# Gas in debris disks:
# A new way to produce patterns?

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#betapic30 (follow on Twitter!), Paris, Sept 8th-12th 2014

Debris disks are

***not*** completely **gas-free**

What is the dynamical effect of this gas?

LETTER

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# Formation of sharp eccentric rings in debris disks with gas but without planets

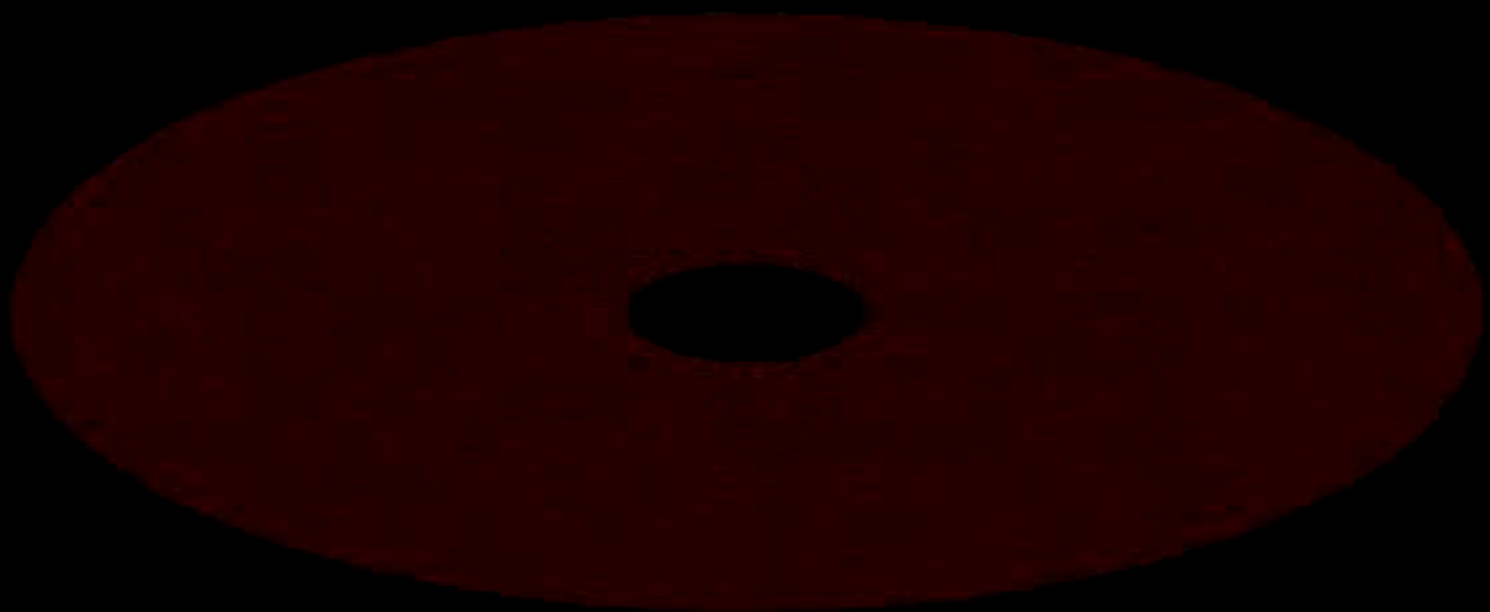
W. Lyra[1,2,3] & M. Kuchner[4]

**'Debris disks' around young stars (analogues of the Kuiper Belt in our Solar System) show a variety of non-trivial structures attributed to planetary perturbations and used to constrain the properties of those planets[1–3]. However, these analyses have largely ignored the fact that some debris disks are found to contain small quantities of gas[4–9], a component that all such disks should contain at some level[10,11]. Several debris disks have been measured with a dust-to-gas ratio of about unity[4–9], at which the effect of hydrodynamics on the structure of the disk cannot be ignored[12,13]. Here we report linear and nonlinear modelling that shows that dust–gas interactions can produce some of the key patterns attributed to planets. We find a robust clumping instability that organizes the dust into narrow, eccentric rings, similar to the Fomalhaut debris disk[14]. The conclusion that such disks might contain planets is not necessarily required to explain these systems.**

Disks around young stars seem to pass through an evolutionary phase when the disk is optically thin and the dust-to-gas ratio $\varepsilon$ ranges from 0.1 to 10. The nearby stars β Pictoris[5,6,15–17], HD32297 (ref. 7), 49 Ceti (ref. 4) and HD 21997 (ref. 9) all host dust disks resembling ordinary debris disks and also have stable circumstellar gas detected in molecular CO, Na I or other metal lines; the inferred mass of gas ranges from lunar masses to a few Earth masses (Supplementary Information). The gas in

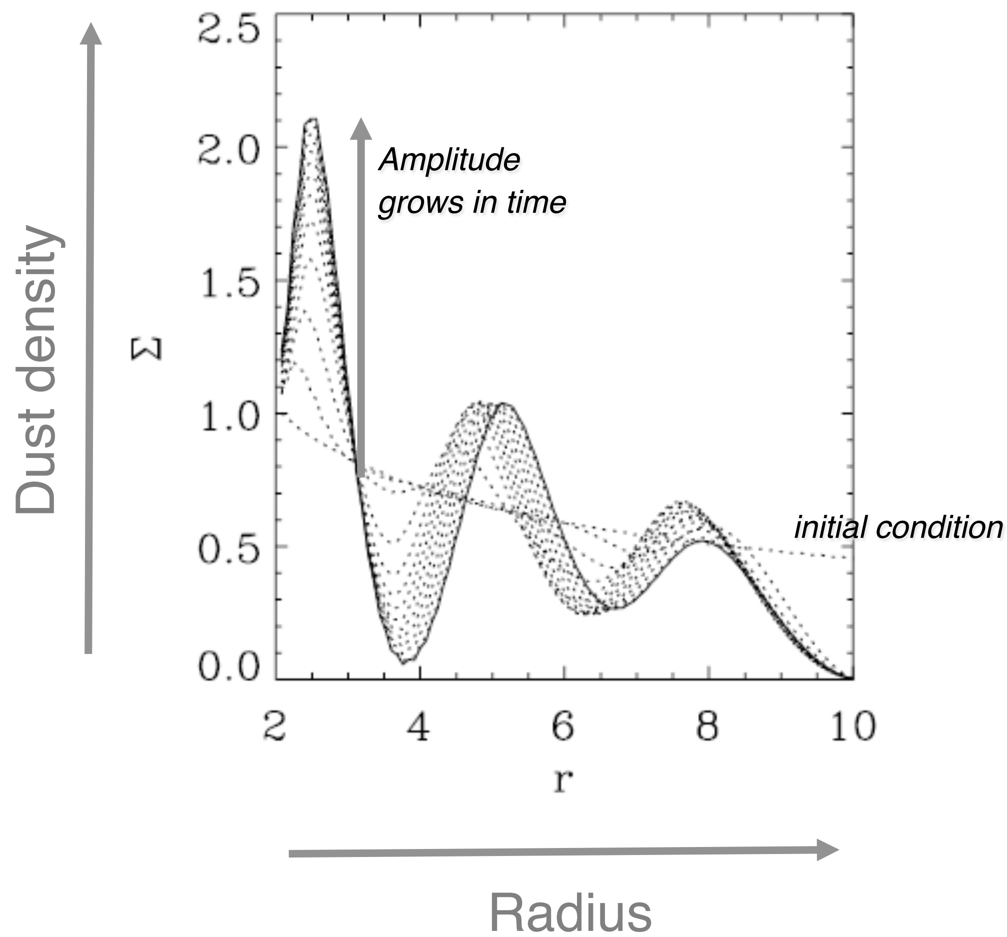
We present simulations of the fully compressible problem, solving for the continuity, Navier–Stokes and energy equations for the gas, and the momentum equation for the dust. Gas and dust interact dynamically through a drag force, and thermally through photoelectric heating. These are parametrized by a dynamical coupling time $\tau_f$ and a thermal coupling time $\tau_T$ (Supplementary Information). The simulations are performed with the Pencil Code[21–24], which solves the hydrodynamics on a grid. Two numerical models are presented: a three-dimensional box embedded in the disk that co-rotates with the flow at a fixed distance from the star; and a two-dimensional global model of the disk in the inertial frame. In the former the dust is treated as a fluid, with a separate continuity equation. In the latter the dust is represented by discrete particles with position and velocities that are independent of the grid.

We perform a stability analysis of the linearized system of equations that should help interpret the results of the simulations (Supplementary Information). We plot in Fig. 1a–c the three solutions that show linear growth, as functions of $\varepsilon$ and $n = kH$, where $k$ is the radial wavenumber and $H$ is the gas scale height ($H = c_s/\sqrt{\gamma}\Omega_K$, where $c_s$ is the sound speed, $\Omega_K$ the Keplerian rotation frequency and $\gamma$ the adiabatic index). The friction time $\tau_f$ is assumed to be equal to $1/\Omega_K$. The left and middle panels show the growth and damping rates. The



Lyra &  Kuchner (2013, Nature, 499, 184)

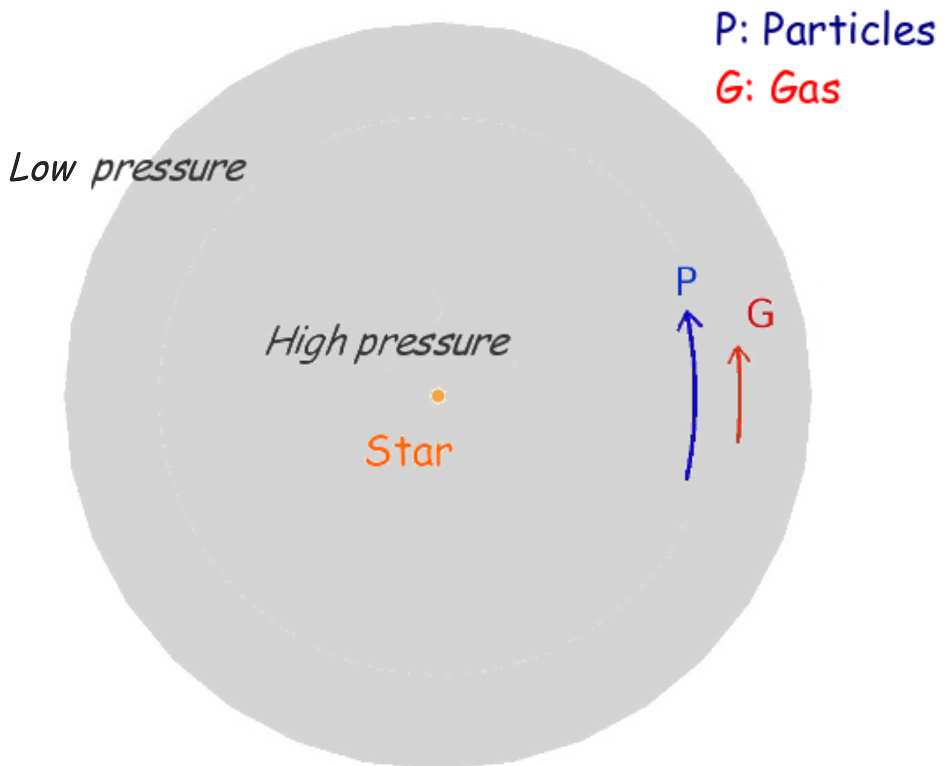
# Dust and gas together leads to instability...



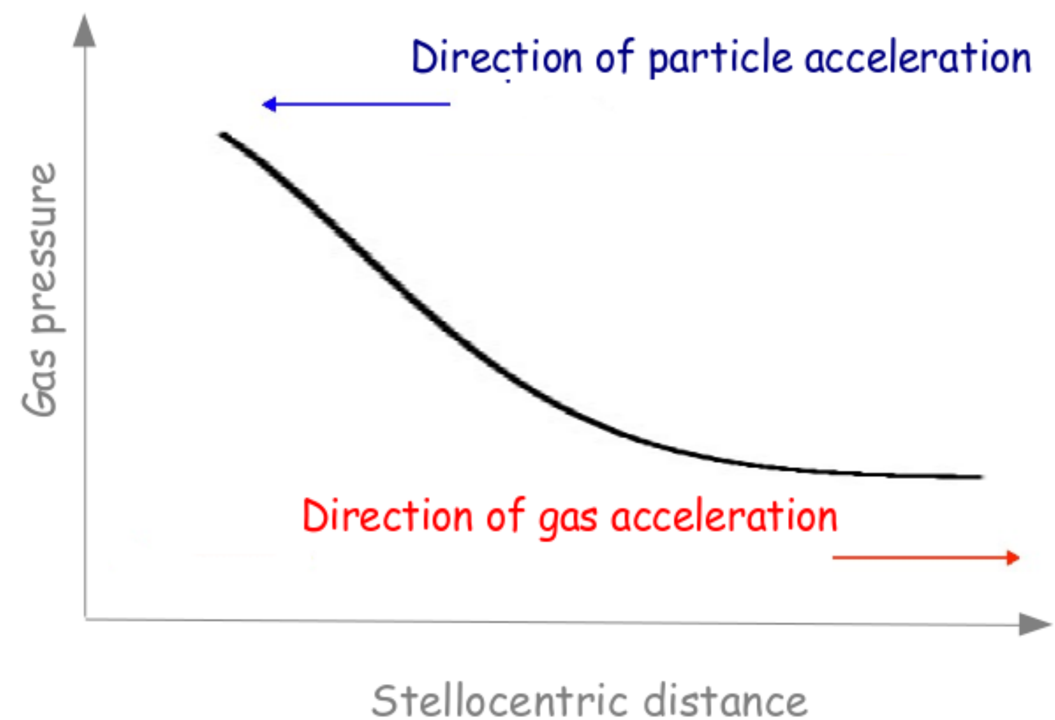

Klahr & Lin (2005)

Suggested that an instability causes dust in debris disks to clump together.

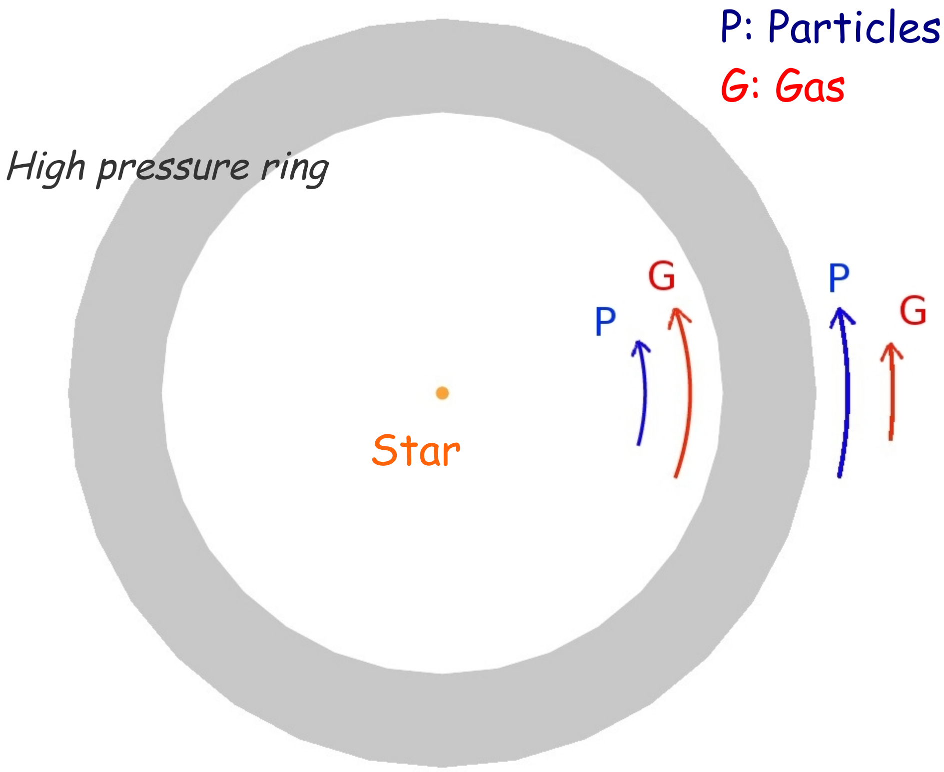
# Particle drift




Adapted from Whipple (1972)



# Pressure Trap

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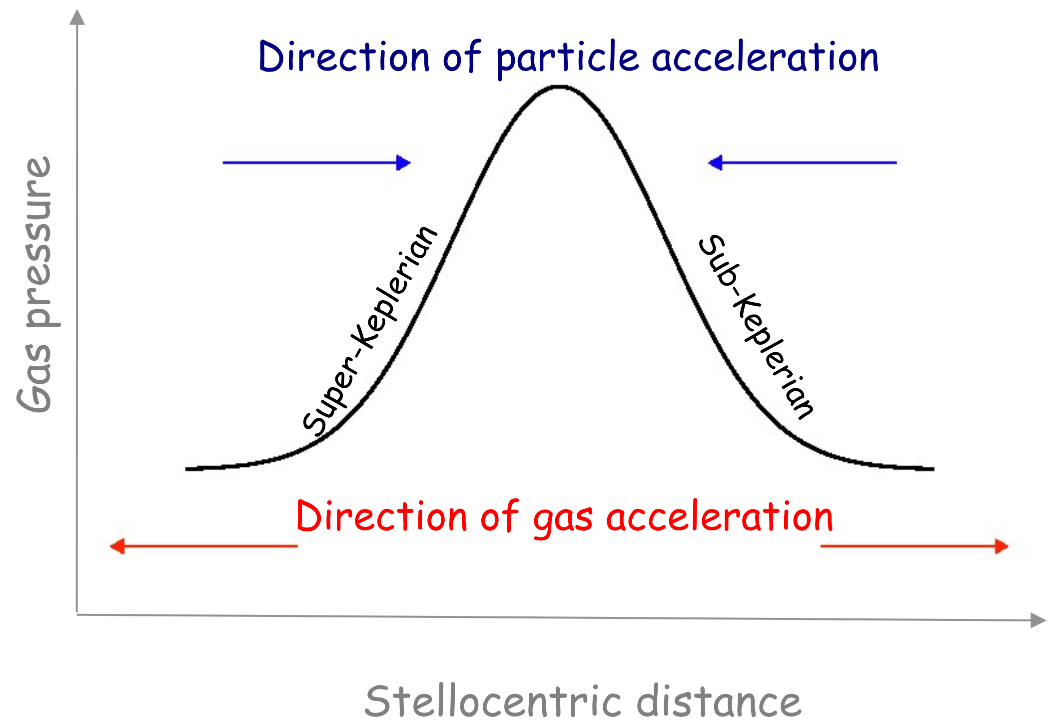


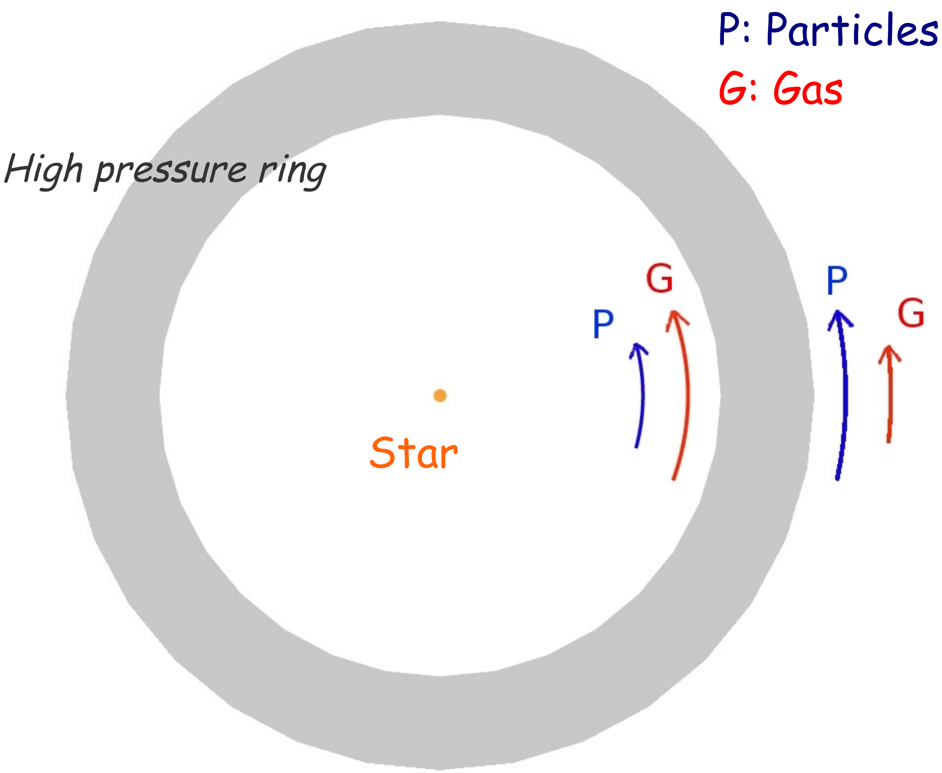
# Pressure Trap

**Gas** $$\frac{D\boldsymbol{u}}{Dt} = -\nabla\Phi - \rho^{-1}\nabla p$$

**Particles** $$\frac{d\boldsymbol{w}}{dt} = -\nabla\Phi - \frac{(\boldsymbol{w}-\boldsymbol{u})}{\tau}$$

$$\boldsymbol{w} = \boldsymbol{u} + \tau\rho^{-1}\nabla p$$

The drag force pushes the particles *toward* the pressure gradient

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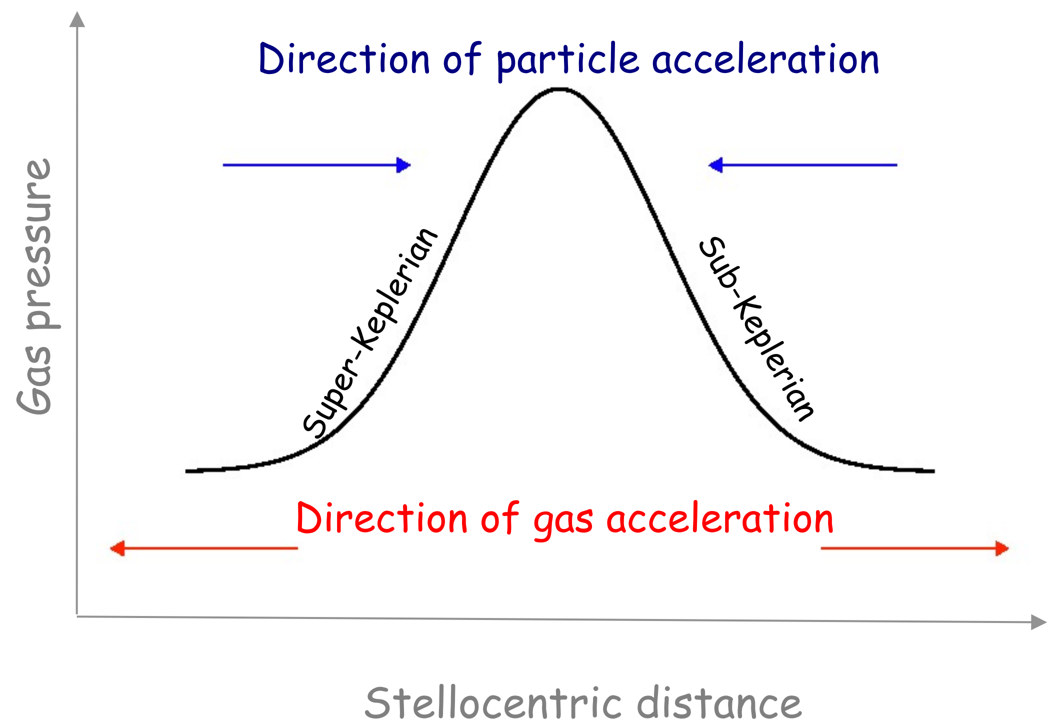
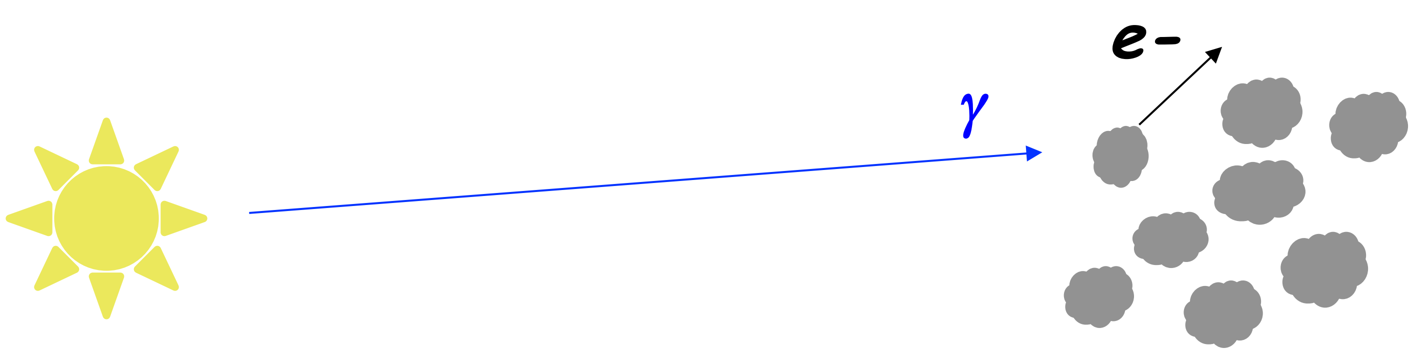


# Photoelectric heating

In optically thin debris disks,
the **dust** is the **main heating agent** for the gas.




Dust intercepts starlight directly,
emits electron, that heats the gas.

**Gas is photoelectrically heated by the dust**

# Runaway process: instability

Dust heats gas

Heated gas = high pressure region

High pressure concentrates dust

# Runaway process: instability

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Heated gas = high pressure region

High pressure concentrates dust

# Model equations

Klahr & Lin (2005) used a simplified, 1-D model.

$$\frac{\partial}{\partial t}\Sigma_d + \frac{1}{r}\frac{\partial}{\partial r} r\Sigma_d v_r = 0.$$

Continuity equation

$$V_\phi = \Omega r + \frac{1}{2\Omega\Sigma_g}\frac{\partial}{\partial r}P$$

Terminal velocity

$$T_g = T_0\left(\frac{\Sigma_d}{\Sigma_0}\right)^\beta,$$

Equation of state

# Model equations

Our simulation adds much more physics, and works in 2D and 3D.

**Klahr & Lin (2005)**

$$\frac{\partial}{\partial t}\Sigma_d + \frac{1}{r}\frac{\partial}{\partial r} r\Sigma_d v_r = 0.$$

$$V_\phi = \Omega r + \frac{1}{2\Omega\Sigma_g}\frac{\partial}{\partial r}P$$

$$T_g = T_0\left(\frac{\Sigma_d}{\Sigma_0}\right)^\beta,$$

**Lyra & Kuchner (2013)**

$$\frac{\partial \Sigma_g}{\partial t} = -(\boldsymbol{u}\cdot\nabla)\Sigma_g - \Sigma_g \nabla\cdot\boldsymbol{u}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \frac{1}{\Sigma_g}\nabla P - \nabla\Phi - \frac{\Sigma_d}{\Sigma_g}\boldsymbol{f}_d$$

$$\frac{\partial S}{\partial t} = -(\boldsymbol{u}\cdot\nabla)S - \frac{c_v}{T}\frac{(T-T_p)}{\tau_T}.$$

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{v}$$

$$\frac{d\boldsymbol{v}}{dt} = -\nabla\Phi + \boldsymbol{f}_d$$

$$\boldsymbol{f}_d = -\frac{(\boldsymbol{v}-\boldsymbol{u})}{\tau_f}$$

$$T_p = T_0\frac{\Sigma_d}{\Sigma_0}.$$

*Inertia for both gas and dust*

*Energy equation*

*Drag force and drag force backreaction*

# Missing physics

**Radiation Forces**

*Radiation pressure*

*Poynting-Robertson drag*

*Photophoresis*

**Collisions**

**Magnetic Fields**

**Detailed treatment of heating and cooling**

**Multiple particle species**

# Linear Analysis

**Dust**

$$D_w \Sigma_d = -\Sigma_d \nabla \cdot \boldsymbol{w}$$

$$D_w w_x = 2\Omega w_y - \frac{1}{\tau_f}(w_x - u_x)$$

$$D_w w_y = -\frac{1}{2}\Omega w_x - \frac{1}{\tau_f}(v_y - u_y)$$

**Gas**

$$D_u \Sigma_g = -\Sigma_g \nabla \cdot \boldsymbol{u}$$

$$D_u u_x = 2\Omega u_y - \frac{1}{\Sigma_g}\frac{\partial P}{\partial x} - \frac{\epsilon}{\tau_f}(u_x - w_x)$$

$$D_u u_y = -\frac{1}{2}\Omega u_x - \frac{1}{\Sigma_g}\frac{\partial P}{\partial y} - \frac{\epsilon}{\tau_f}(u_y - w_y)$$

$$\lim_{\tau_T \to 0} P = c_v(\gamma - 1) T_0 \Sigma_g \Sigma_d / \Sigma_0$$

$$\psi = \psi_0 + \psi'$$

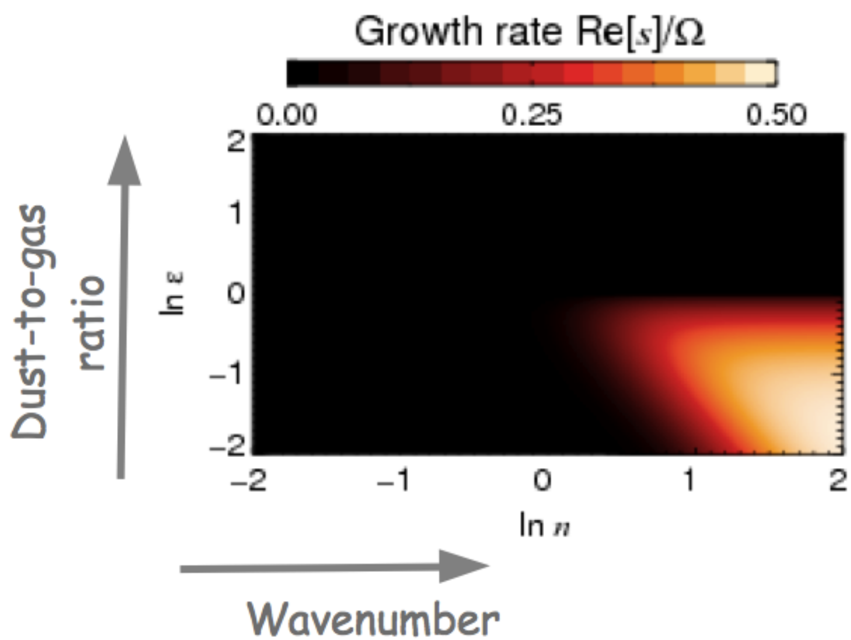
$$\psi' = \hat{\psi} \exp(ikx + st)$$

**Dispersion relation**

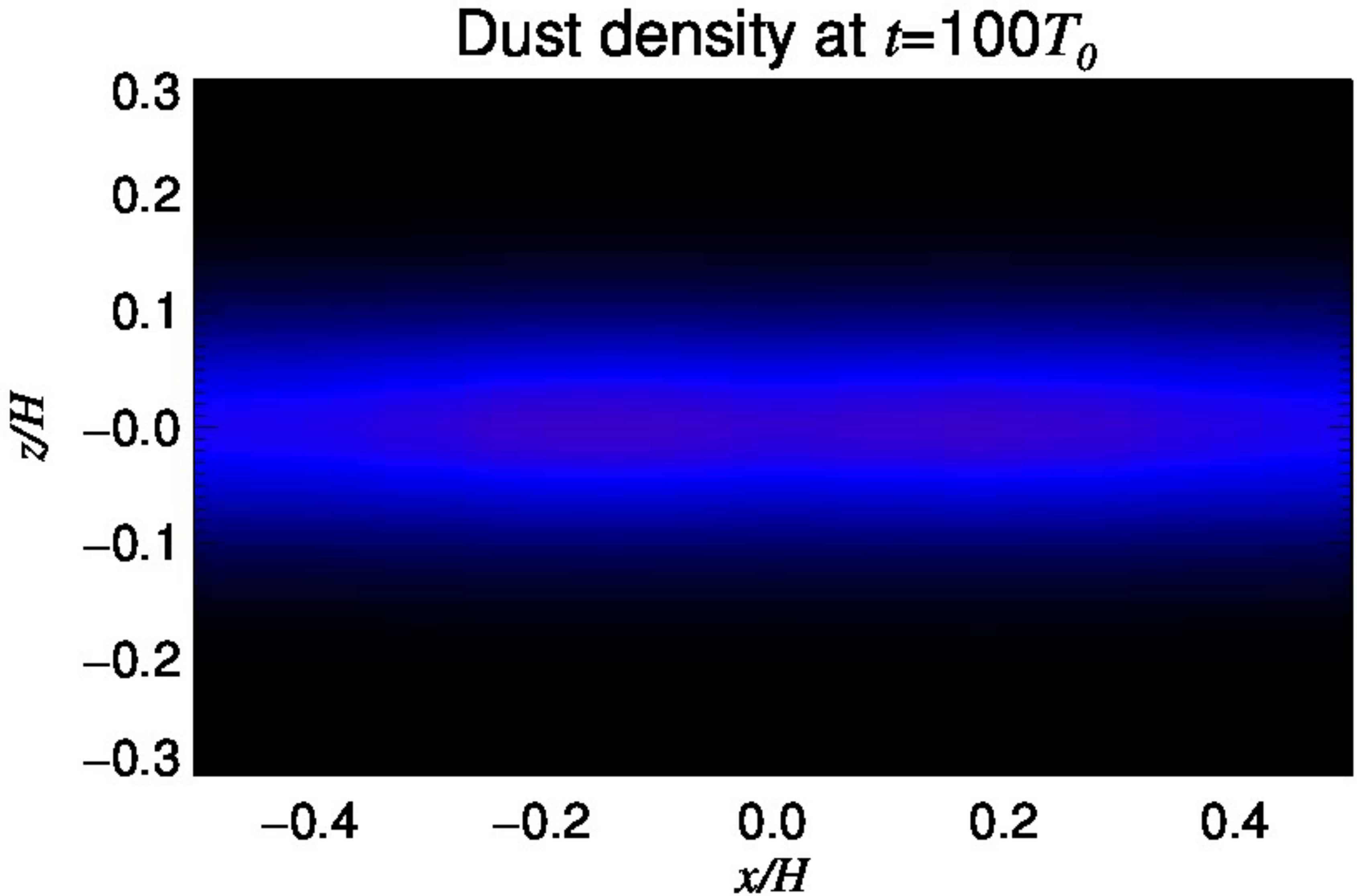
$$A\omega^5 + B\omega^4 + C\omega^3 + D\omega^2 + E\omega + F = 0$$

$$A = 1$$
$$B = 2\epsilon + 2$$
$$C = \epsilon^2 + \epsilon(n^2 + 2) + 3$$
$$D = \epsilon^2 n^2 + \epsilon(3n^2 + 2) + 2$$
$$E = \epsilon^2(2n^2 + 1) + \epsilon(3n^2 + 2) + 2$$
$$F = \epsilon^2 n^2 - \epsilon n^2$$

$\epsilon = \Sigma_d / \Sigma_g$ $\quad n = kH$ $\quad \omega = s/\Omega$



## Photoelectric instability - 3D stratified local box



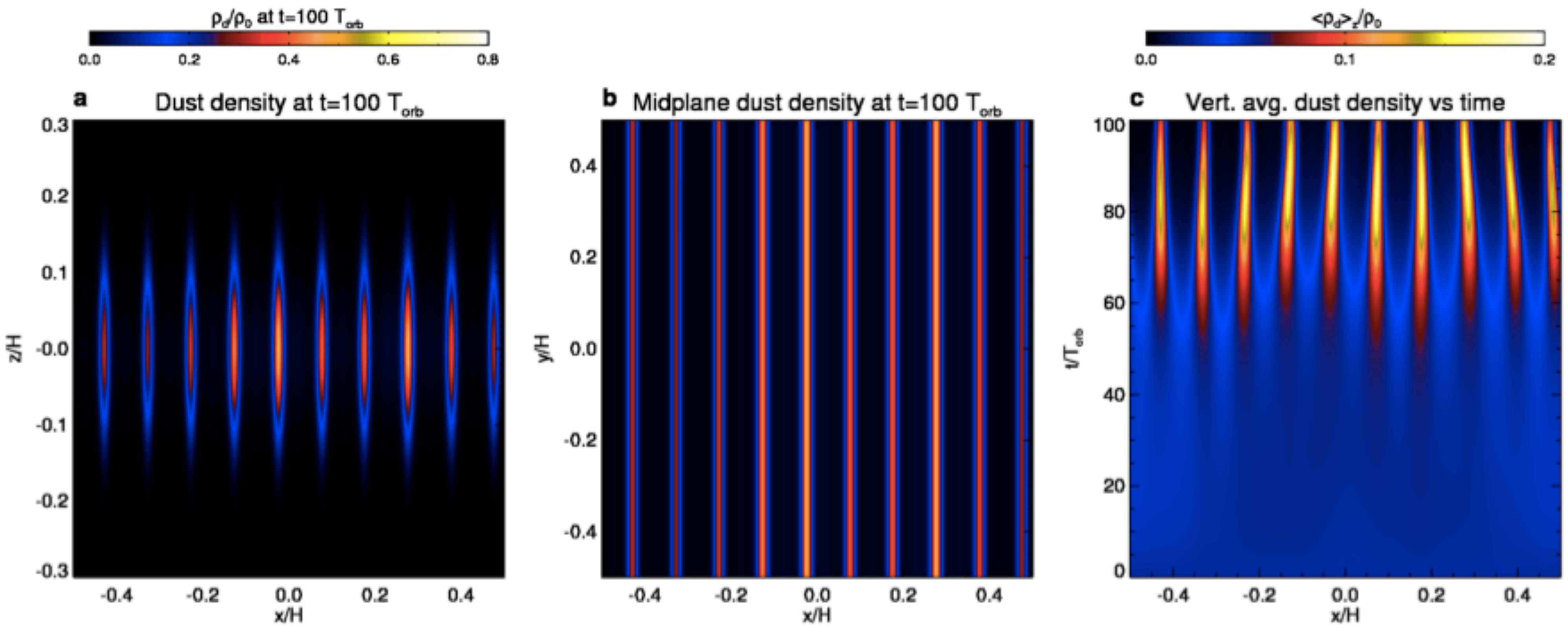
# Photoelectric Instability

Dust heats gas

Heated gas = high pressure region

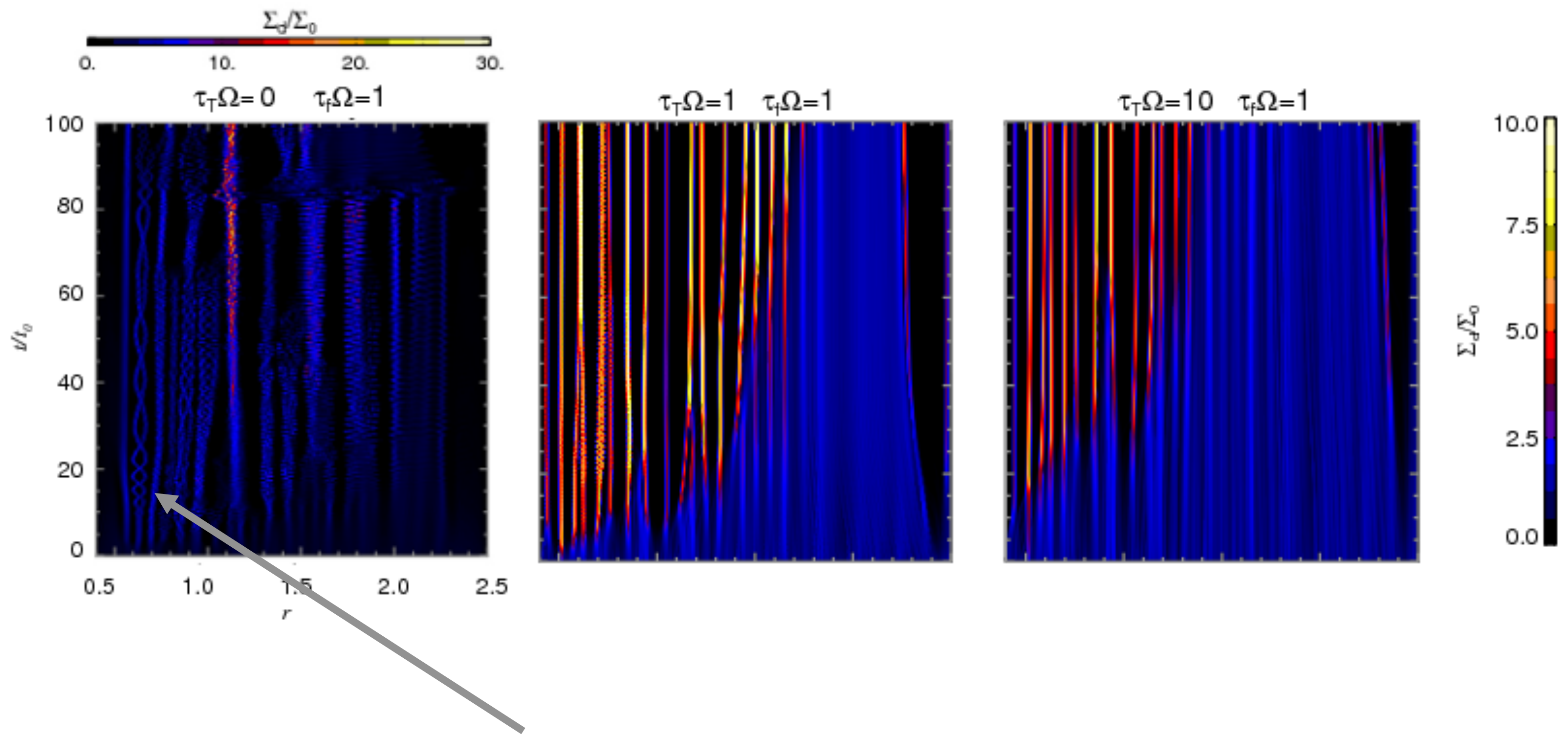
High pressure concentrates dust

## 3D Stratified



# Oscillations

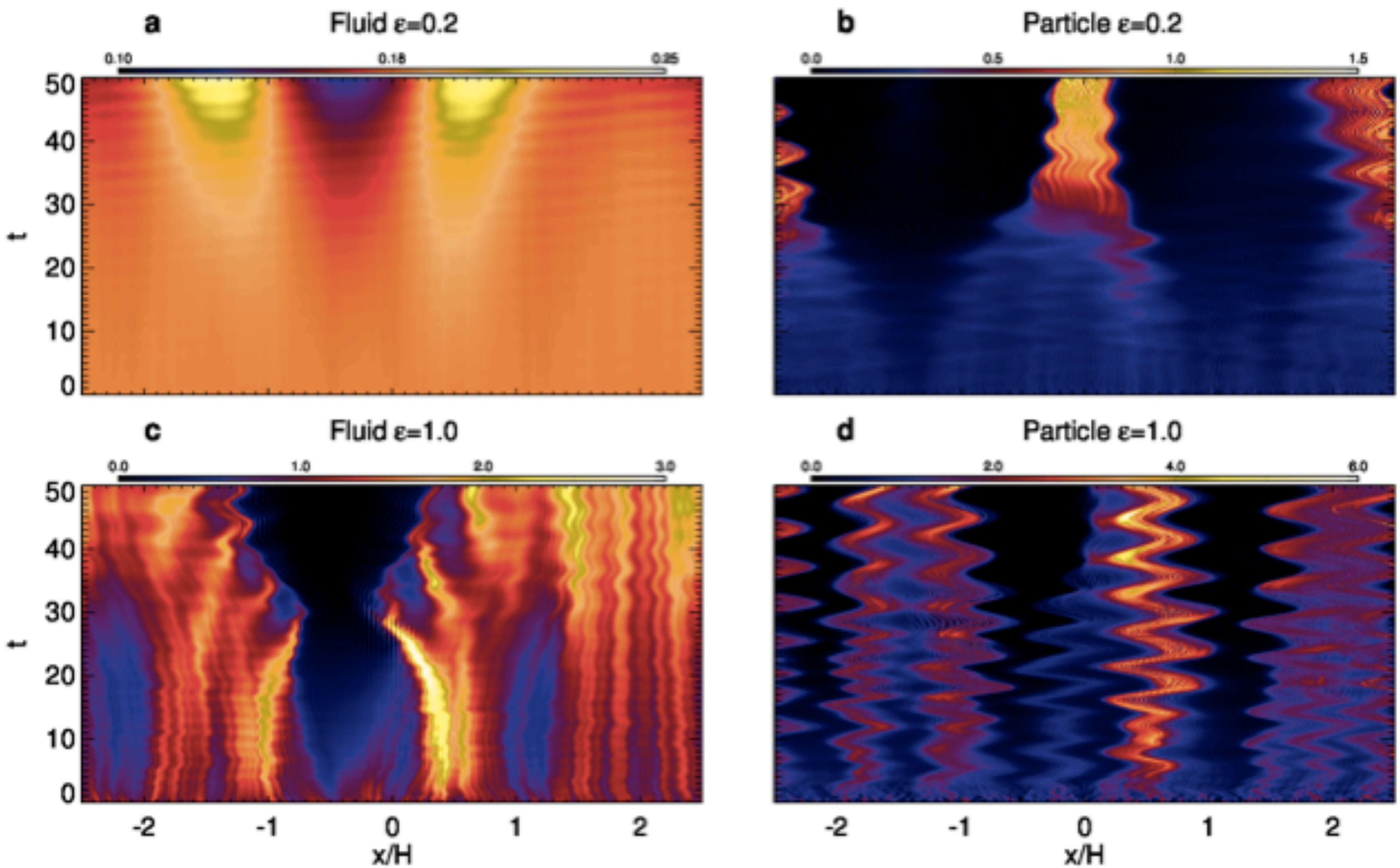
Thermal coupling time




Oscillations appear
with decreasing thermal time.

# Oscillations

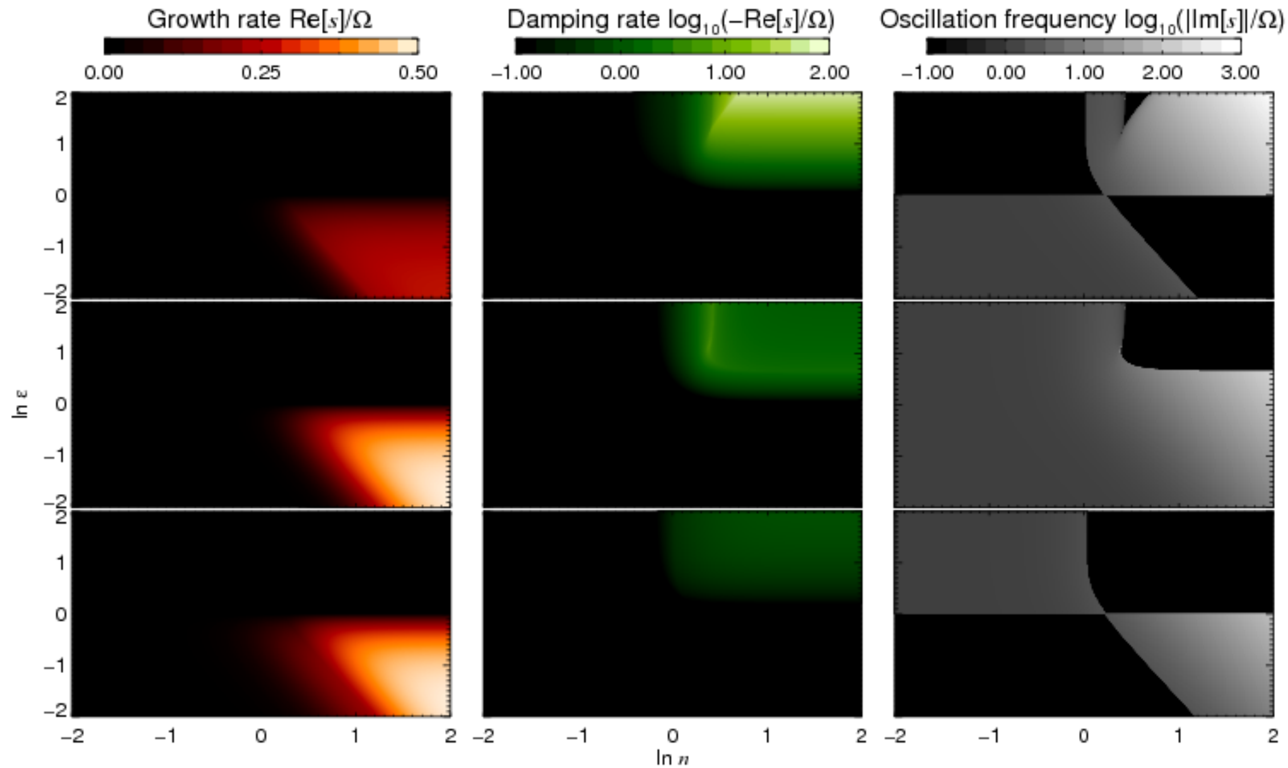
*Low Reynolds number* *High Reynolds number*




**Epicyclic oscillations**

**clear at high Reynolds numbers!**

# Solutions




The dispersion relation is a 5th order polynomium, so there are five roots!

## *Dispersion relation*

$$A\omega^5 + B\omega^4 + C\omega^3 + D\omega^2 + E\omega + F = 0$$

$$A = 1$$
$$B = 2\epsilon + 2$$
$$C = \epsilon^2 + \epsilon(n^2+2) + 3$$
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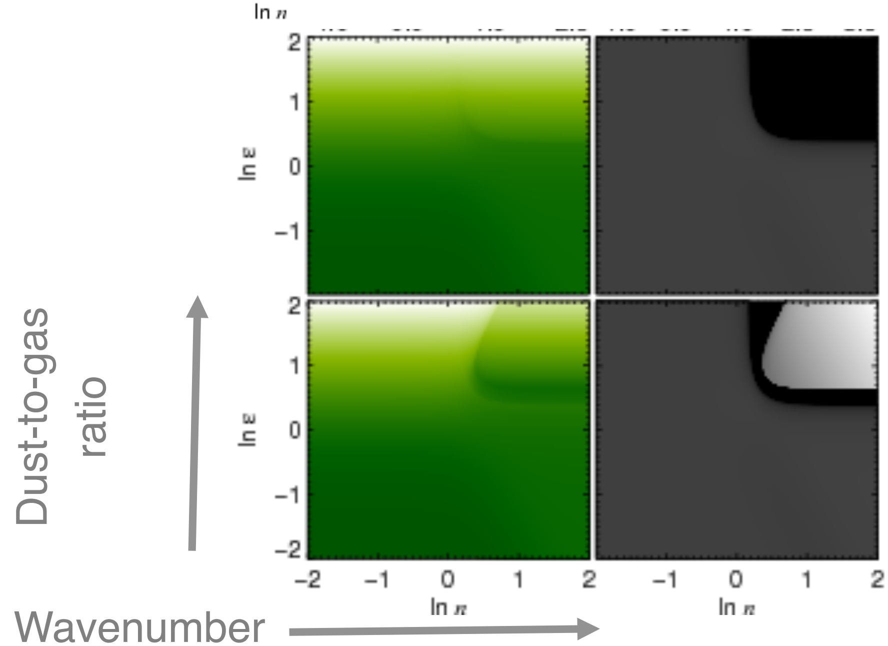
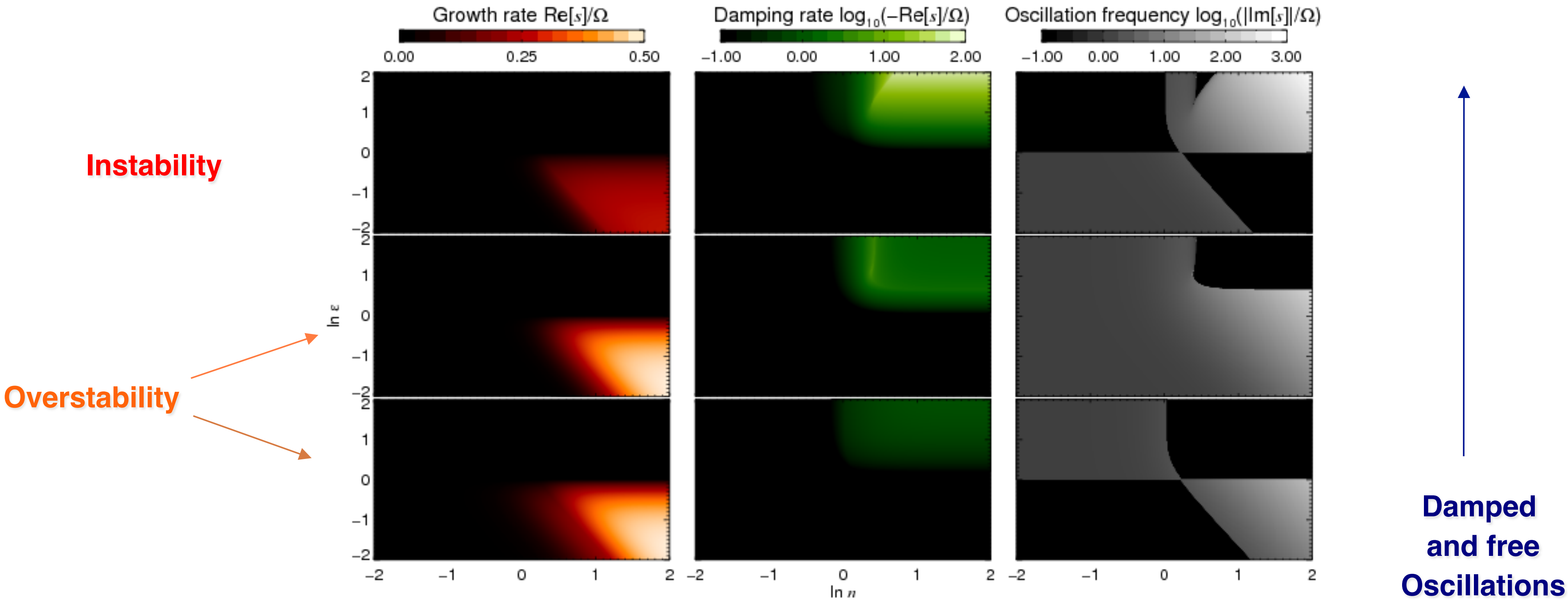
$$E = \epsilon^2(2n^2+1) + \epsilon(3n^2+2) + 2$$
$$F = \epsilon^2 n^2 - \epsilon n^2$$

$\epsilon = \Sigma_d / \Sigma_g$ $\quad n = kH$ $\quad \omega = s/\Omega$



# Solutions

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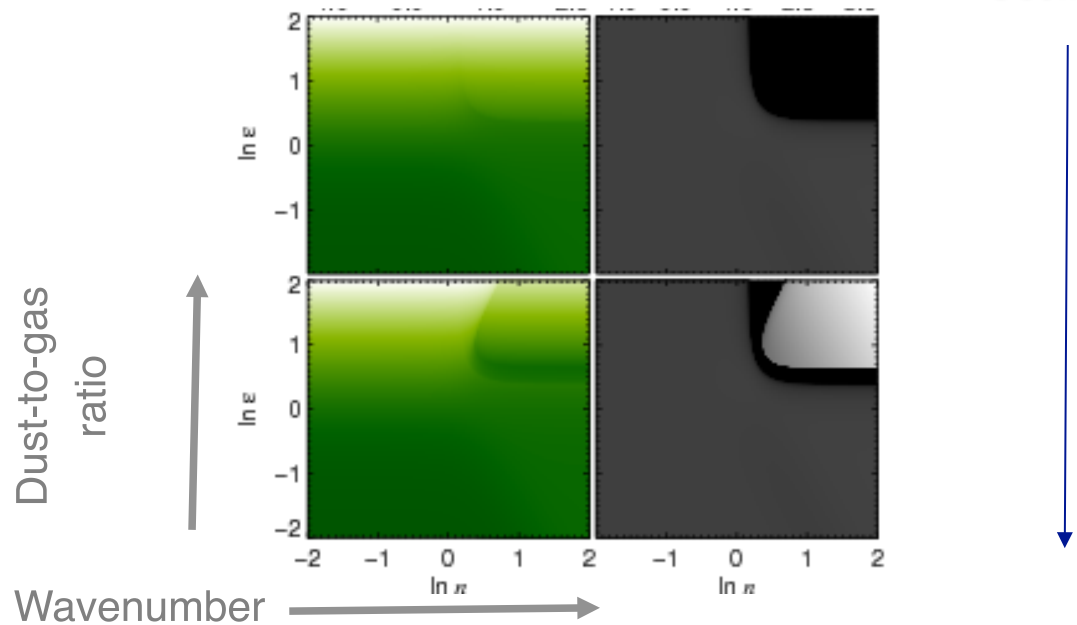
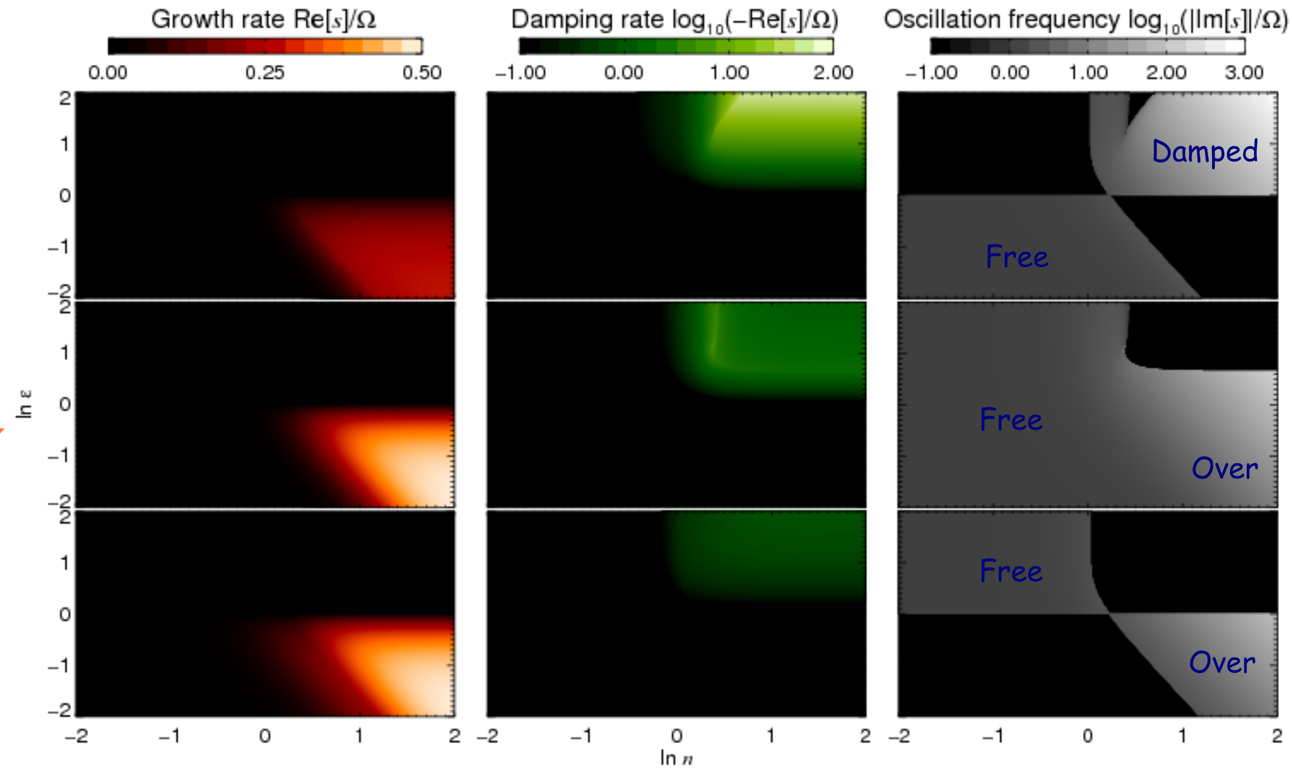


# Solutions

**Instability**

**Overstability**



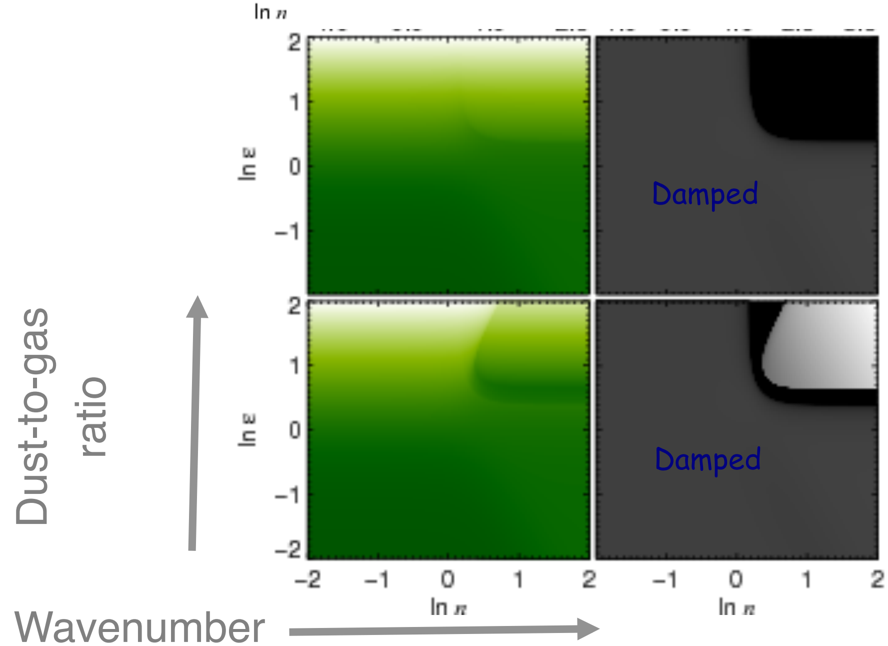

**Damped and free Oscillations**

*Dispersion relation*

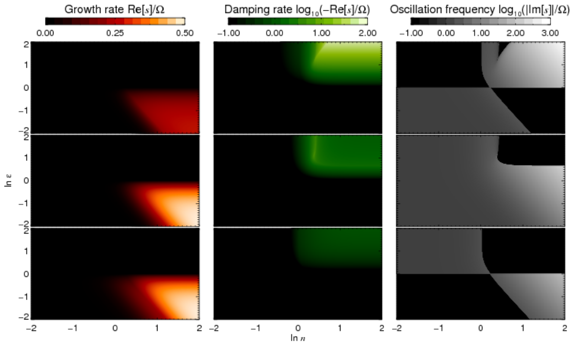
$$A\omega^5 + B\omega^4 + C\omega^3 + D\omega^2 + E\omega + F = 0$$

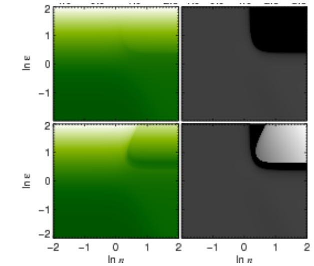
$$A = 1$$
$$B = 2\epsilon + 2$$
$$C = \epsilon^2 + \epsilon(n^2+2) + 3$$
$$D = \epsilon^2 n^2 + \epsilon(3n^2+2) + 2$$
$$E = \epsilon^2(2n^2+1) + \epsilon(3n^2+2) + 2$$
$$F = \epsilon^2 n^2 - \epsilon n^2$$

$\epsilon = \Sigma_d / \Sigma_g$ $\quad n = kH \quad$ $\omega = s/\Omega$



# Solutions

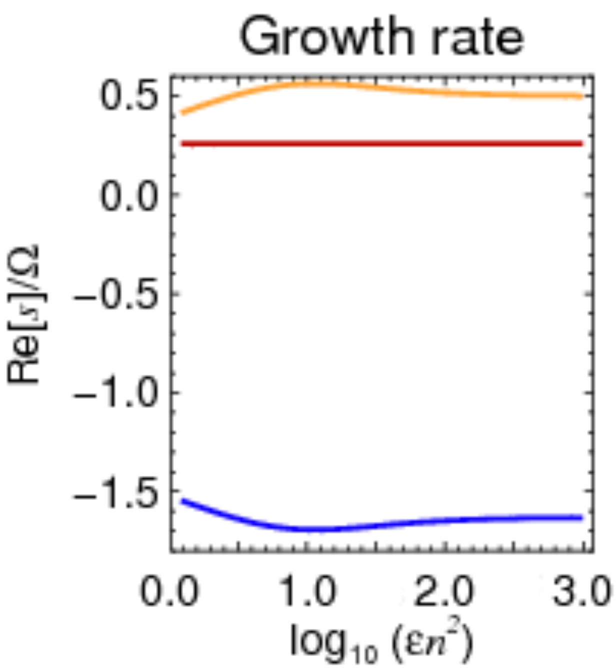


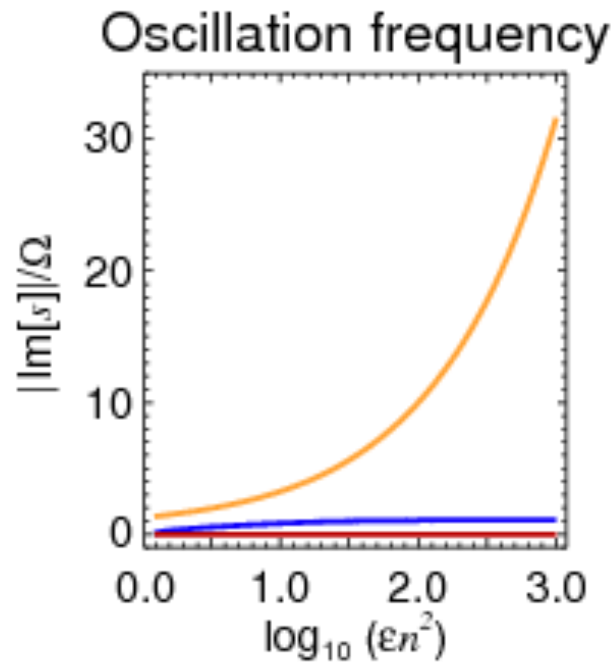



**Overstability**

**Instability**

**Oscillations**




Max growth rate: *Ω/2*.

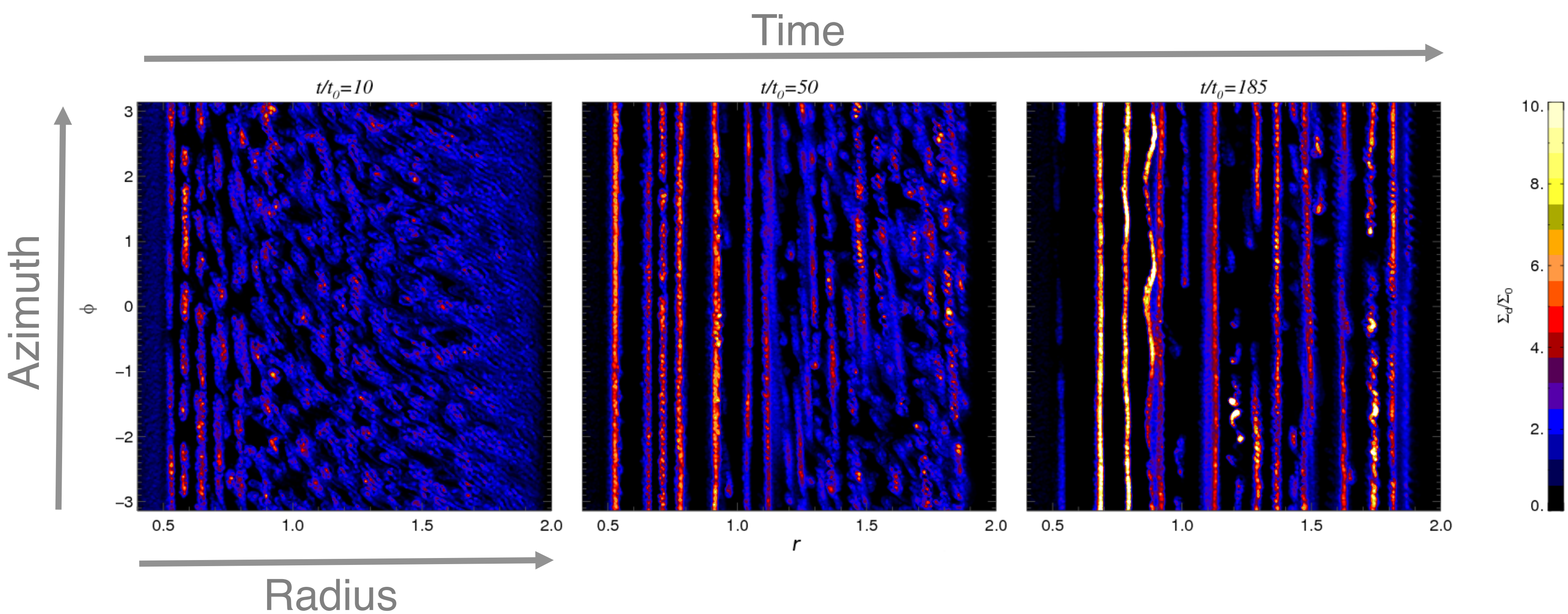
**Million-fold amplification in five orbits!**

A very powerful instability.

# The model in r-$\phi$: Eccentric rings

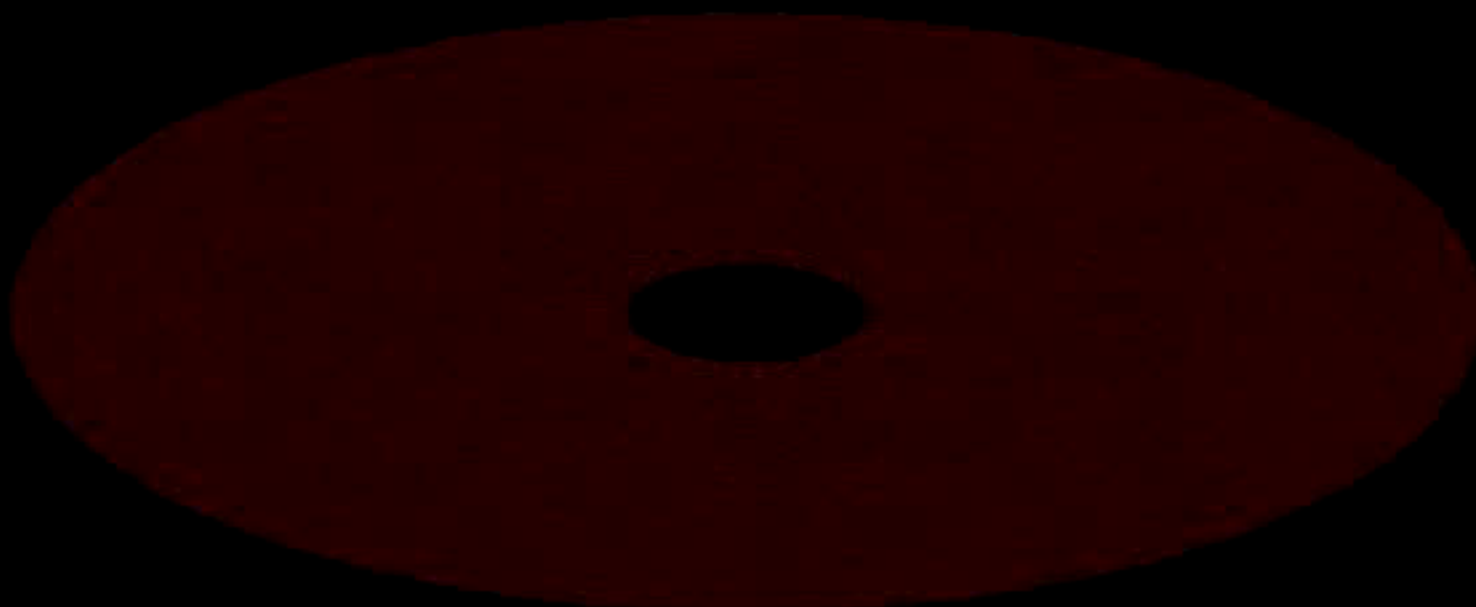
**Growth** of **axisymmetric** modes
+
**Damping** of **nonaxissymetric** modes.

= ***Rings !!!***



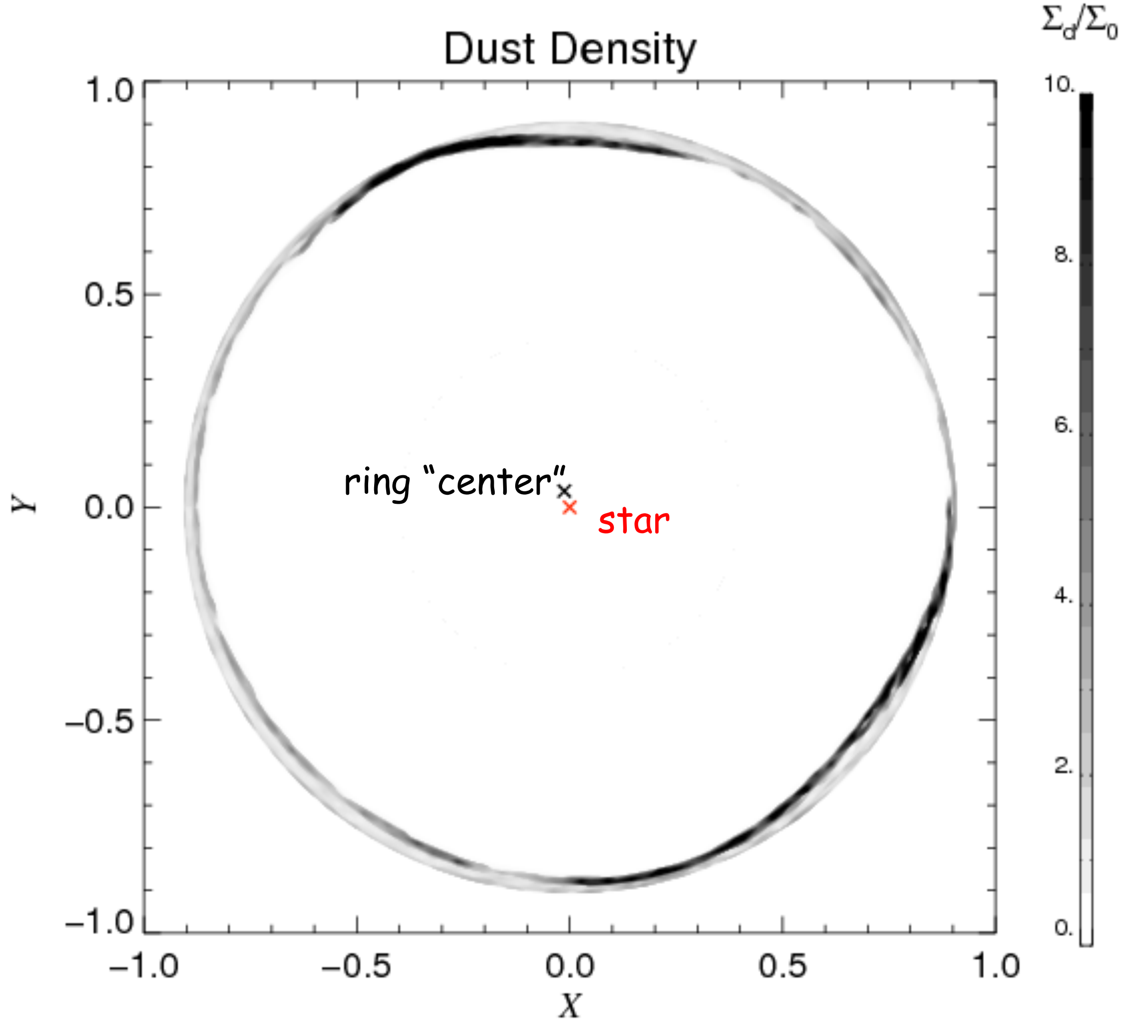

Epicyclic oscillations

make the ring appear ***eccentric !!!***



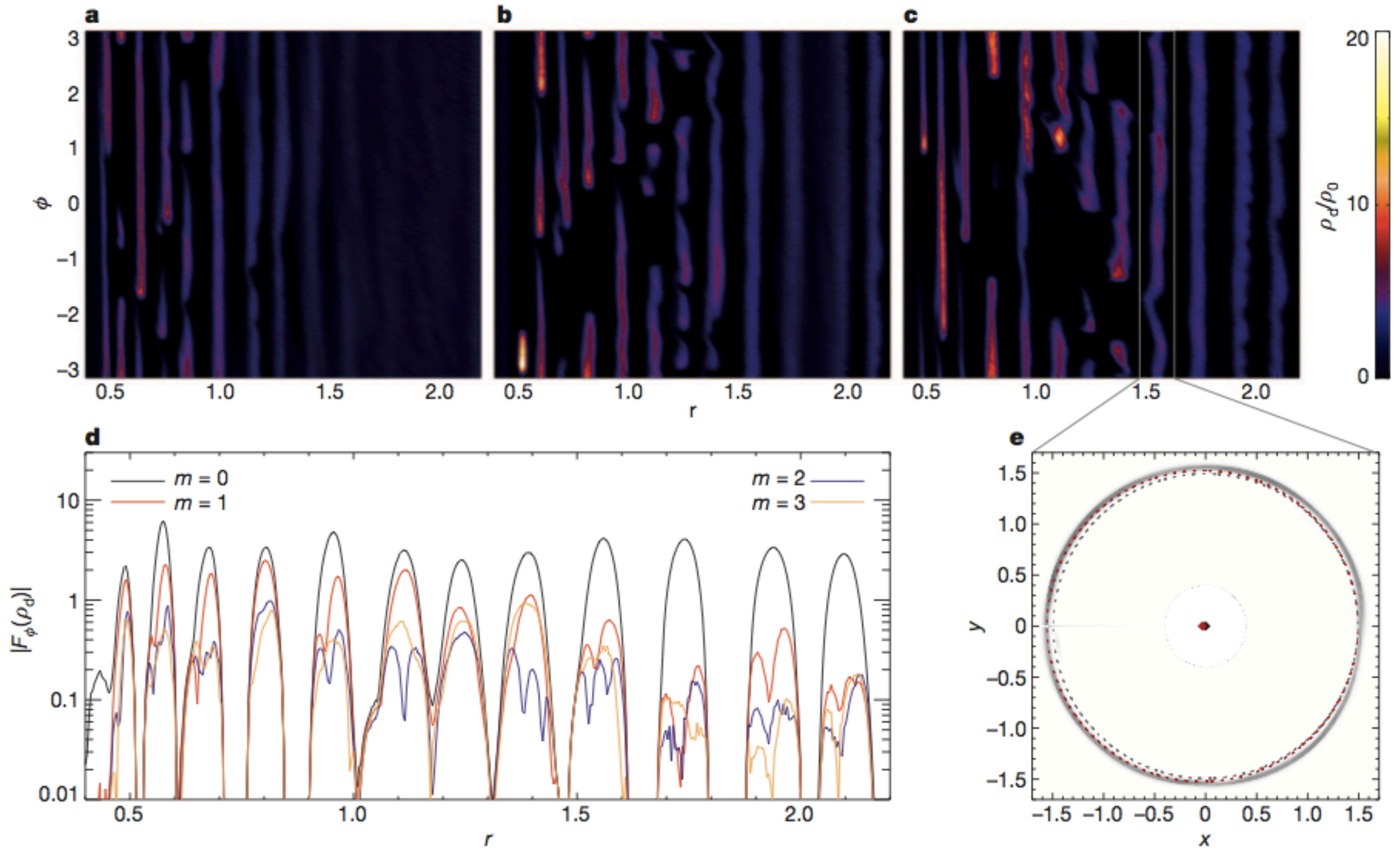
Lyra &  Kuchner (2013, Nature, 499, 148)

# Ring Offset

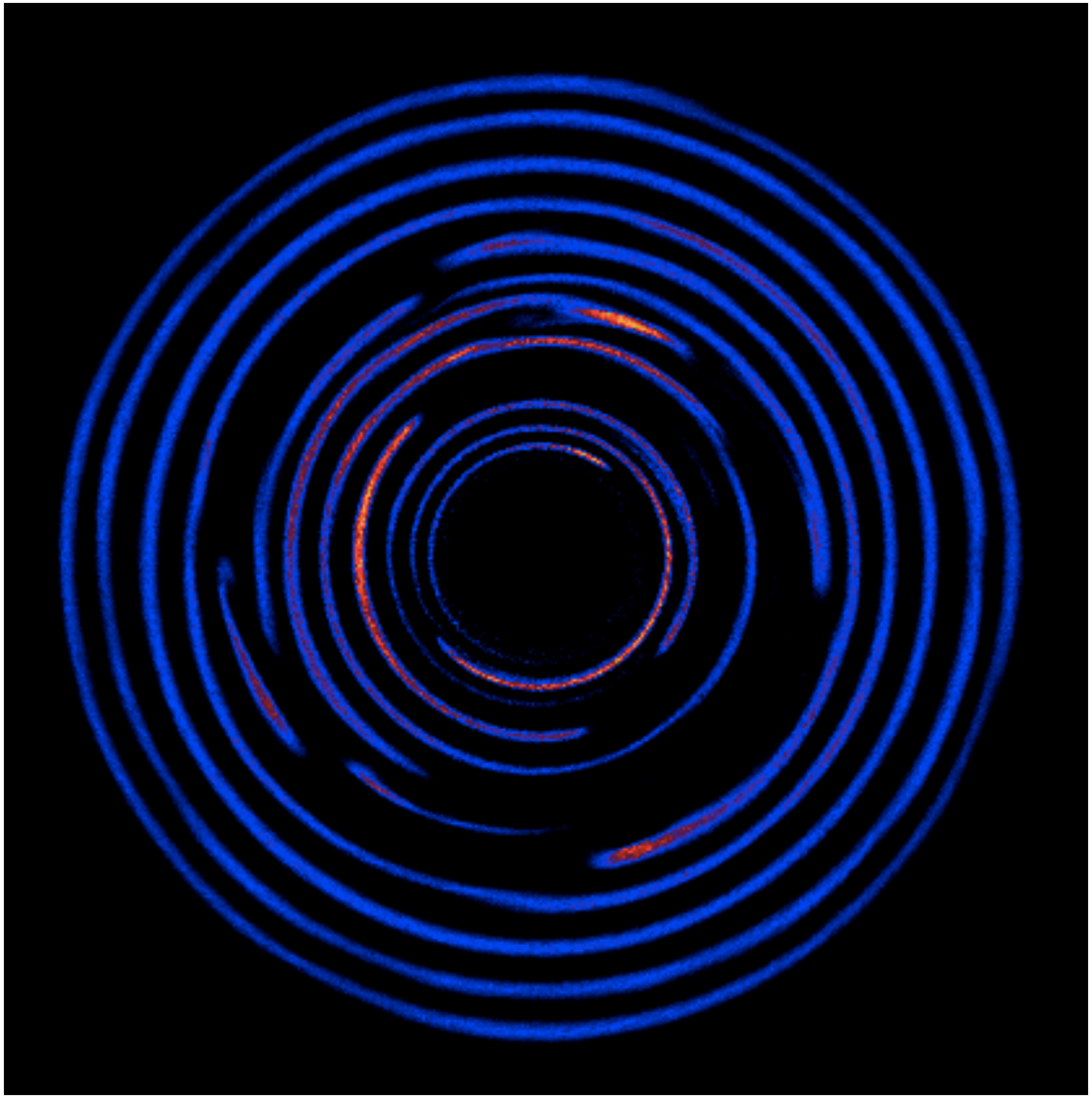



Eccentricity e=0.04

# Ring Shape: eccentric or off-centered?



# Prediction(?): Incomplete rings (arcs)



# Conclusions

There is a robust ring-forming

***photoelectric instability***

in optically thin gas-dust disks

Reproduces gross properties of observed systems

(rings, sharp edges, eccentricity)

Maximum for gas-to-dust ratio ~ 5

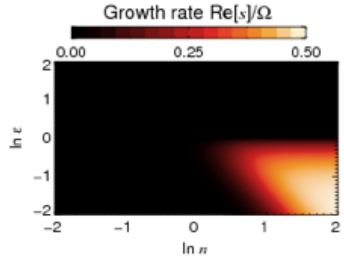
(probably more applicable to transitional disks
and gas-rich debris disks such as 49 Ceti)

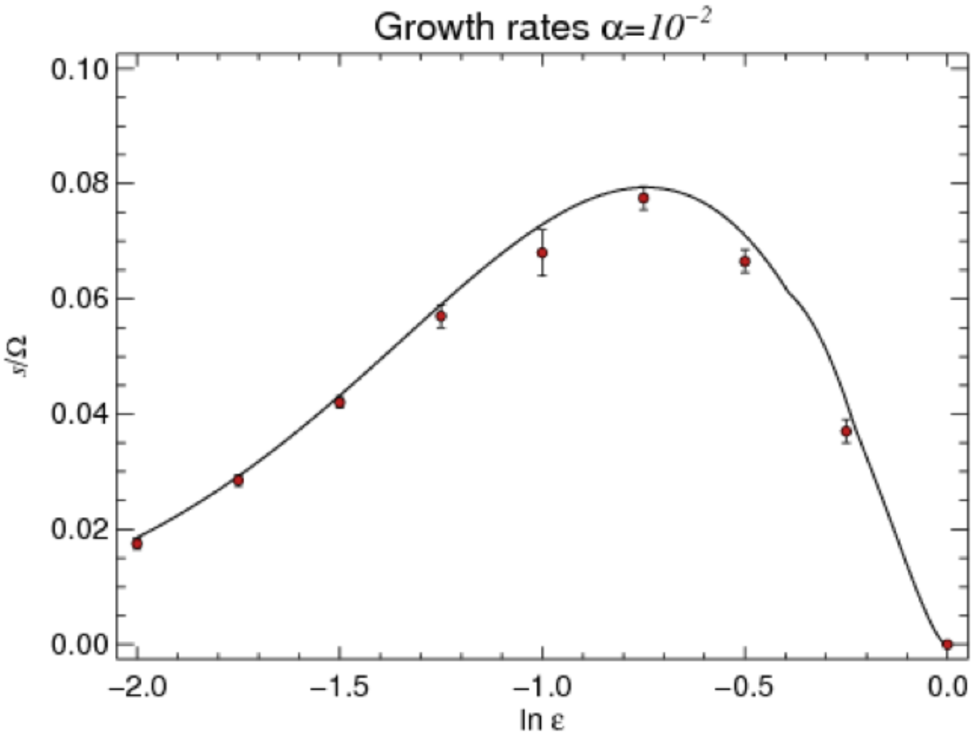
**Future work**:

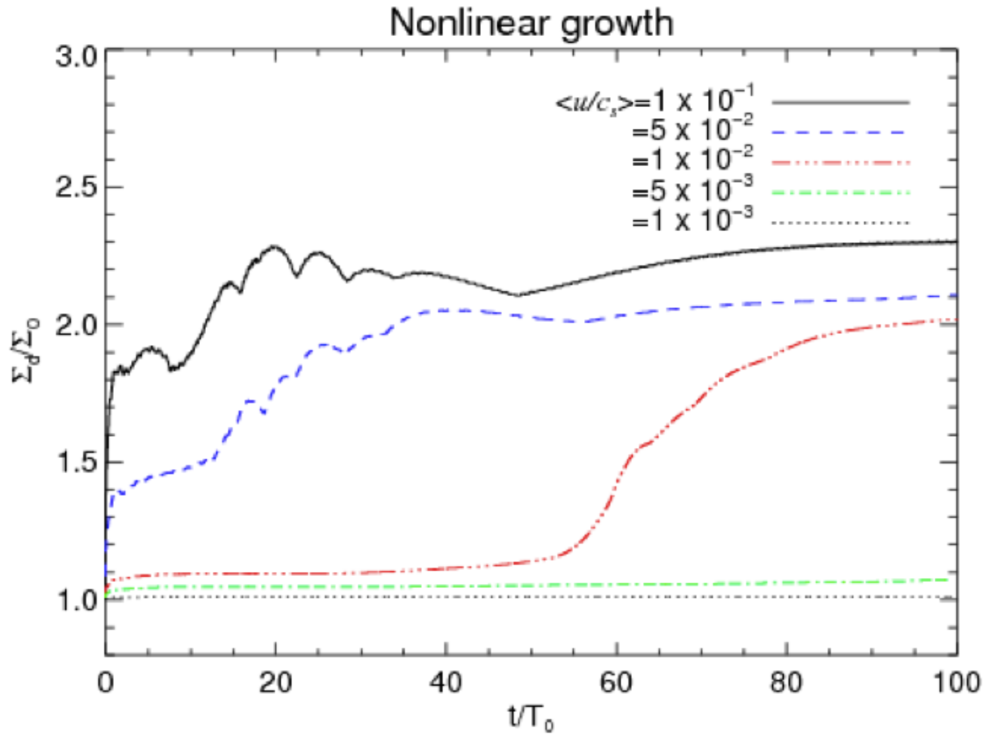
3D turbulence, Radiation forces, Collisions….

*…. (suggestions?)*

# Linear and nonlinear growth



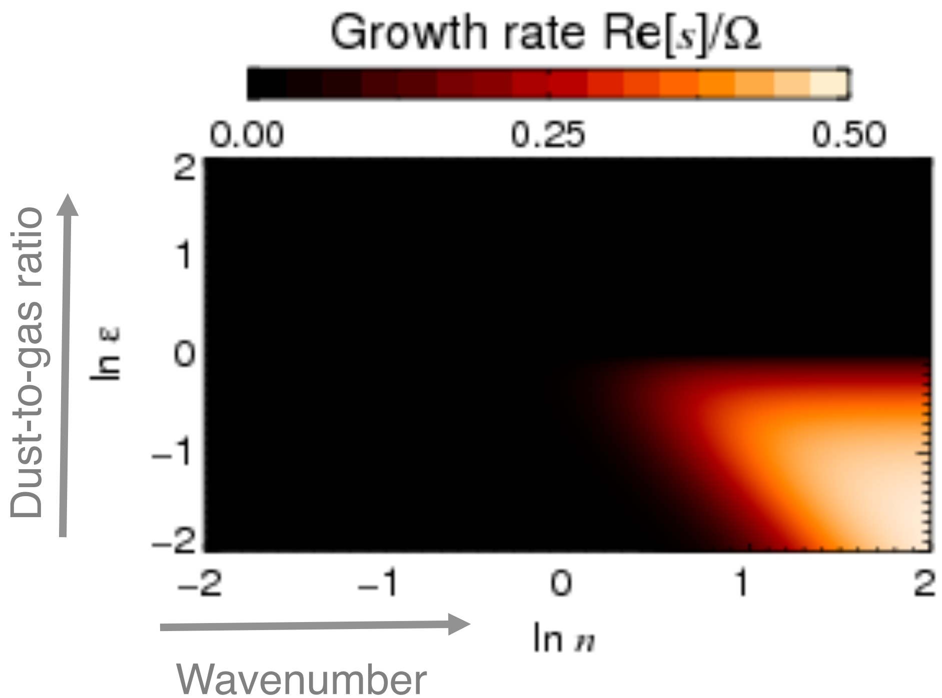



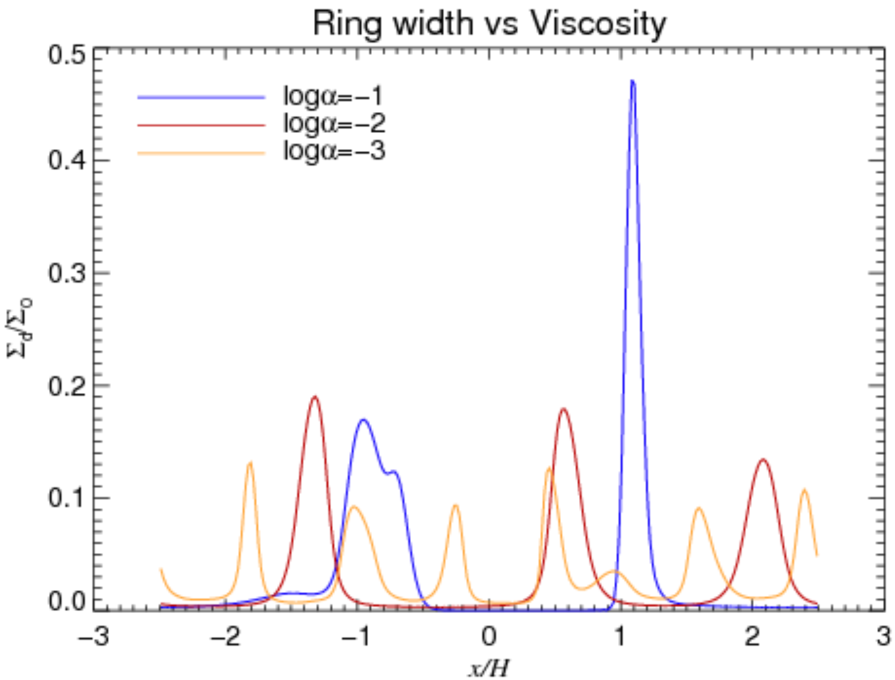

Linear growth only exists for e < 1

But there is
nonlinear growth
beyond !

# Ring Spacing

Ring spacing is determined
by the wavelength of maximum growth.




Which in turn is determined by viscosity

**Ring spacing ~ 10 Kolmogorov lengths**

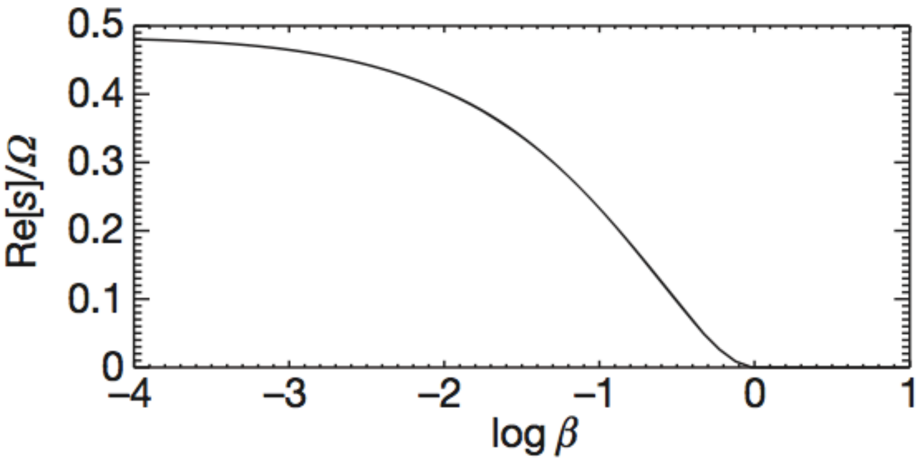
# Photoelectric Instability

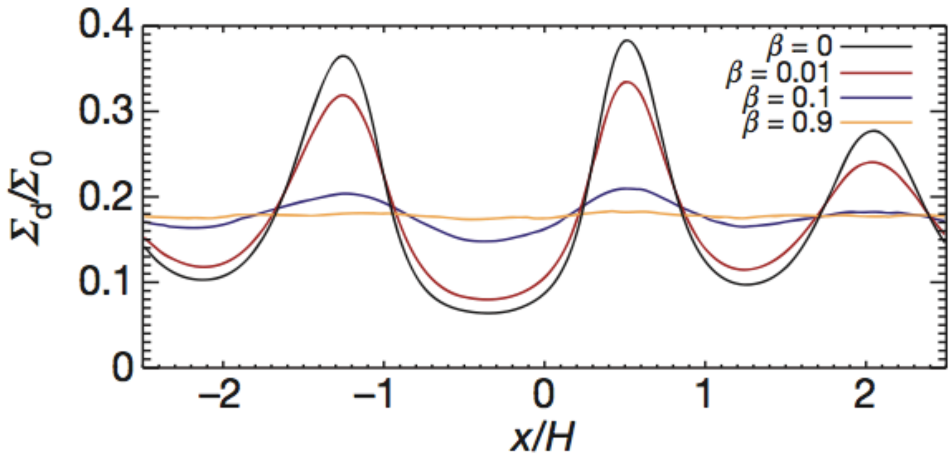
Other heating sources

$$p = \rho_g c_b^2 + p(\rho_d)$$

All other sources Photoelectric

$$\beta = \gamma \left( \frac{c_b}{c_s} \right)^2$$

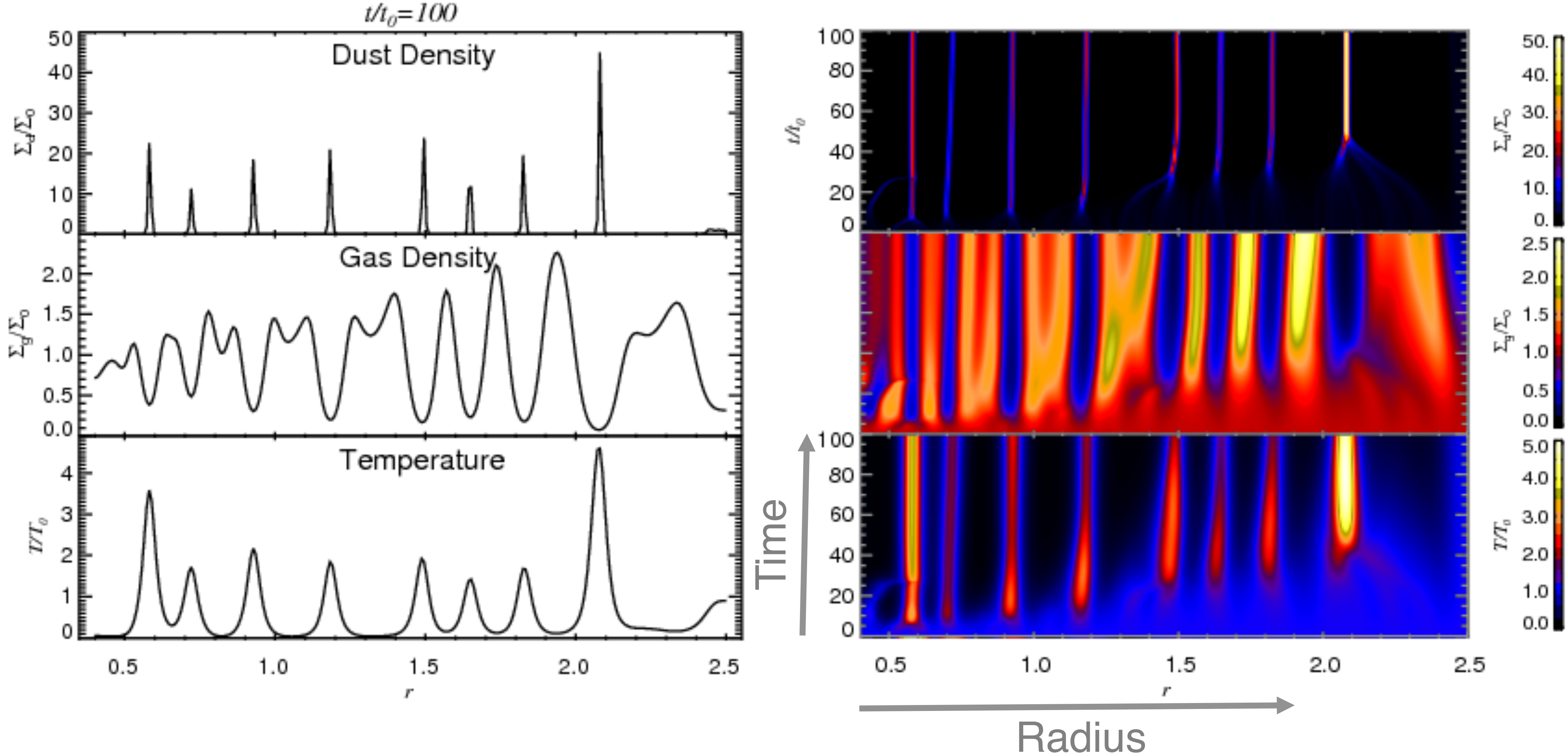



Instability exists *only* when photoelectric heating dominates.

# Photoelectric Instability – Nonlinear evolution in 1D

Dust heats gas
Heated gas = high pressure region
High pressure concentrates dust




**Narrow hot dust rings**
**Cold gas collects between rings**

# Robustness

Growth over 4 orders of magnitude in dust-gas coupling time (friction time)