Polydisperse Pebble Accretion

Doing away with planetesimal accretion

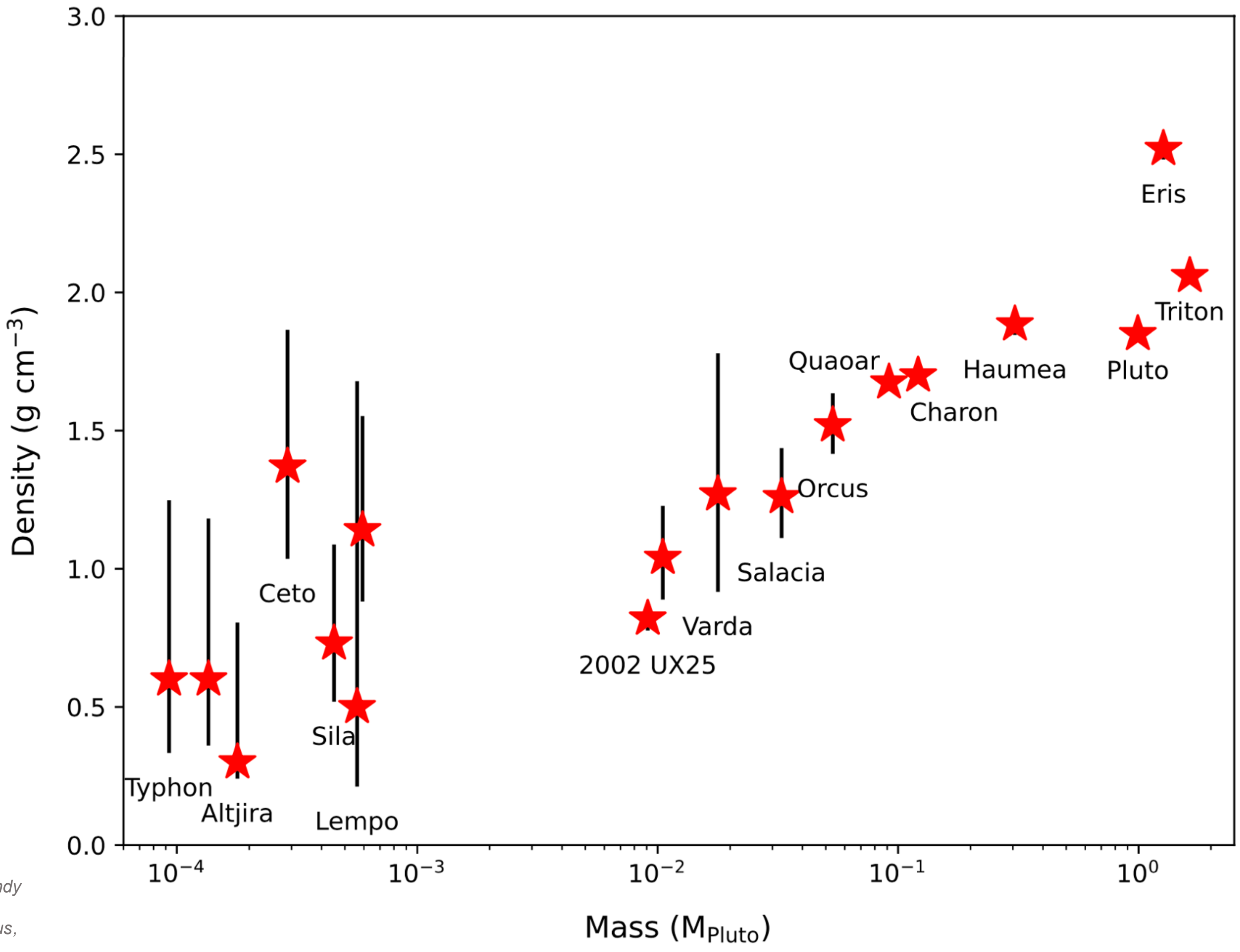
Wladimir Lyra

New Mexico State University

Las Cruces NM, USA

Workshop on Planets Around M-Dwarfs – Lund, Sweden – Feb 3rd-5th, 2026

The size-density relationship of Kuiper Belt objects



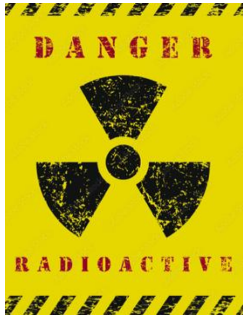
Data; *Thomas (2000), Stansberry et al. (2006), Grundy et al. (2007), Brown et al. (2011), Stansberry et al. (2012), Brown (2013), Fornasier et al. (2013), Vilenius, et al. (2014), Nimmo et al. (2016), Ortiz et al. (2017), Brown and Butler (2017), Grundy et al. (2019), Morgado et al. (2023), Pereira et al. (2023).*

Cañas & Lyra + (2024)

Previous best bet:
Porosity removal by gravitational compaction

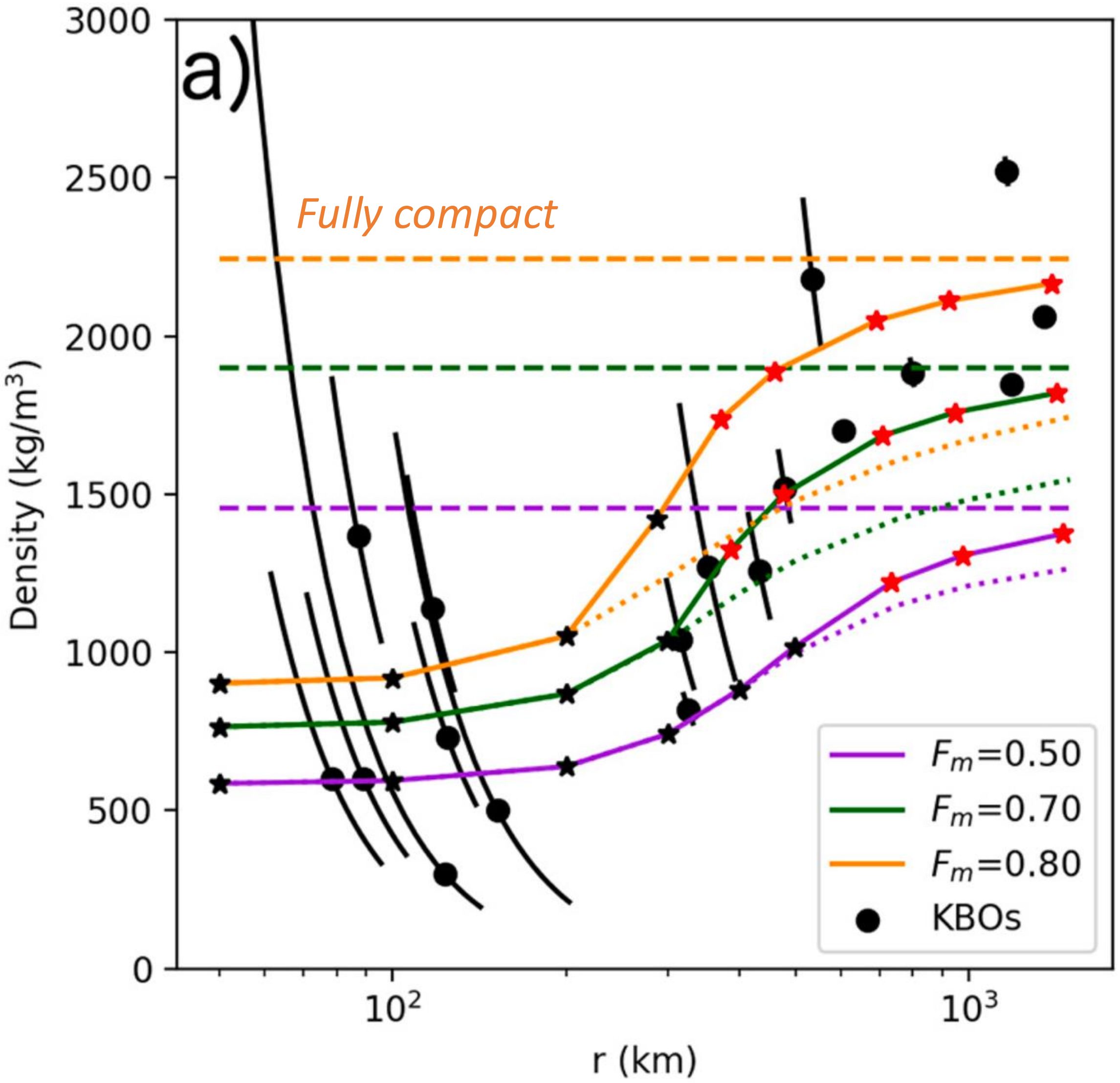
Problem

- Timing! ^{26}Al would melt if formed within 4 Myr



Assumptions

- ~~Constant composition at birth and growth~~
- ~~Growth by planetesimal accretion~~

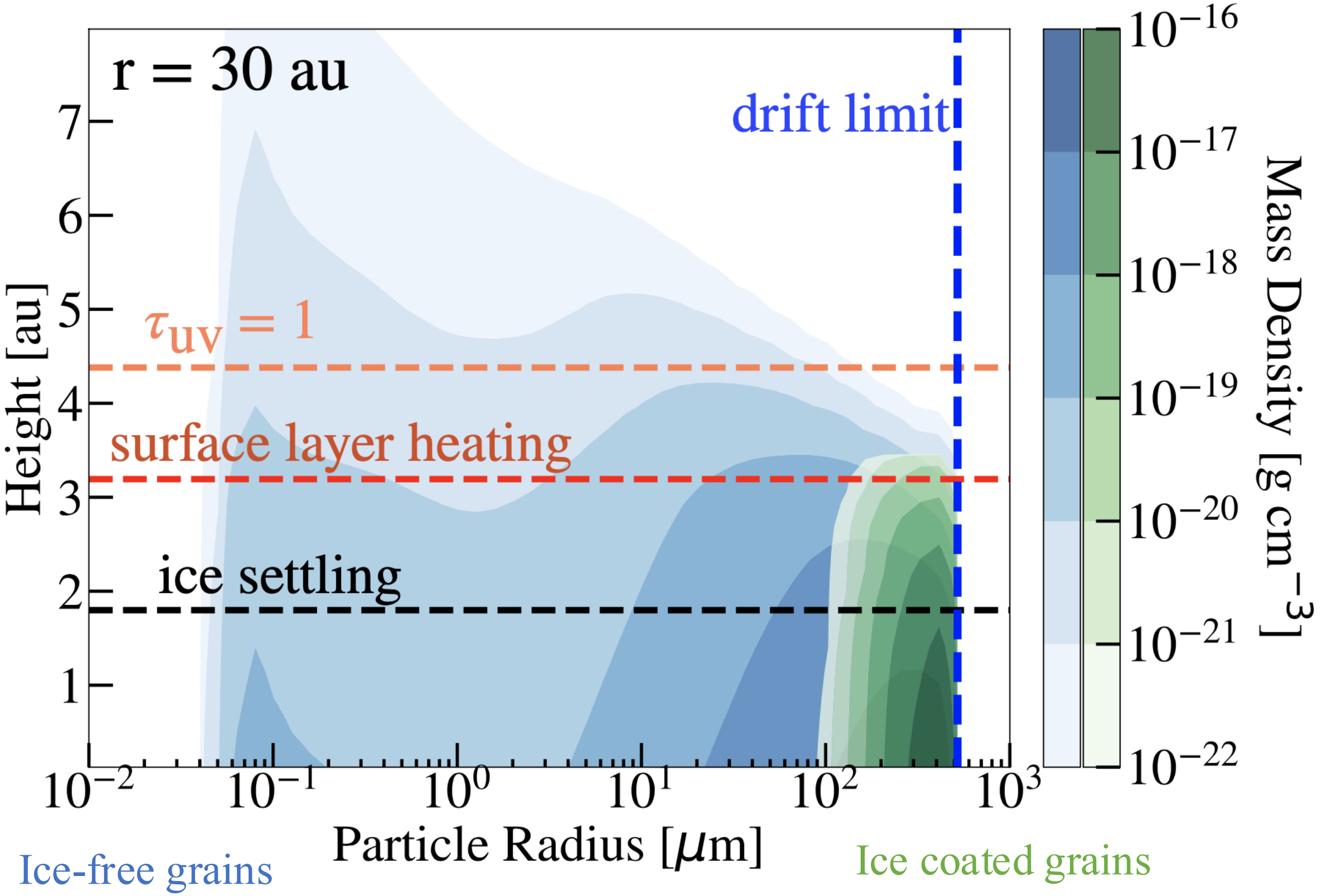


F_m = rock mass fraction

Bierson & Nimmo (2019)

Thermal and photo-desorption

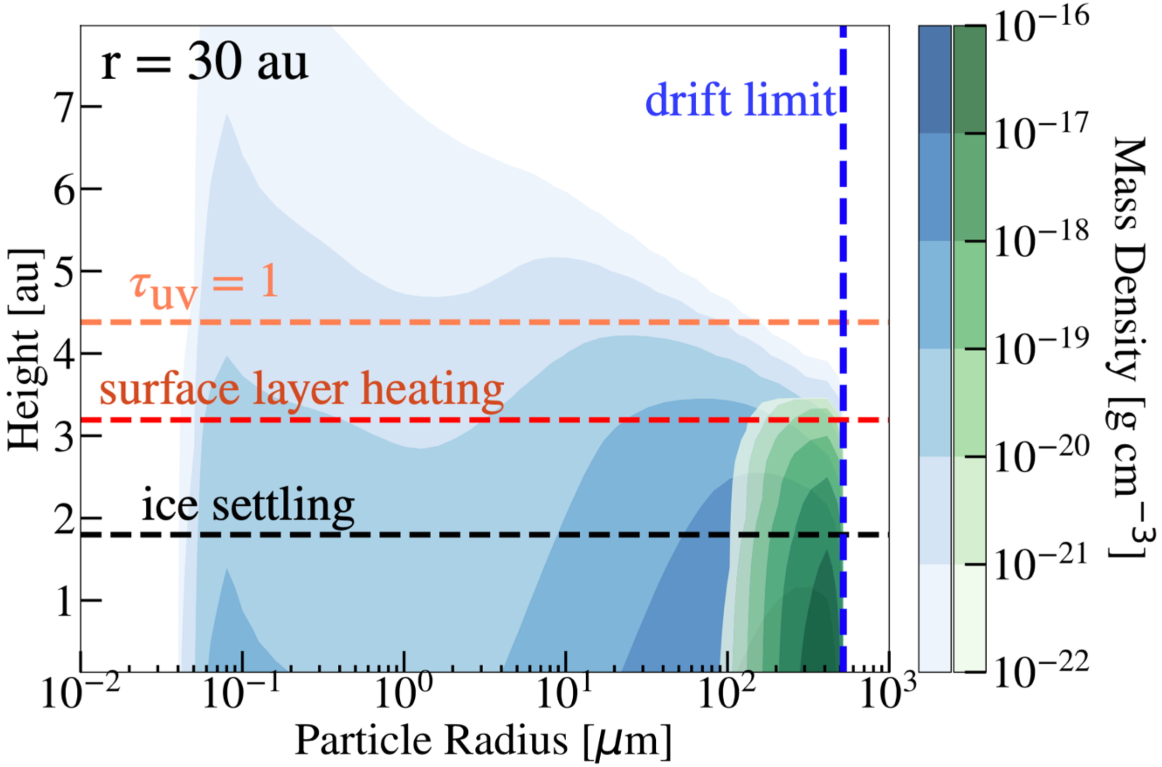
Heating and UV irradiation remove ice
on Myr timescales (Harrison & Schoen 1967).



Powell et al. (2022)

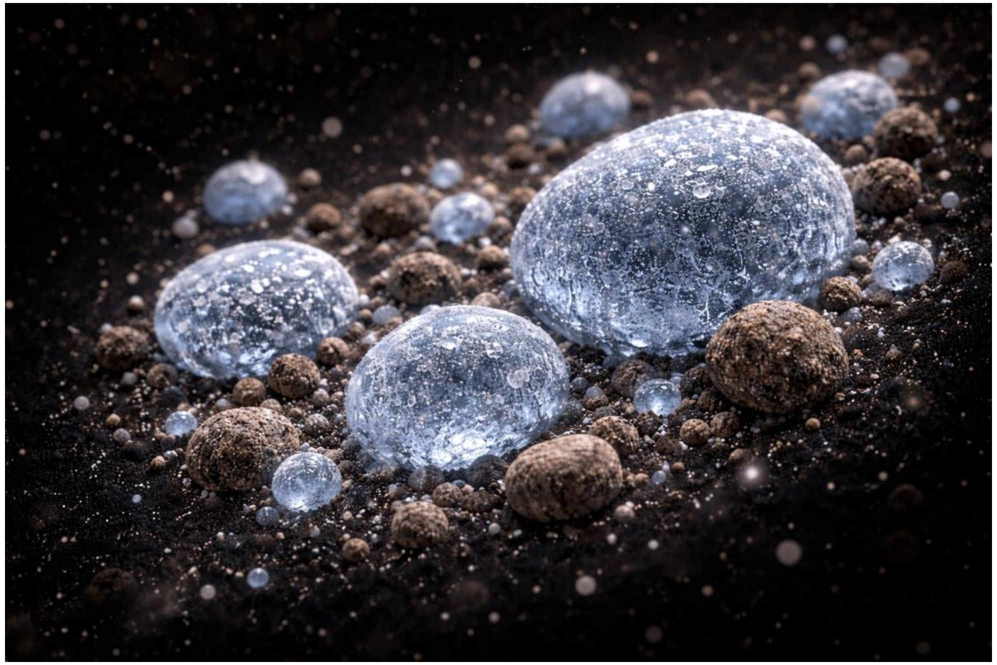
Proof-of-concept model

Small grains lofted in the atmosphere lose ice
Big grains are shielded and remain icy.



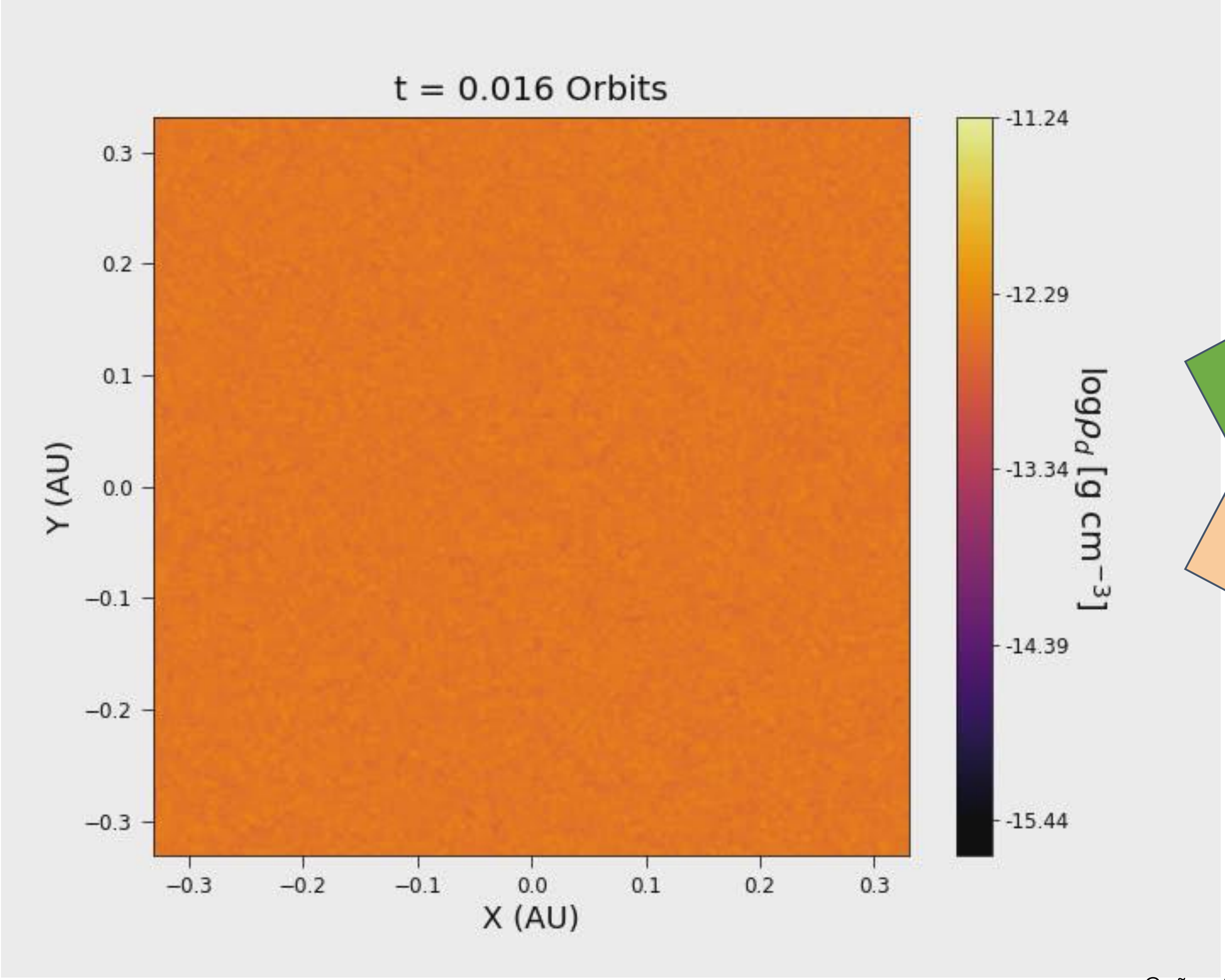
Ice-free grains

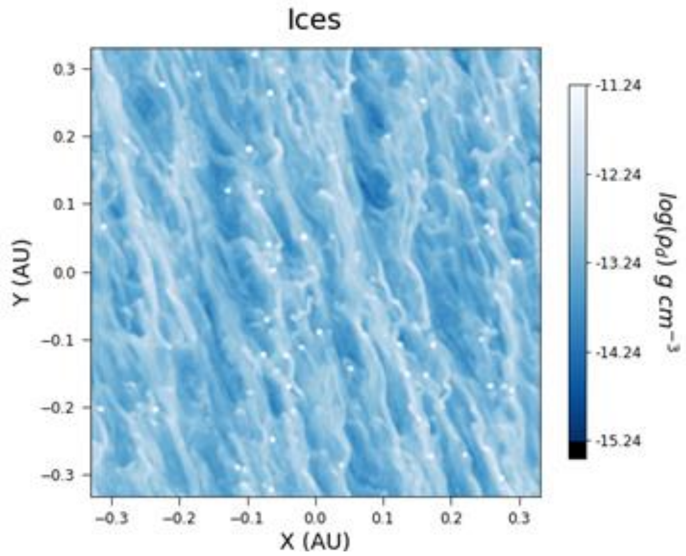
Ice coated grains

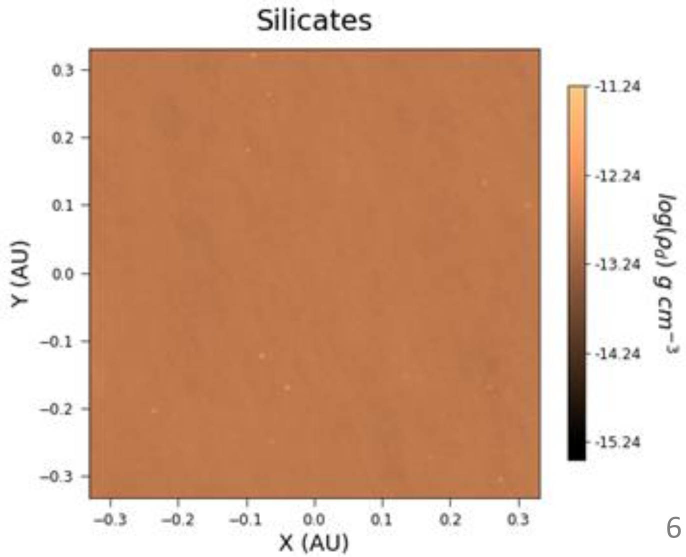


Powell et al. (2022)

Split into icy and silicate pebbles

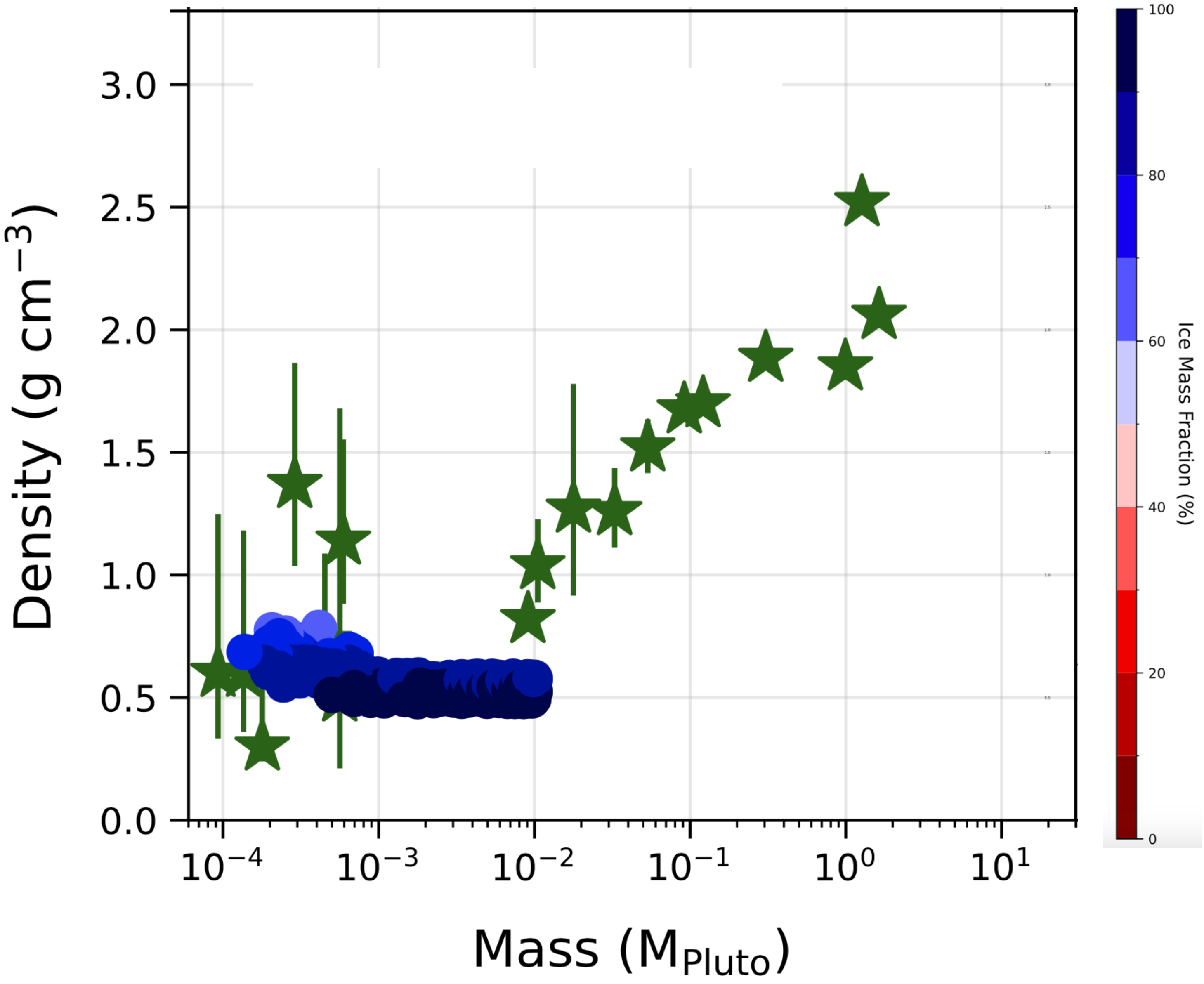






Cañas & Lyra + (2024)

The first planetesimals are icy



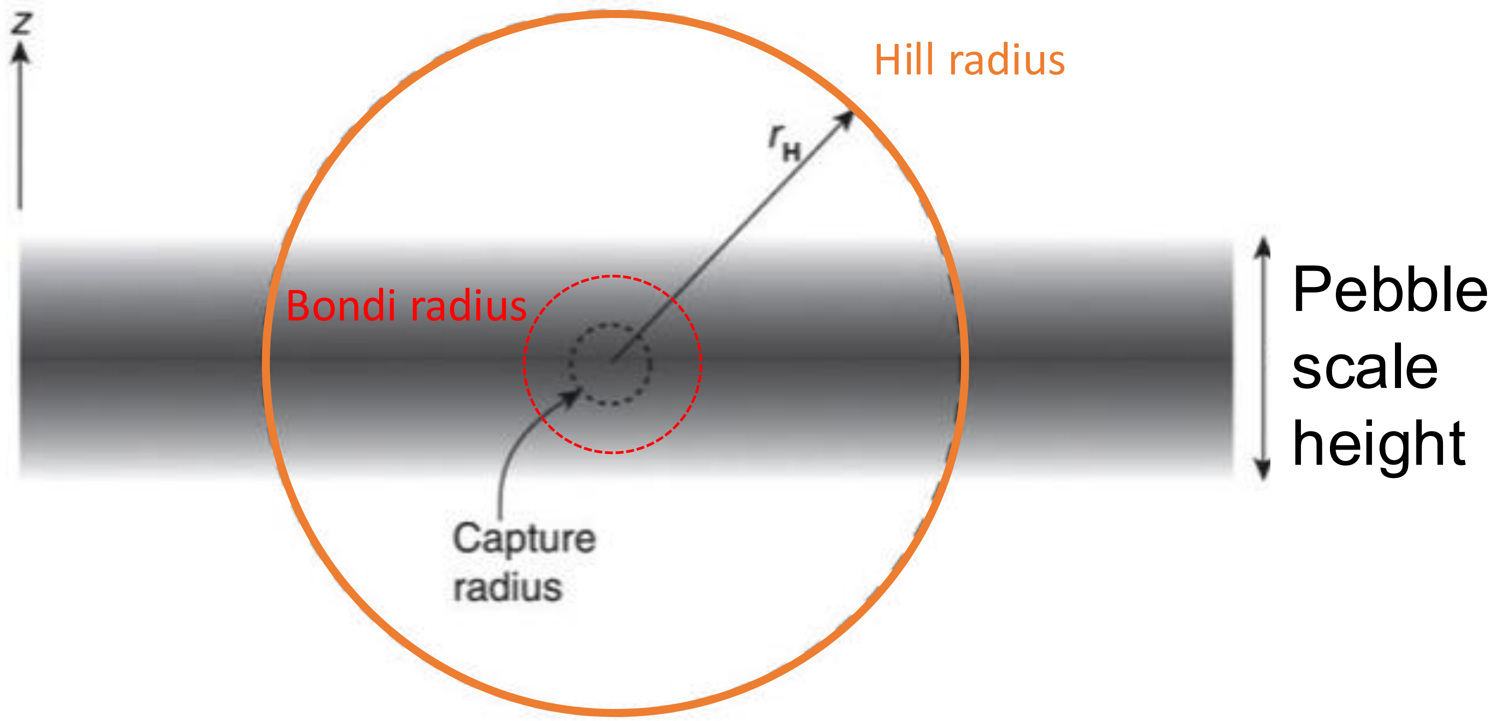
Cañas & Lyra + (2024)

Pebble Accretion: Geometric, Bondi, and Hill regime

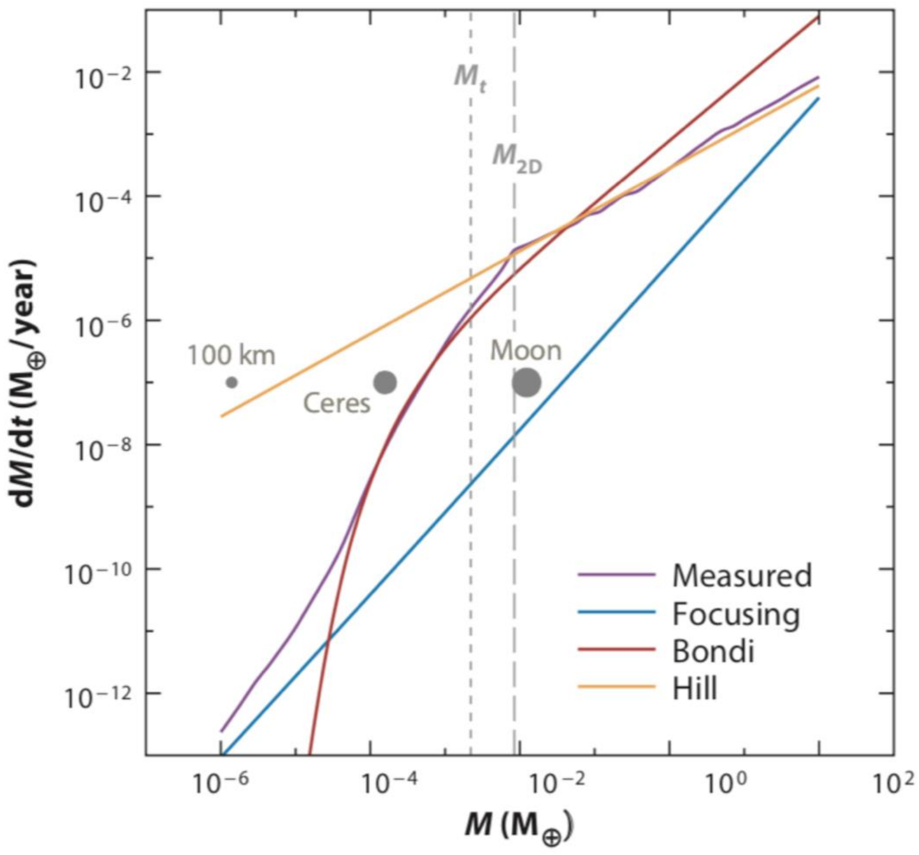
Bondi accretion - Bound against headwind

Hill accretion - Bound against stellar tide

$$\xi \equiv \left(\frac{R_{acc}}{2H_d}\right)^2 \qquad \dot{M}_{3D} = \lim_{\xi \to 0} \dot{M} = \pi R_{acc}^2 \rho_{d0} \delta v, \qquad \dot{M}_{2D} = \lim_{\xi \to \infty} \dot{M} = 2 R_{acc} \Sigma_d \delta v,$$

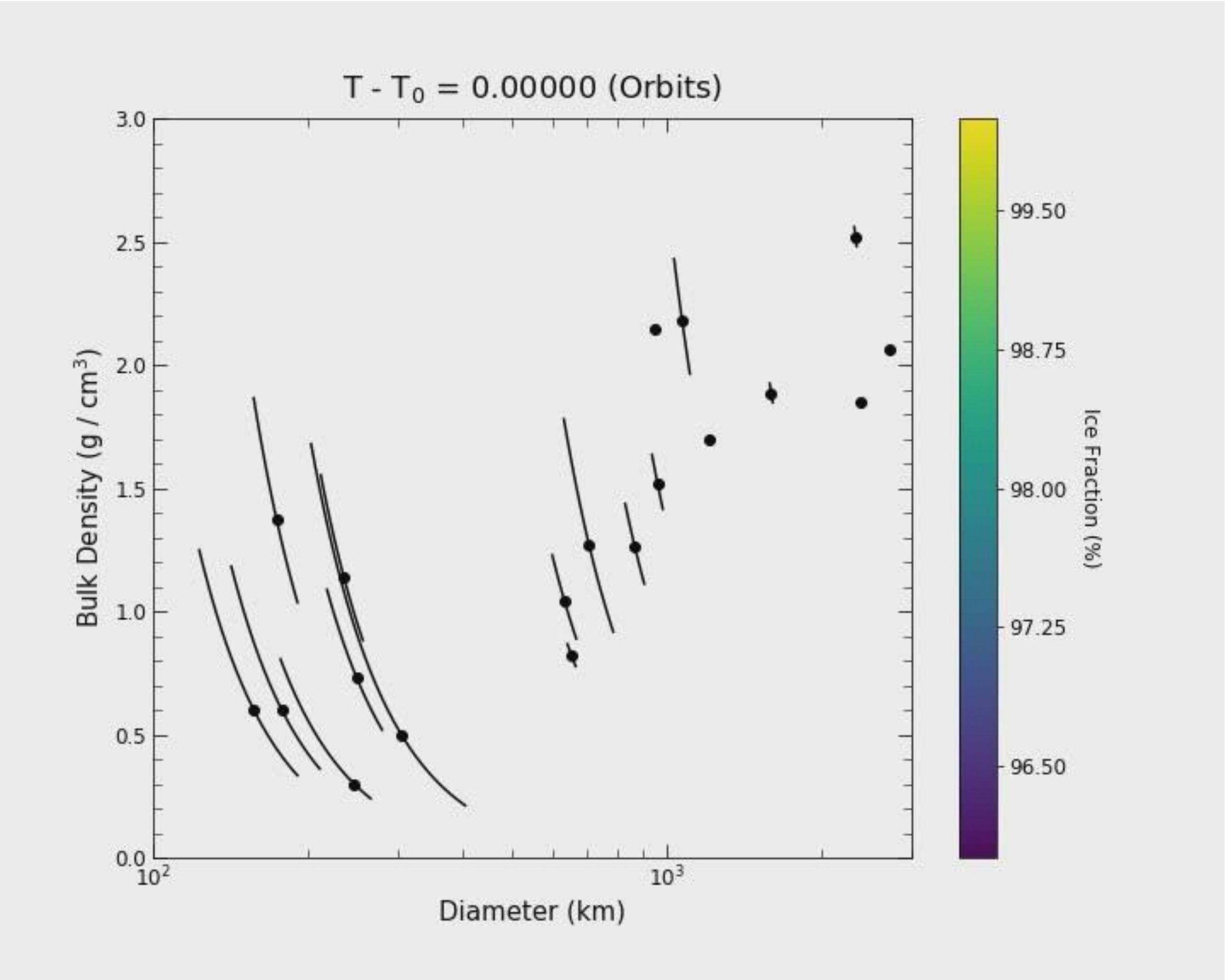


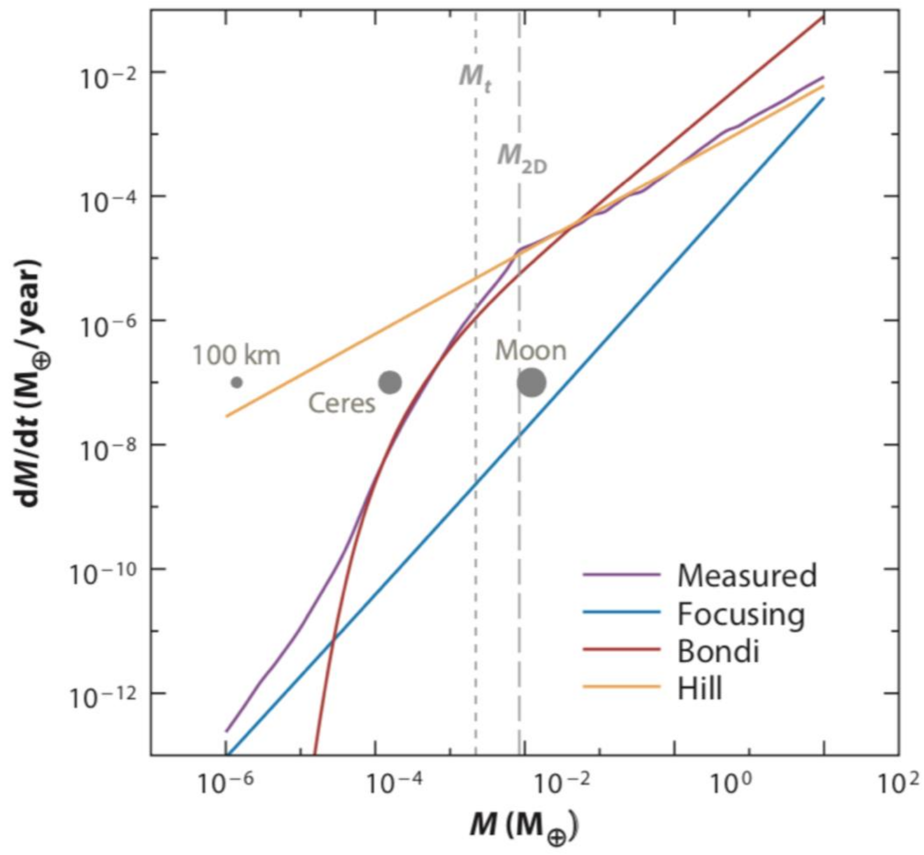
Mass Accretion rates



Johansen & Lambrechts (2017)

Integrate pebble accretion



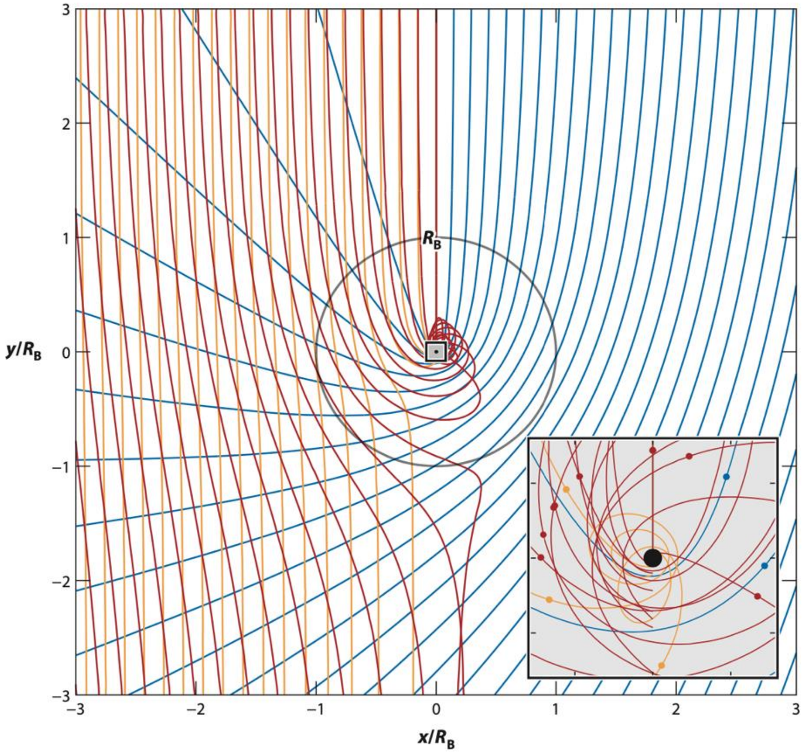


Pebble Accretion: Pebbles of different size accrete differently

Large
Medium
Small

"Goldilocks effect" in the Bondi regime

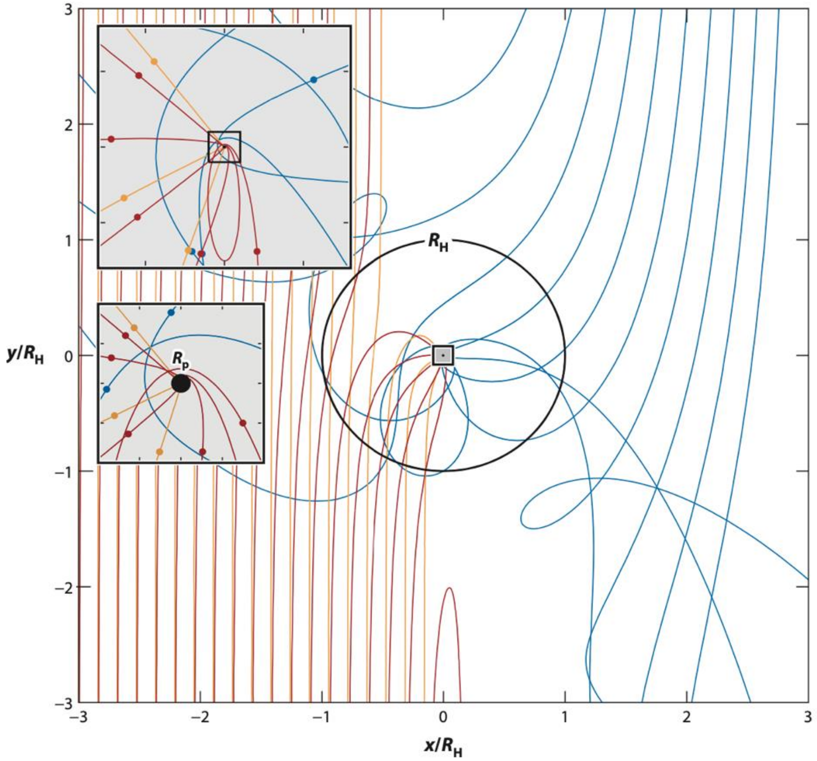
Bondi Regime



Best accreted pebble

Drag time ~ Bondi Time

Hill Regime



Best accreted pebble

Drag time ~ Orbital Time

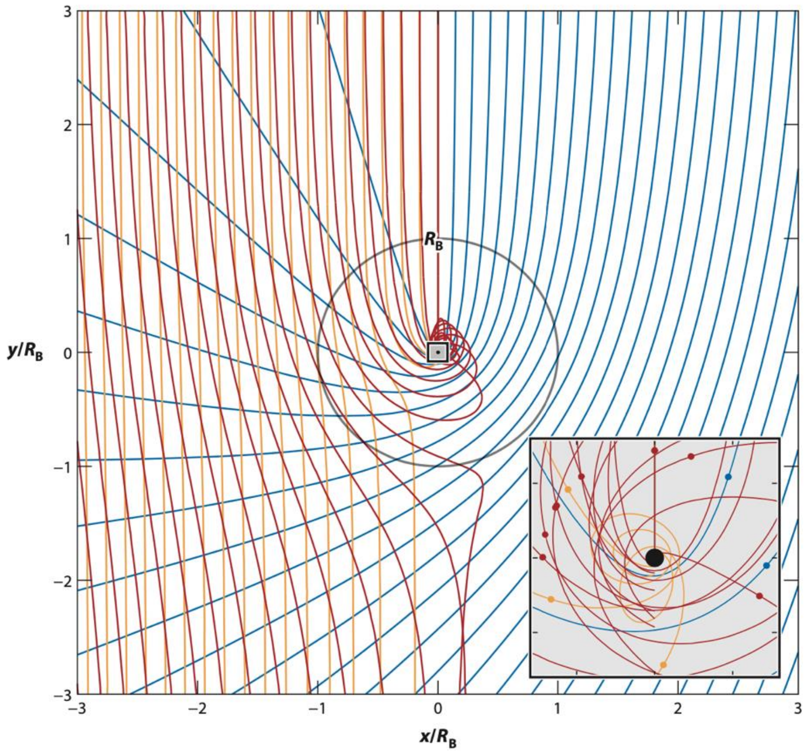
Johansen & Lambrechts (2017); Lyra & Johansen + (2023)

Pebble Accretion: Pebbles of different size accrete differently

“Goldilocks effect”

Bondi Regime

- Large ($t_s >> t_b$)
- Medium ($t_s \sim t_b$)
- Small ($t_s << t_b$)



Pebble size that accretes optimally

Drag time t_s ~ Time to cross Bondi sphere t_b

$$t_b = GM/\Delta v^3$$

Johansen & Lambrechts (2017); Lyra & Johansen + (2023)

Polydisperse (Multi-Species) Pebble Accretion

$$\rho_d(a, z) = \int_0^a m(a')\, F(a', z)\, da'.$$

$$F(a, z) \equiv f(a)\, e^{-z^2/2H_d^2},$$

$$f(a) = \frac{3(1-p)Z\Sigma_g}{2^{5/2}\pi^{3/2}H_g\rho_\bullet^{(0)}a_{\max}^{4-k}}\sqrt{1 + a\frac{\pi}{2}\frac{\rho_\bullet(a)}{\Sigma_g\alpha}}\; a^{-k}.$$

$$\dot{M}(a) = \int_0^a \frac{\partial \dot{M}(a')}{\partial a'} da',$$

$$\frac{\partial \dot{M}(a)}{\partial a} = \pi R_{\rm acc}^2(a)\,\delta v(a)\, S(a) m(a) f(a).$$

$$S \equiv \frac{1}{\pi R_{\rm acc}^2}\int_{-R_{\rm acc}}^{R_{\rm acc}} 2\sqrt{R_{\rm acc}^2 - z^2}\,\exp\left(-\frac{z^2}{2H_d^2}\right) dz,$$

$$W(a) = \frac{3(1-p)Z\Sigma_g}{4\pi\rho_\bullet^{(0)}a_{\max}^{4-k}}\, a^{-k},$$

$$\hat{R}_{\rm acc}^{\rm (Bondi)} = \left(\frac{4\tau_f}{t_{\rm B}}\right)^{1/2} R_{\rm B},$$

$$\delta v \equiv \Delta v + \Omega R_{\rm acc},$$

$$\hat{R}_{\rm acc}^{\rm (Hill)} = \left(\frac{\rm St}{0.1}\right)^{1/3} R_{\rm H},$$

$$R_{\rm acc} \equiv \hat{R}_{\rm acc}\exp[-\chi(\tau_f/t_p)^\gamma],$$

$$\frac{\partial \Sigma_d(a)}{\partial a} \propto a^{-p};$$

$$\rho_\bullet \propto a^{-q};$$

$$t_p \equiv \frac{GM_p}{(\Delta v + \Omega R_{\rm H})^3}$$

$$\dot{M}_{\rm 2D,\,Hill} = 2\times 10^{2/3}\Omega R_H^2 \int_0^{a_{\max}} {\rm St}(a)^{2/3}\, m(a)\, W(a)\, da.$$

$$\dot{M}_{\rm 3D,\,Bondi} = \frac{4\pi R_{\rm B}\Delta v^2}{\Omega} \times \int_0^{a_{\max}} {\rm St}\, e^{-2\psi} m(a) f(a) \times \left[1 + 2\left({\rm St}\frac{\Omega R_{\rm B}}{\Delta v}\right)^{1/2} e^{-\psi}\right] da,$$

$$\psi \equiv \chi[{\rm St}/(\Omega t_p)]^\gamma.$$

Lyra & Johansen + (2023)

Analytical theory of polydisperse (multi-species) pebble accretion

$\xi \equiv \left(\frac{R_{acc}}{2H_d}\right)^2$

Monodisperse (single species)

$$\dot{M}_{3D} = \lim_{\xi \to 0} \dot{M} = \pi R_{acc}^2 \rho_{d0} \delta v,$$

$$\dot{M}_{2D} = \lim_{\xi \to \infty} \dot{M} = 2 R_{acc} \Sigma_d \delta v,$$

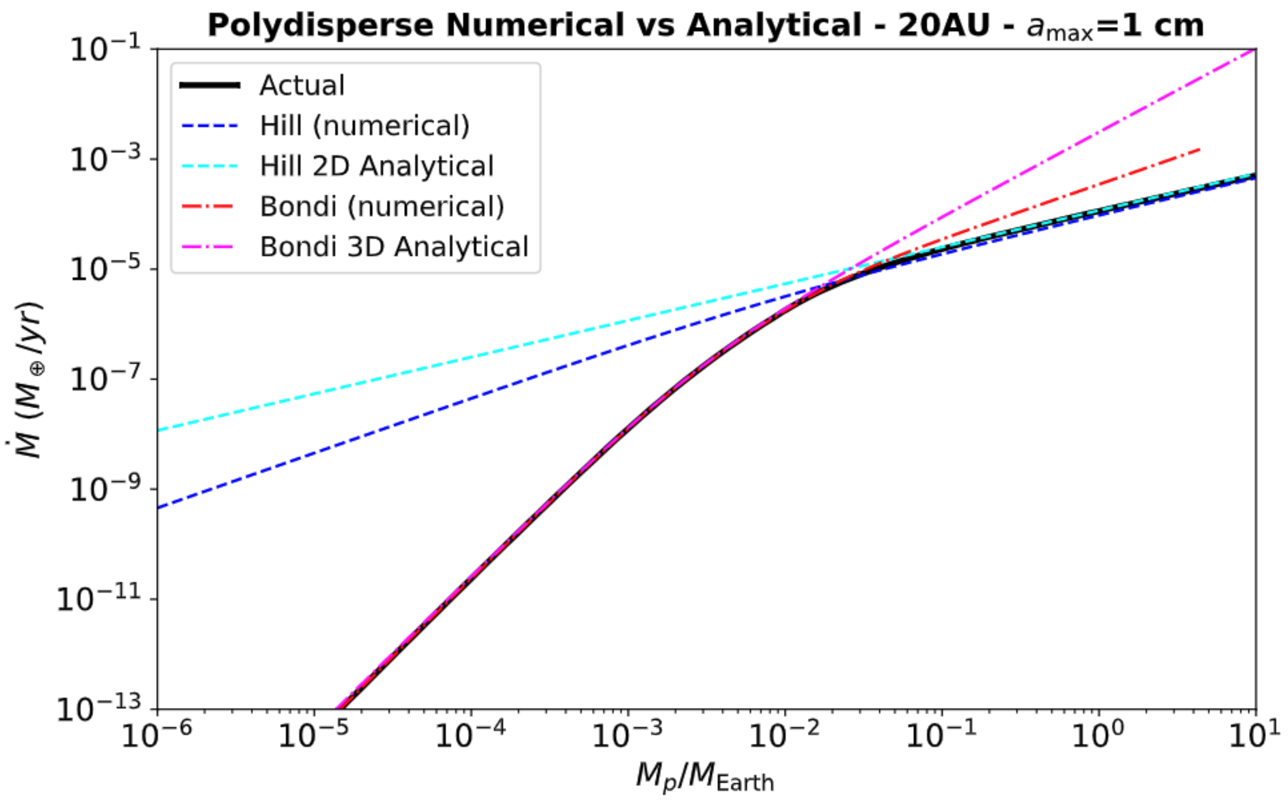
Lambrechts & Johansen (2012)

Polydisperse (multiple species)

$$\dot{M}_{2D,Hill} = \frac{6(1-p)}{14-5q-3k} \left(\frac{St_{max}}{0.1}\right)^{2/3} \Omega R_H^2 Z \Sigma_g.$$

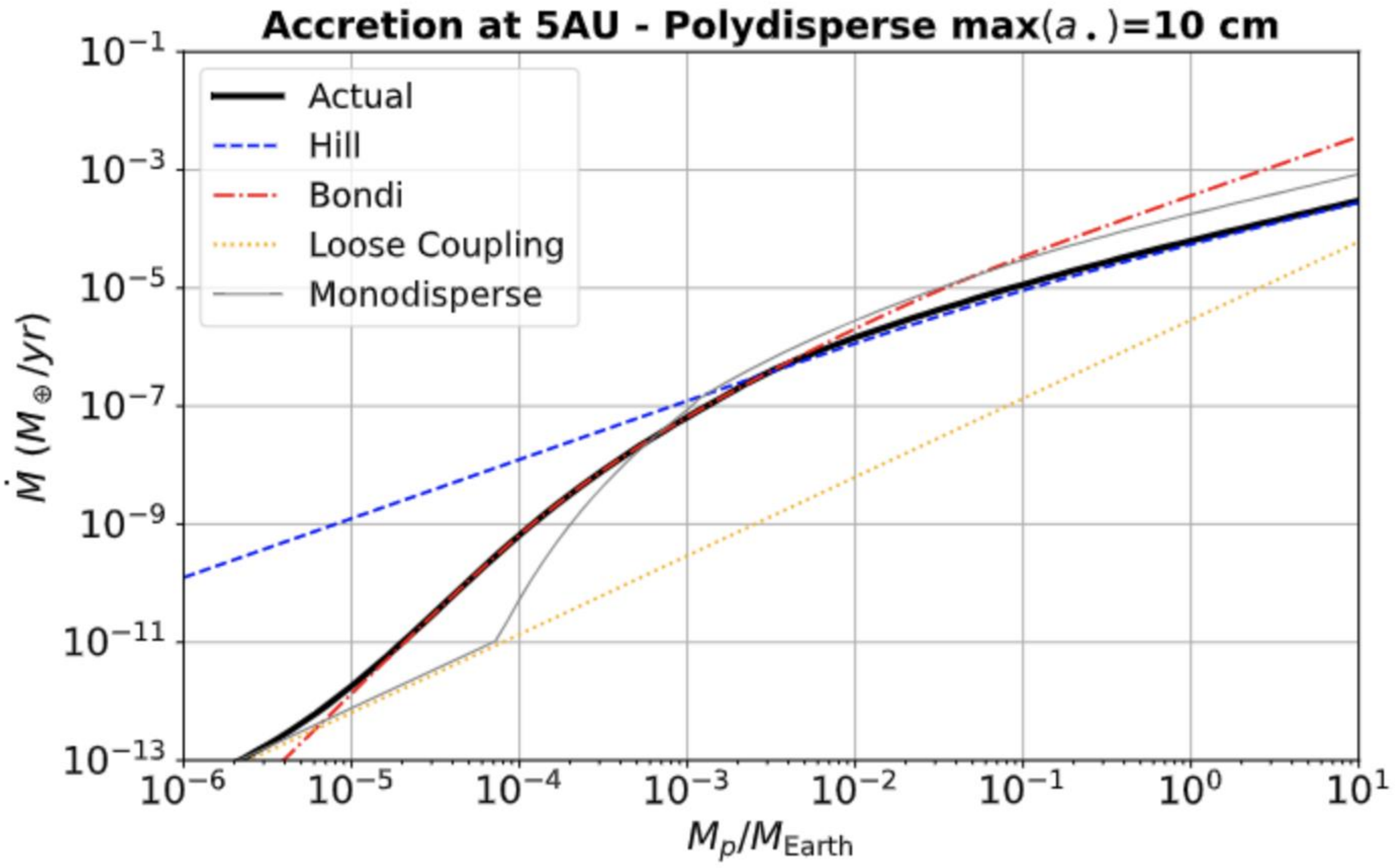
$$\dot{M}_{3D,Bondi} \approx C_1 \frac{\gamma_l\left(\frac{b_1+1}{s}, j_1 a_{max}^s\right)}{s j_1^{(b_1+1)/s}} + C_2 \frac{\gamma_l\left(\frac{b_2+1}{s}, j_2 a_{max}^s\right)}{s j_2^{(b_2+1)/s}} + C_3 \frac{\gamma_l\left(\frac{b_3+1}{s}, j_3 a_{max}^s\right)}{s j_3^{(b_3+1)/s}} + C_4 \frac{\gamma_l\left(\frac{b_4+1}{s}, j_4 a_{max}^s\right)}{s j_4^{(b_4+1)/s}},$$

Lyra & Johansen + (2023)



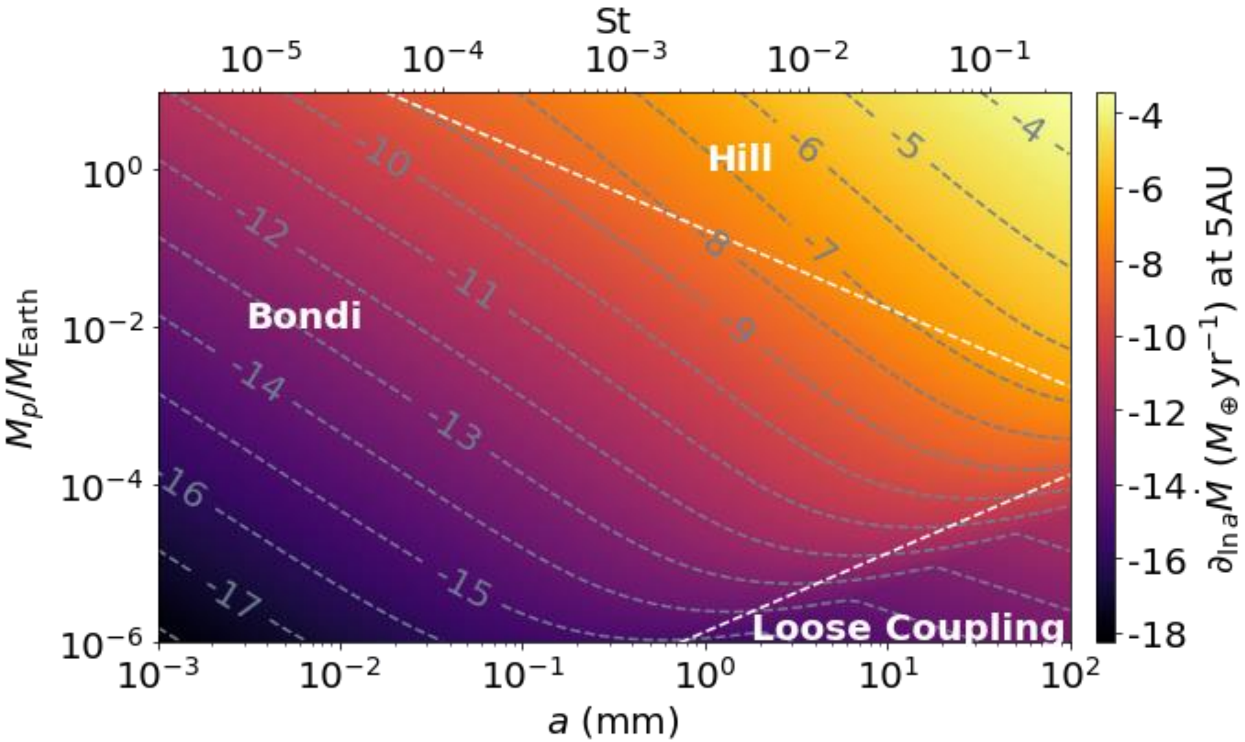
Lyra & Johansen + (2023)

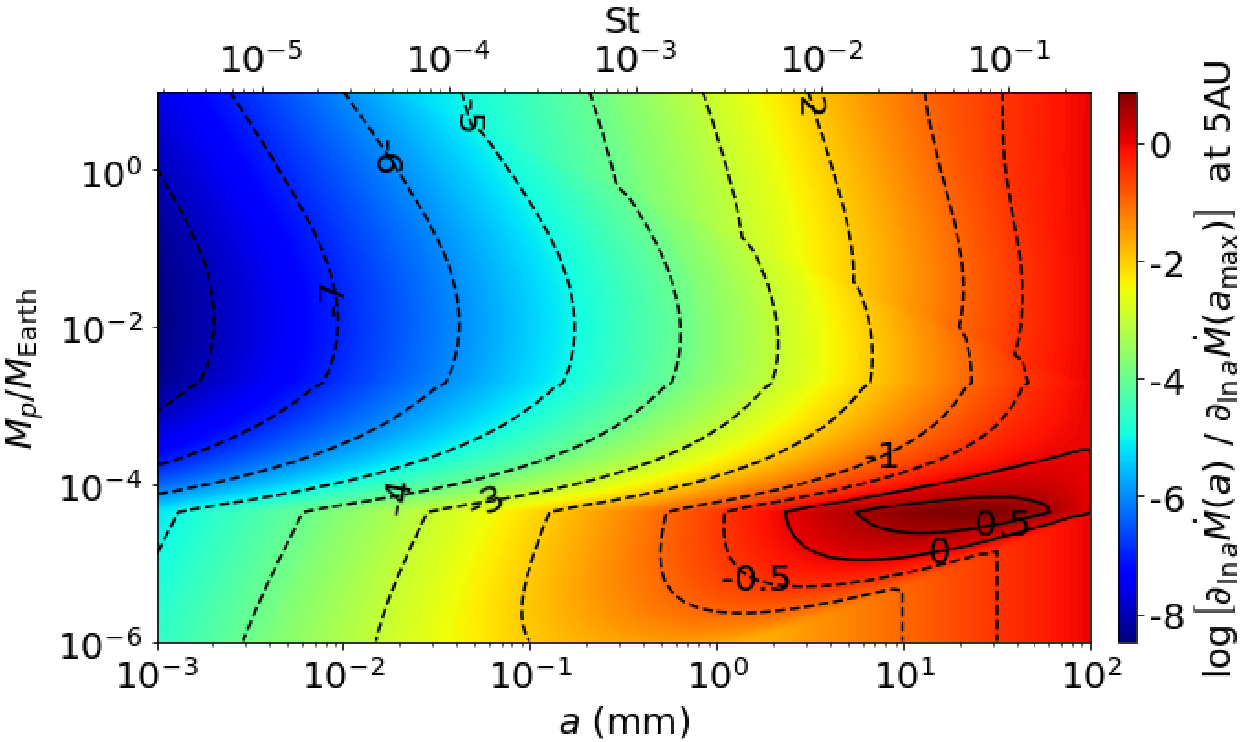
Accretion Rates



Lyra & Johansen + (2023)

Differential Accretion Rates

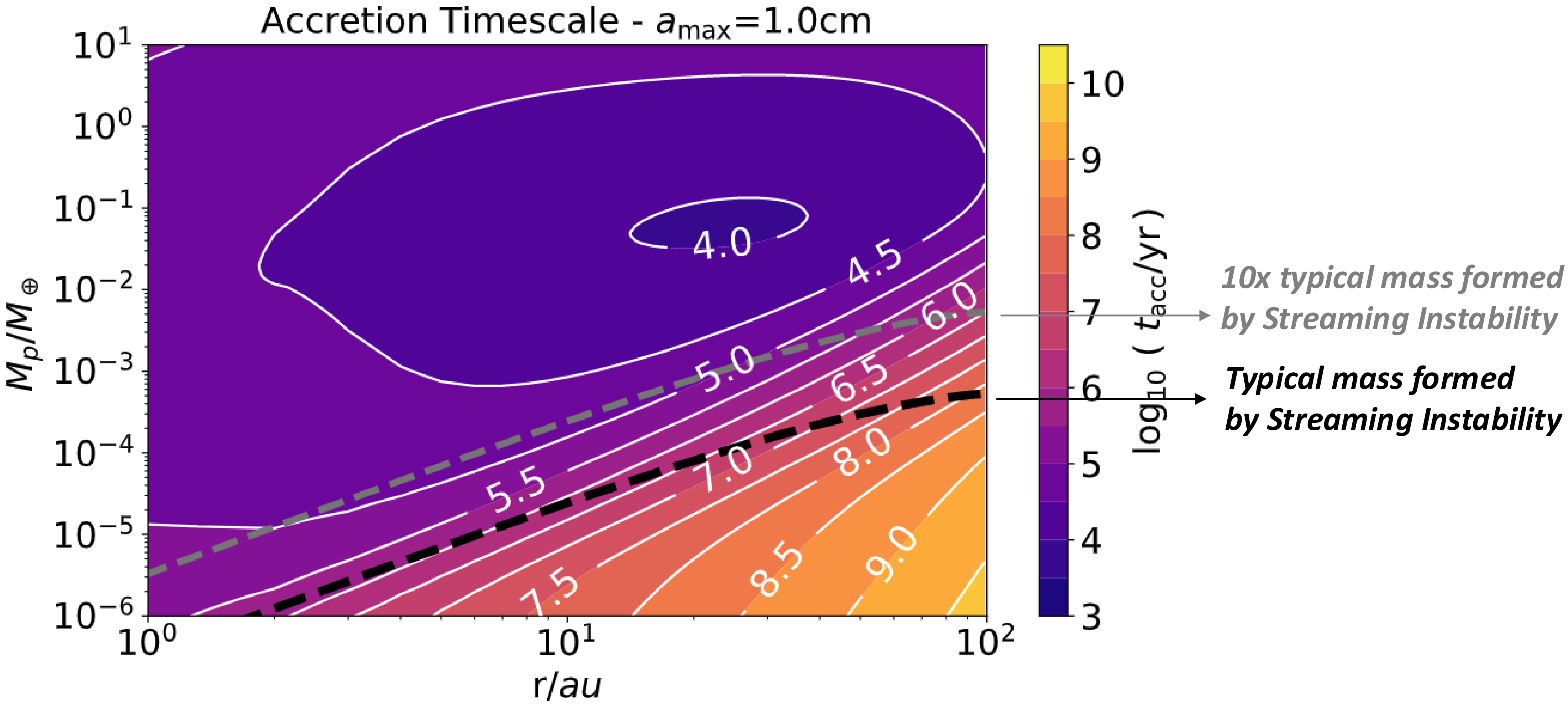




Lyra & Johansen + (2023)

Accretion Timescales

Myr accretion timescales possible on top of planetesimals produces by Streaming Instability



Lyra & Johansen + (2023)

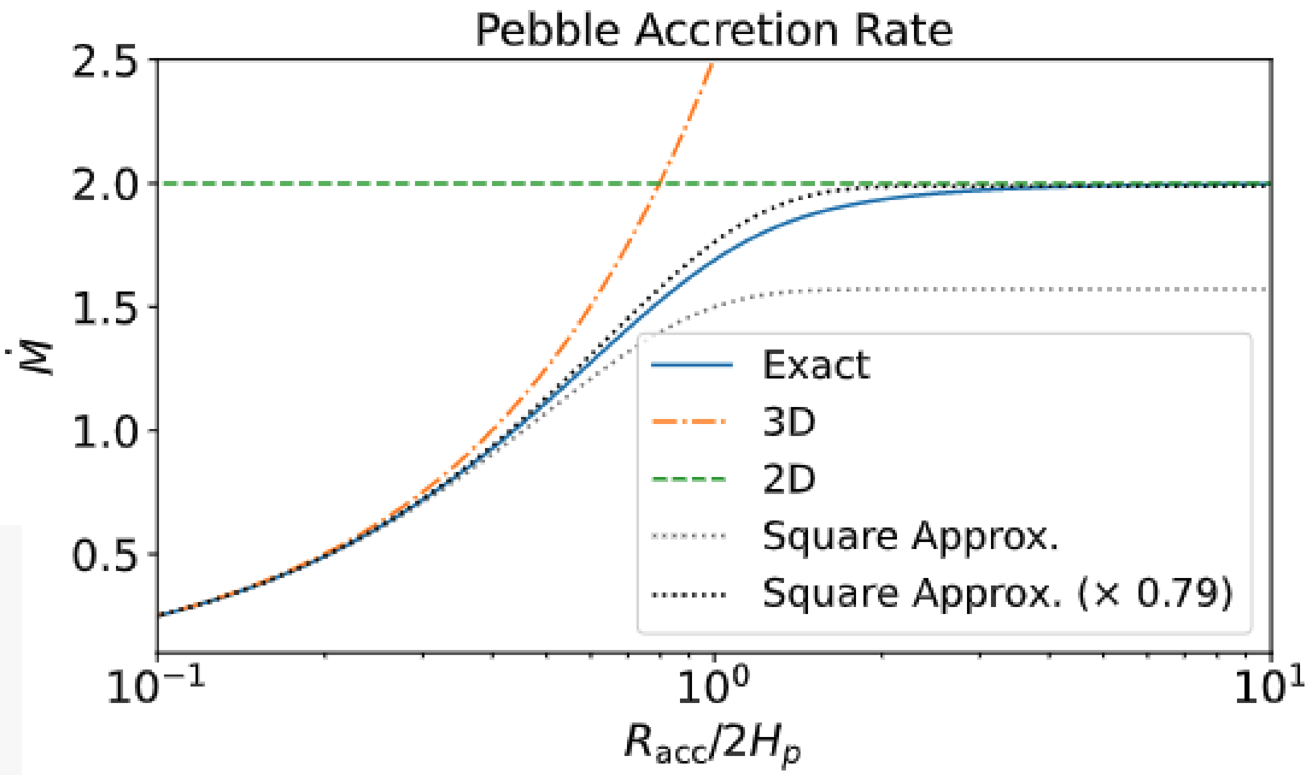
Analytical Solution for
General Monodisperse (single species) Pebble Accretion

$$\dot{M} = \pi R_{\rm acc}^2 \rho_{d0}\, S\, \delta v.$$

$$S \equiv \frac{1}{\pi R_{\rm acc}^2} \int_{-R_{\rm acc}}^{R_{\rm acc}} 2\sqrt{R_{\rm acc}^2 - z^2}\, \exp\left(-\frac{z^2}{2H_d^2}\right)\, dz,$$

$$S = e^{-\xi}\left[I_0(\xi) + I_1(\xi)\right], \qquad \xi \equiv \left(\frac{R_{\rm acc}}{2H_d}\right)^2$$

```
y = (x/2)**2

# Modified Bessel function of the first kind of real order.
I0 = sp.special.iv(0, y)
I1 = sp.special.iv(1, y)

Sint = np.exp(-y) * (I0 + I1)
rho_int = rhop * Sint
Mdot = pi*r**2 * rho_int * deltav
```



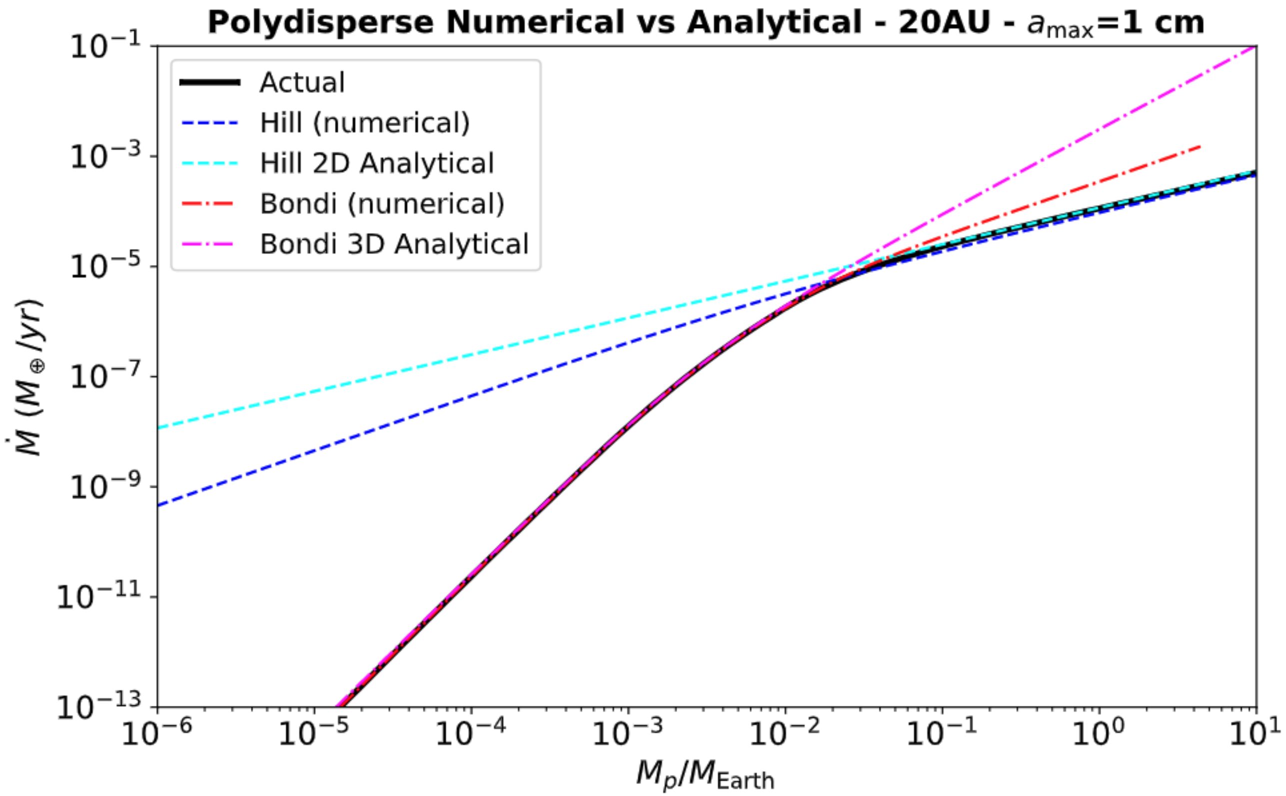
Lyra & Johansen + (2023)

Analytical Solutions for
2D and 3D Polydisperse (multi-species) Pebble Accretion

$$\dot{M}_{\rm 2D,Hill} = \frac{6(1-p)}{14-5q-3k}\left(\frac{\rm St_{max}}{0.1}\right)^{2/3}\Omega R_H^2 Z \Sigma_g.$$

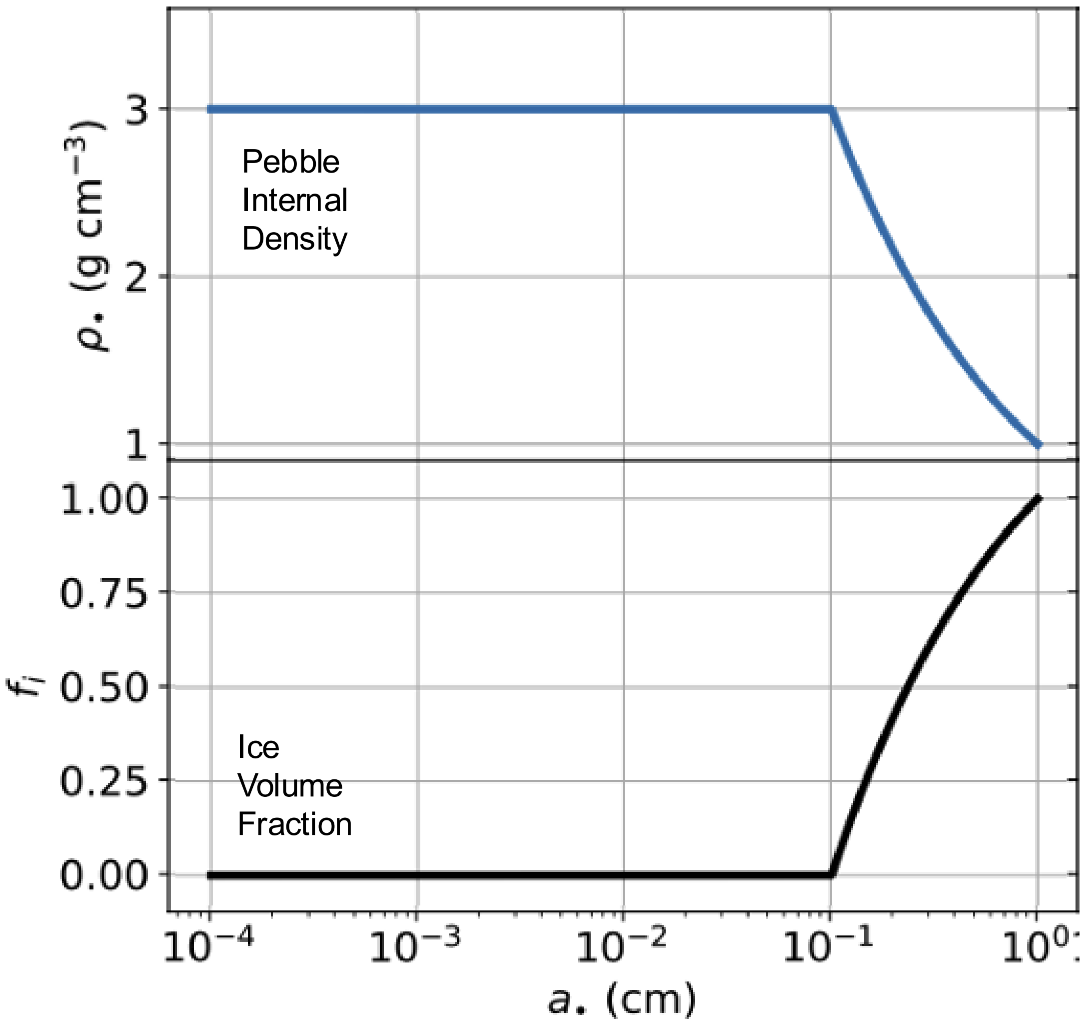
$$\dot{M}_{\rm 3D,Bondi} \approx C_1\frac{\gamma_l\left(\frac{b_1+1}{s}, j_1 a_{\rm max}^s\right)}{s\, j_1^{(b_1+1)/s}} + C_2\frac{\gamma_l\left(\frac{b_2+1}{s}, j_2 a_{\rm max}^s\right)}{s\, j_2^{(b_2+1)/s}} + C_3\frac{\gamma_l\left(\frac{b_3+1}{s}, j_3 a_{\rm max}^s\right)}{s\, j_3^{(b_3+1)/s}} + C_4\frac{\gamma_l\left(\frac{b_4+1}{s}, j_4 a_{\rm max}^s\right)}{s\, j_4^{(b_4+1)/s}}$$

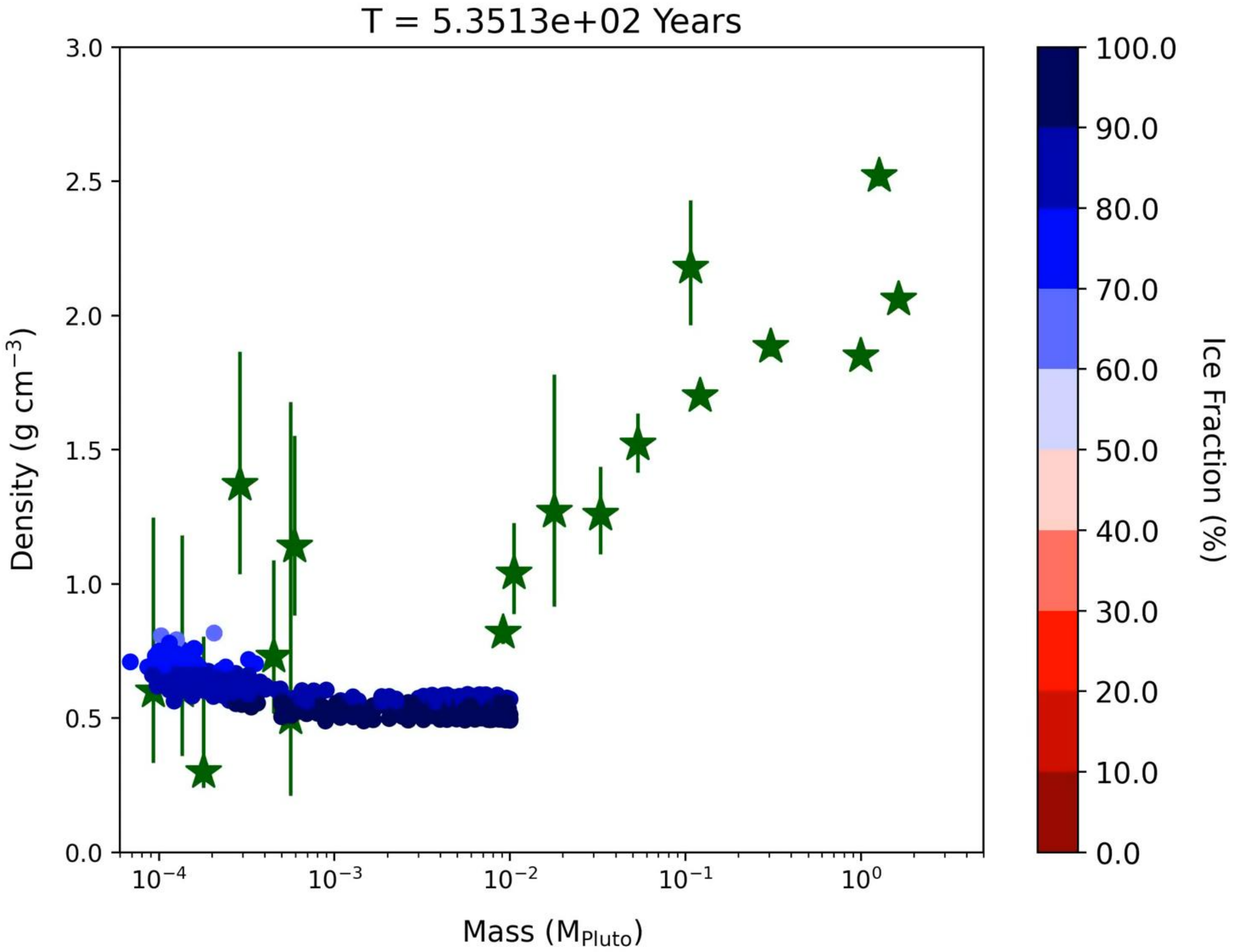
```
gamma11 = sp.special.gammainc((b1+1)/s,j1*a**s)*sp.special.gamma((b1+1)/s)
gamma12 = sp.special.gammainc((b2+1)/s,j2*a**s)*sp.special.gamma((b2+1)/s)
gamma13 = sp.special.gammainc((b3+1)/s,j3*a**s)*sp.special.gamma((b3+1)/s)
gamma14 = sp.special.gammainc((b4+1)/s,j4*a**s)*sp.special.gamma((b4+1)/s)

G1 = C1*gamma11/s/j1**((b1+1)/s)
G2 = C2*gamma12/s/j2**((b2+1)/s)
G3 = C3*gamma13/s/j3**((b3+1)/s)
G4 = C4*gamma14/s/j4**((b4+1)/s)

Mbondi3d = G1 + G2 + G3 + G4
```



Lyra & Johansen + (2023)

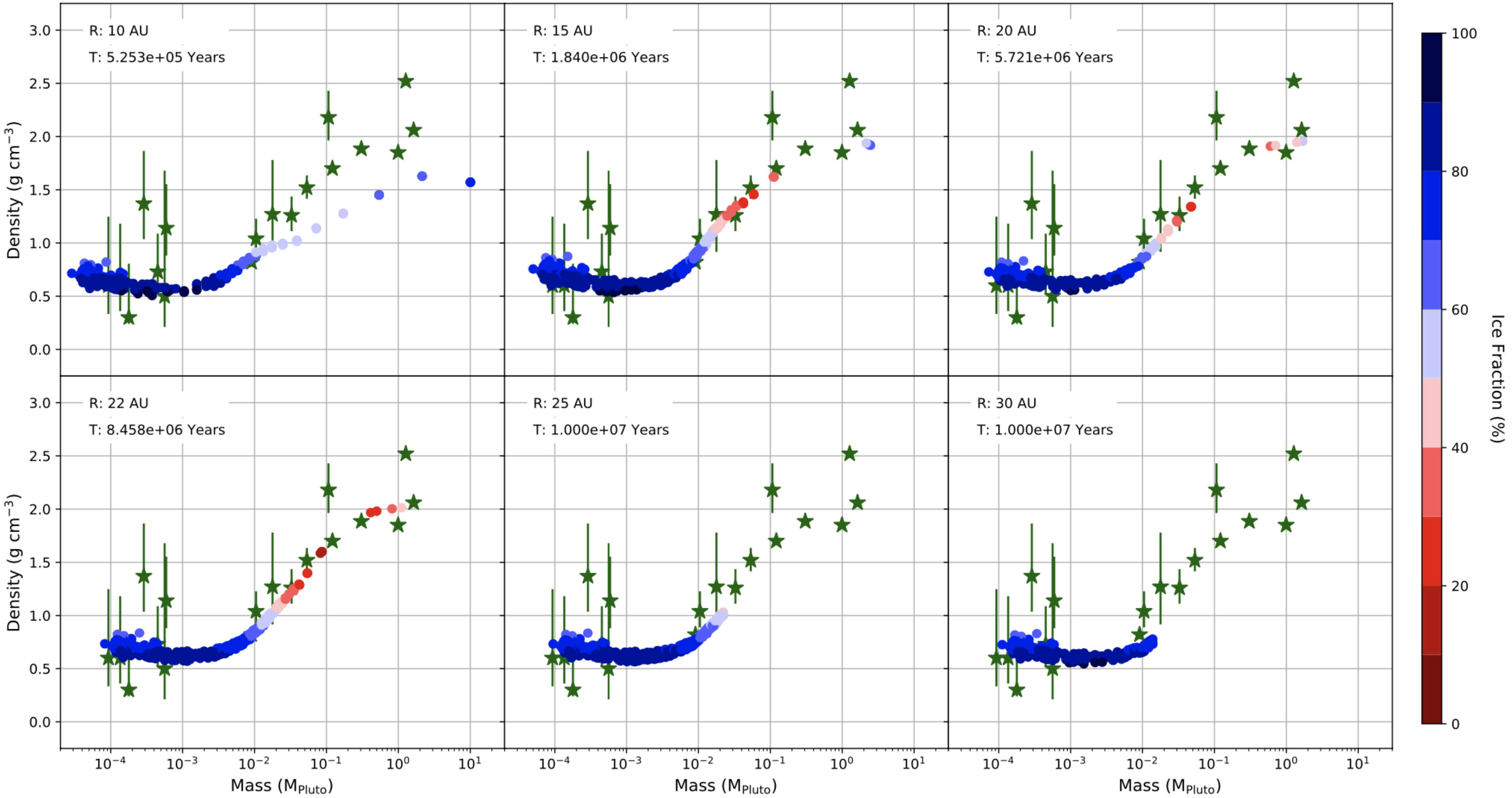
Growing Pluto by silicate pebble accretion





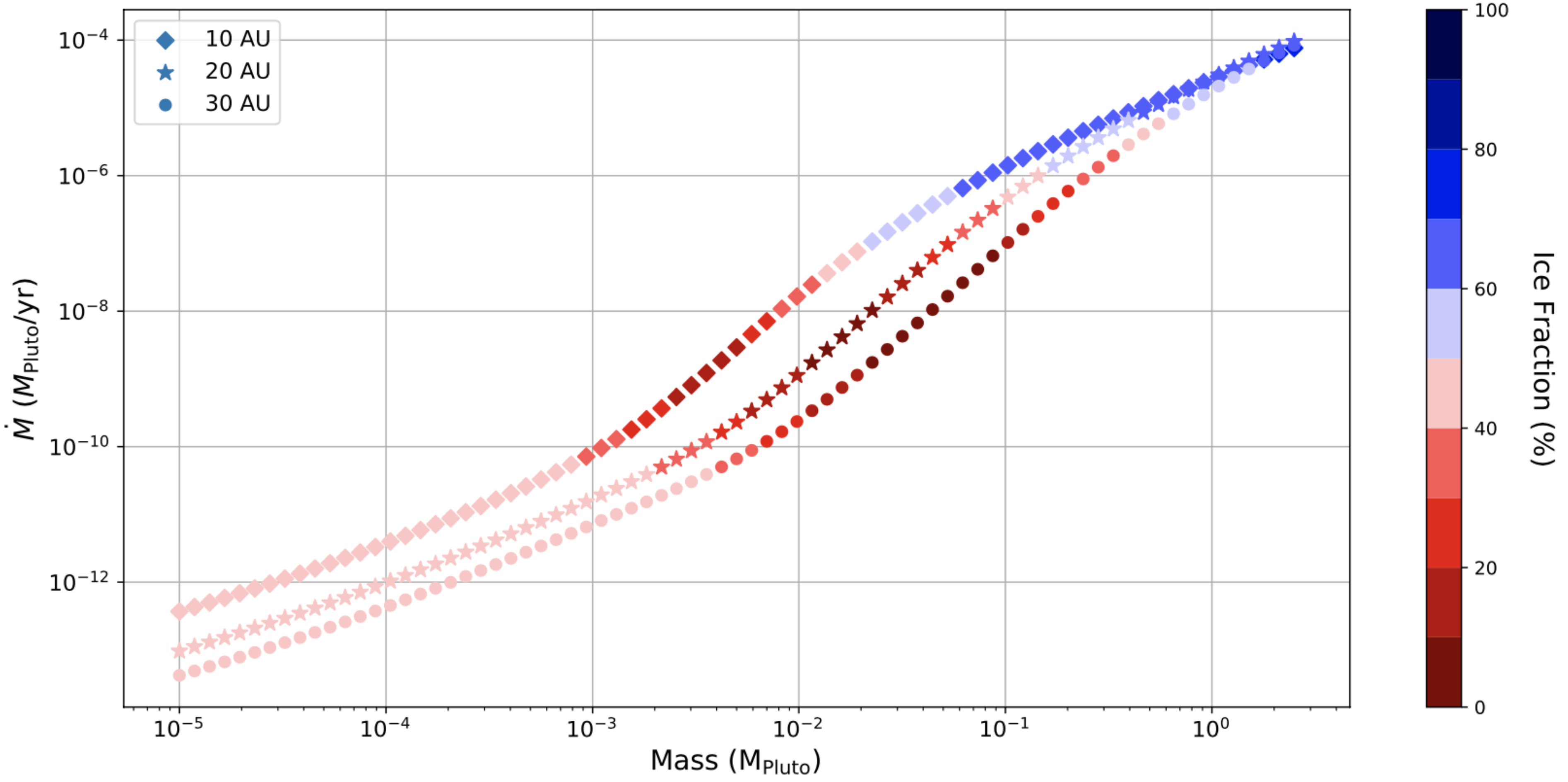
Canas & Lyra + (2024)

Distance Range
15 - 25AU



Canas & Lyra + (2024)

The window of silicate accretion



Canas & Lyra + (2024)

Conclusions

- Polydisperse Bondi accretion 1-2 orders of magnitude more efficient than monodisperse
  - Best accreted pebbles are those of drag time ~ Bondi time, not the largest ones
  - The largest ones dominate the mass budget, but accrete poorly
- Onset of Bondi accretion 1-2 orders of magnitude lower in mass compared to monodisperse
  - Bondi accretion possible on top of Streaming Instability planetary embryos within disk lifetime
  - Reaches 100-350km objects within Myr timescales
- Analytical solution to
  - Monodisperse general case
  - Polydisperse 2D Hill and 3D Bondi
- KBO density problem:
  - Two different pebble populations, maintained by ice desorption off small grains
  - Streaming instability: icy-rich small objects; nearly uniform composition
  - Polydisperse pebble accretion: silicate-rich larger objects; varied composition
  - Melting avoided by
    - ice-rich formation
    - ^{26}Al incorporated mostly in long (>Myr) phase of silicate accretion
  - KBOs best reproduced between 15-25 AU