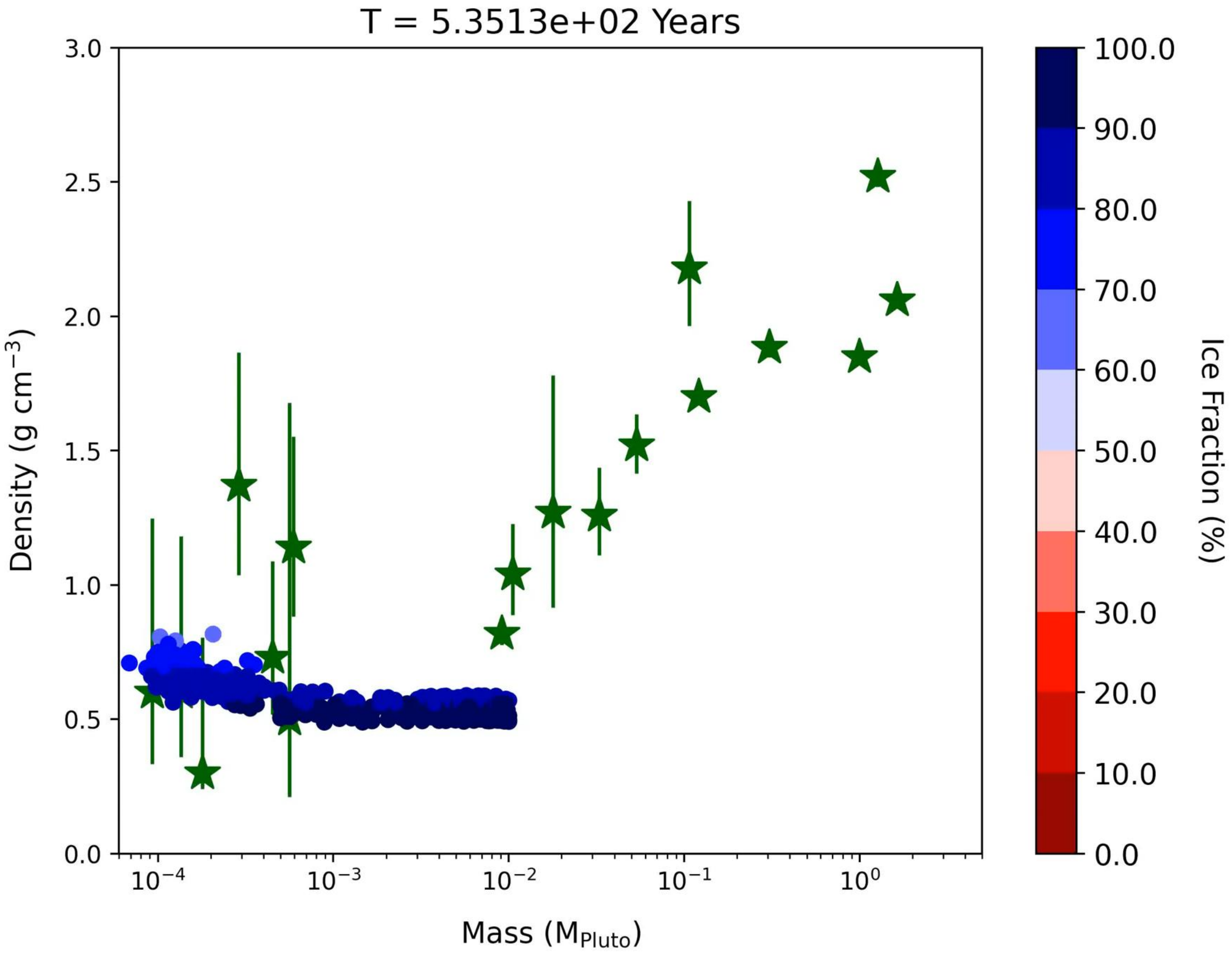
Growing Pluto by silicate pebble accretion



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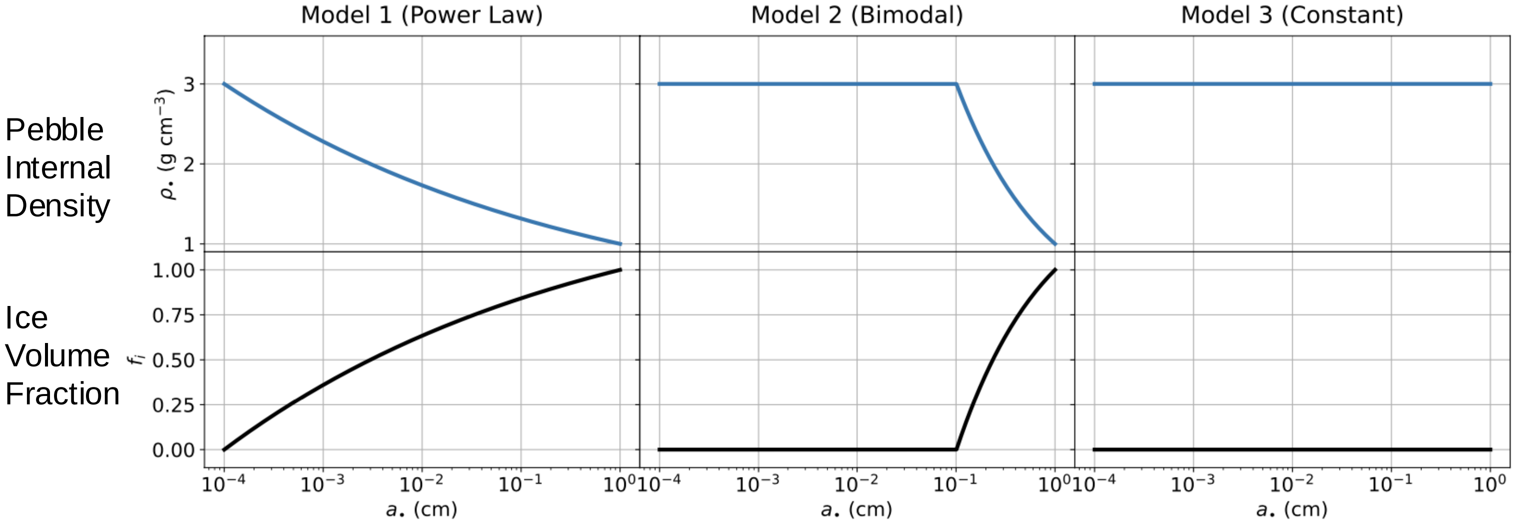
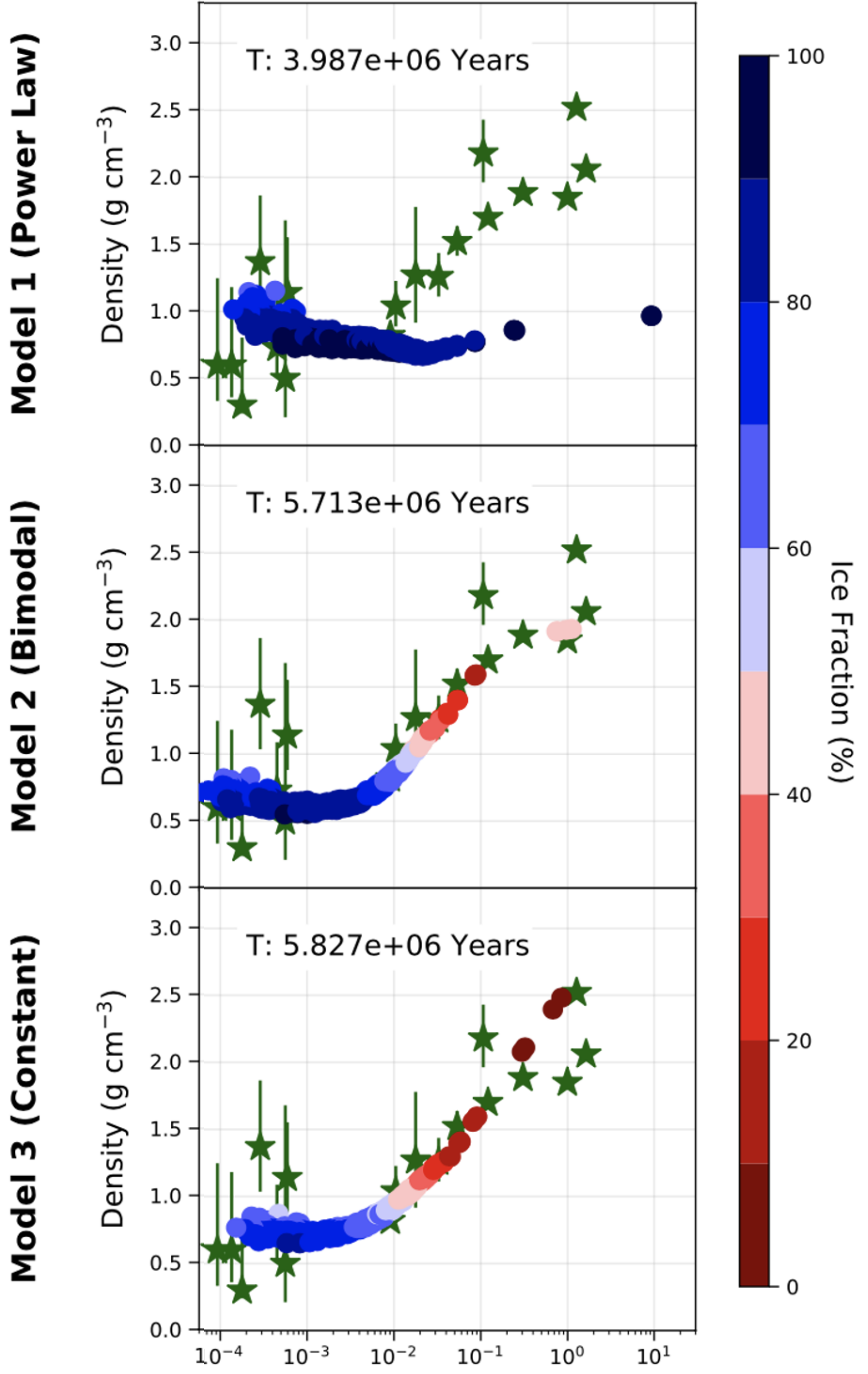
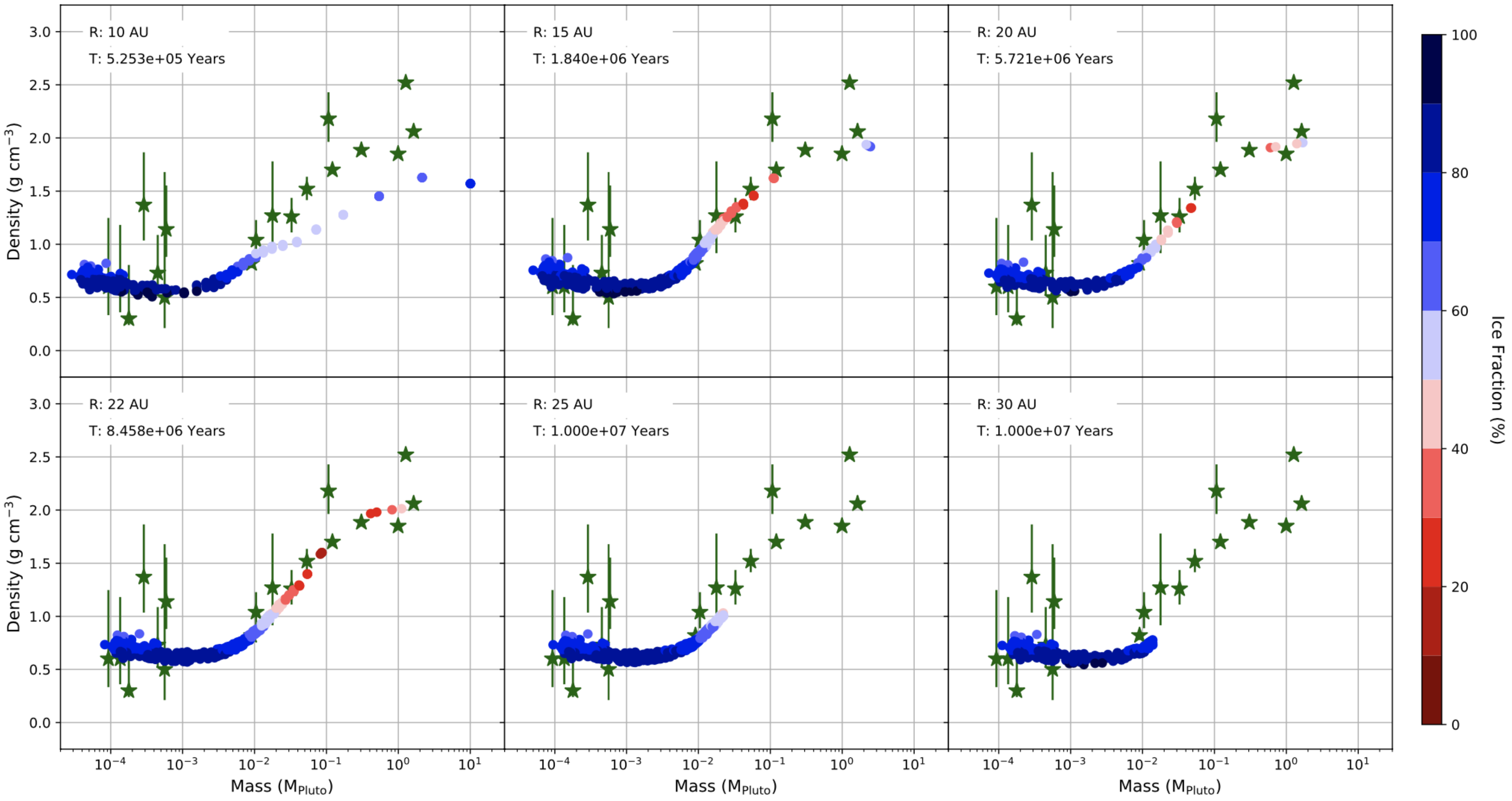
Resulting Densities vs Mass relations

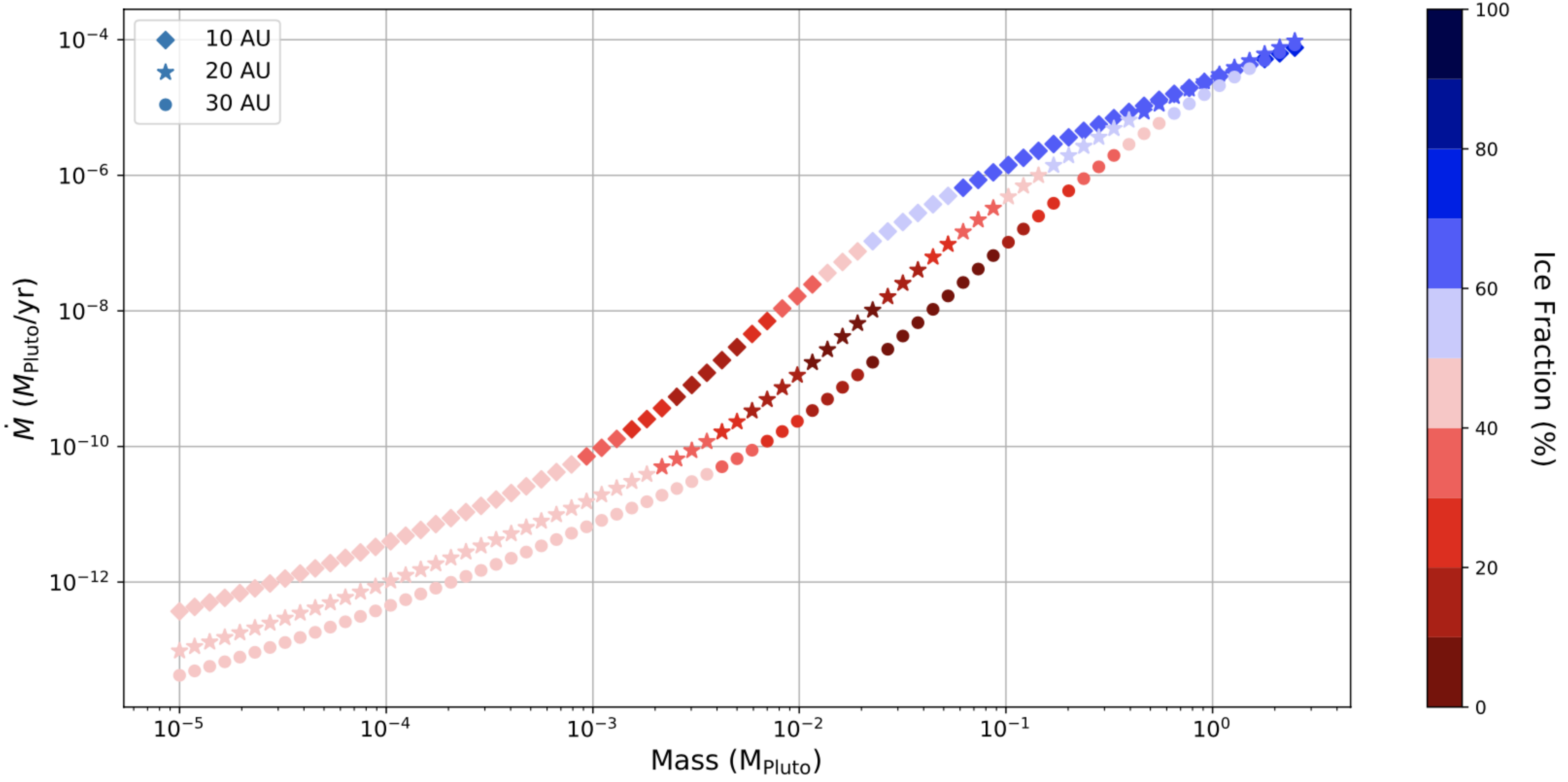




Distance Range
15 - 25AU



The window of silicate accretion



Conclusions

- Polydisperse Bondi accretion 1-2 orders of magnitude more efficient than monodisperse
  - Best accreted pebbles are those of drag time ~ Bondi time, not the largest ones
  - The largest ones dominate the mass budget, but accrete poorly
- Onset of Bondi accretion 1-2 orders of magnitude lower in mass compared to monodisperse
  - Bondi accretion possible on top of Streaming Instability planetary embryos within disk lifetime
  - Reaches 100-350km objects within Myr timescales
- Analytical solution to
  - Monodisperse general case
  - Polydisperse 2D Hill and 3D Bondi
- KBO density problem:
  - Two different pebble populations, maintained by ice desorption off small grains
  - Streaming instability: icy-rich small objects; nearly uniform composition
  - Polydisperse pebble accretion: silicate-rich larger objects; varied composition
  - Melting avoided by
    - ice-rich formation
    - ^{26}Al incorporated mostly in long (>Myr) phase of silicate accretion
  - KBOs best reproduced between 15-25 AU