# Contextualizing the *(insert agreed name)* in the Layered Accretion paradigm


**Wlad Lyra**

NSF Fellow (2010, AMNH)

Carl Sagan Fellow (2011, JPL)

***American Museum of Natural History***

***Max-Planck Institute for Astronomy***


Ringberg, June 2011

Collaborators: Hubert Klahr, Kryzstof Mizerski

# Contextualizing the Baroclinic Instability in the Layered Accretion paradigm

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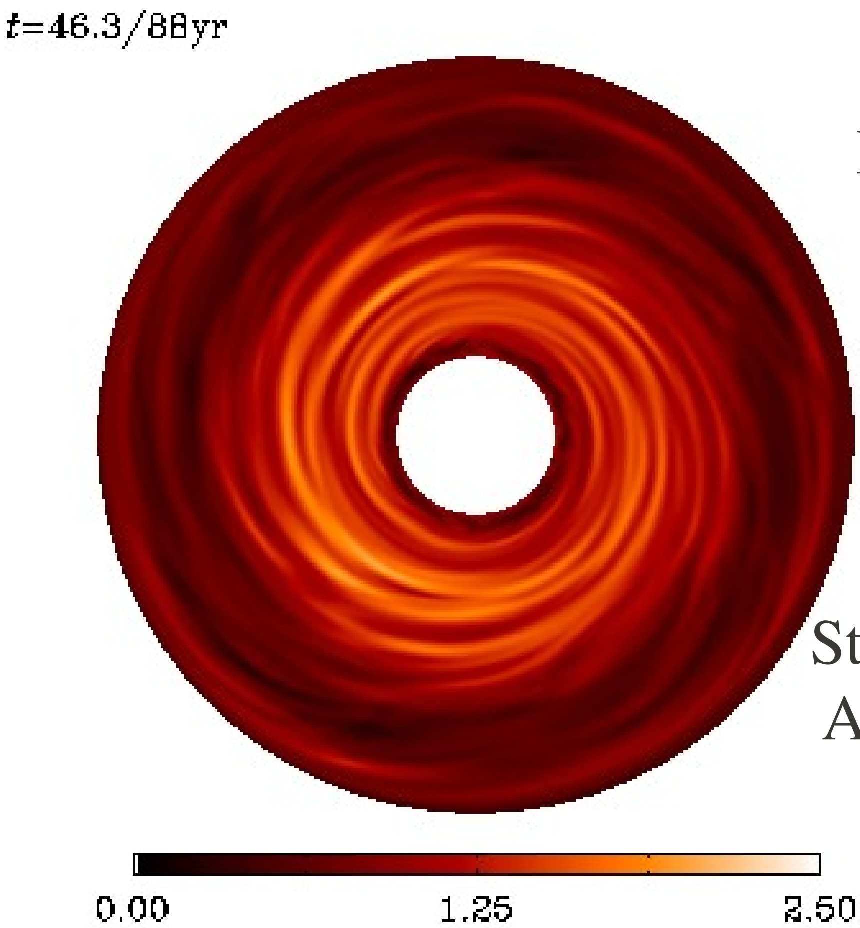
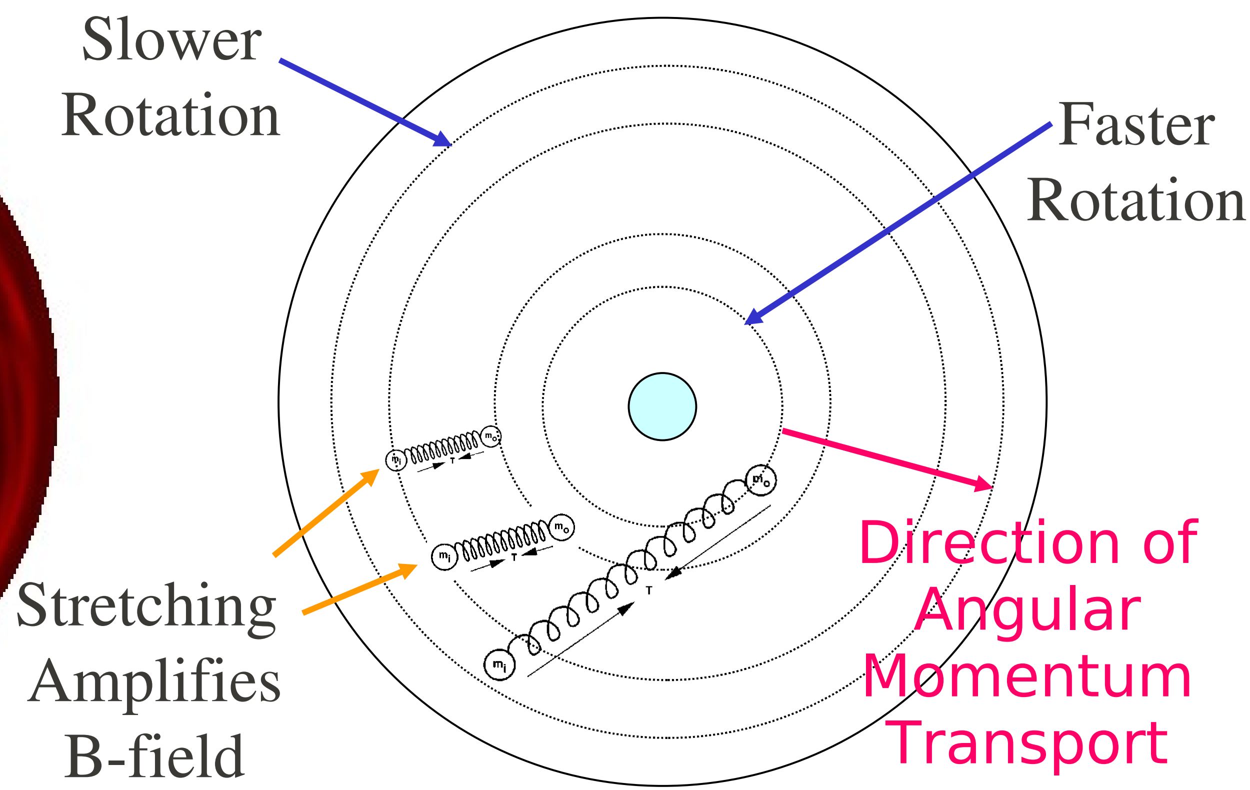
# *Accretion in disks occurs via turbulent viscosity*

Turbulence in disks is enabled by
the Magneto-Rotational Instability

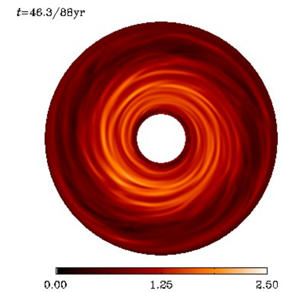



MRI sketch



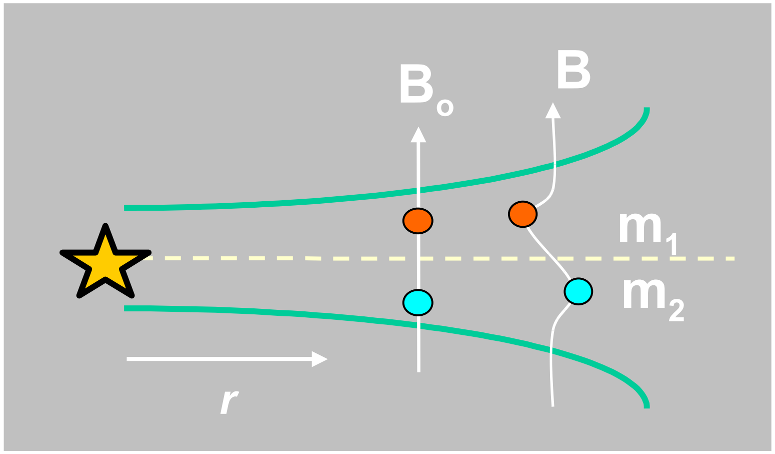

Unstable if angular
velocity decreases
outward

# Accretion in disks occurs via turbulent viscosity



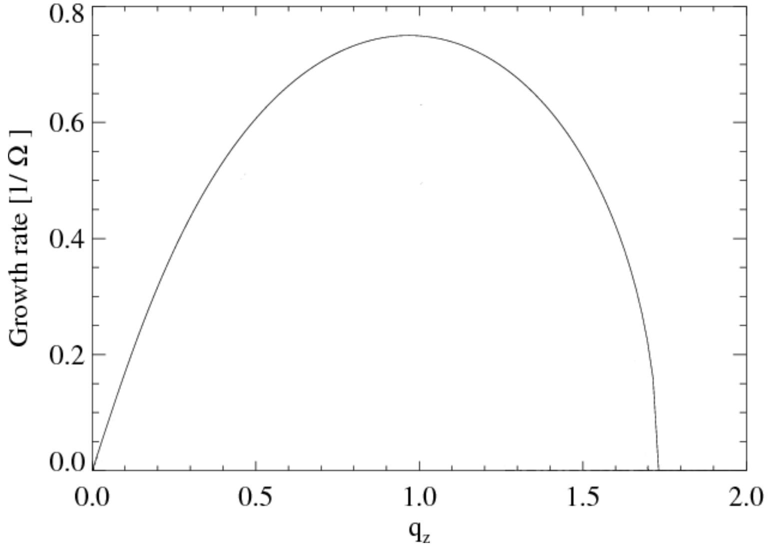

Turbulence in disks is enabled by
the Magneto-Rotational Instability

## Unstable Wavelength




$$k_{BH} = \frac{\Omega}{V_A}$$

## Dispersion Relation



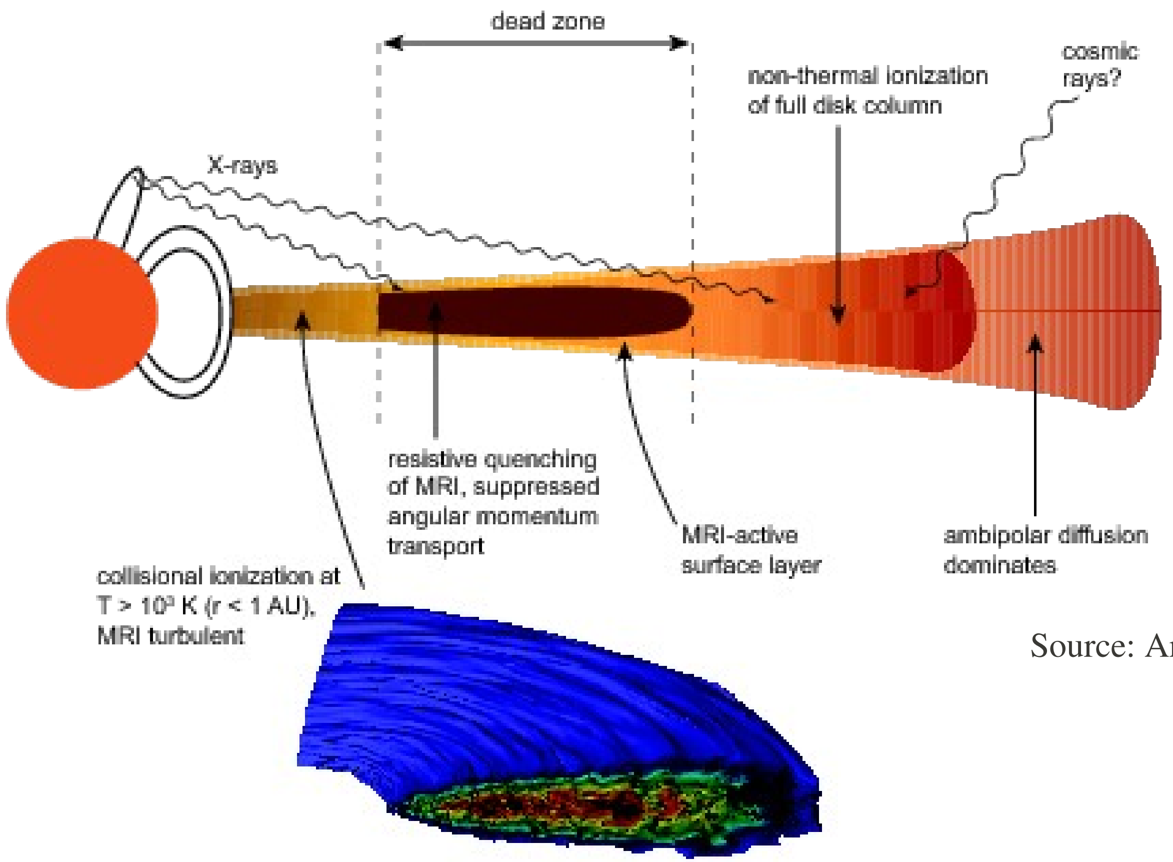

$$q = k / k_{BH}$$

After: Balbus & Hawley (1991)
Hawley & Balbus (1992)

***Properties***

| | |
|---|---|
| Unstable range: | 0 < q < sqrt(3) |
| Most unstable wavelength: | q ~ 1 |
| Maximum growth rate: | ~0.75 $\Omega$ |

# ***Alas... Dead zones are robust features of accretion disks***




Source: Armitage (2010)

Therefore....
The search for hydrodynamical routes
for turbulence continues.

# *Theoretical Modelling*

**Continuity eq. (density)**

$$\frac{\partial \rho}{\partial t} = -(\boldsymbol{u}\cdot\nabla)\rho - \rho\nabla\cdot\boldsymbol{u}$$

**Navier-Stokes eq. (momentum)**

$$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \nabla\Phi - \rho^{-1}\left(\nabla p + \boldsymbol{J}\times\boldsymbol{B}\right)$$

**Induction eq. (magnetic field)**

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times\left(\boldsymbol{u}\times\boldsymbol{B} - \eta\mu_0\boldsymbol{J}\right)$$

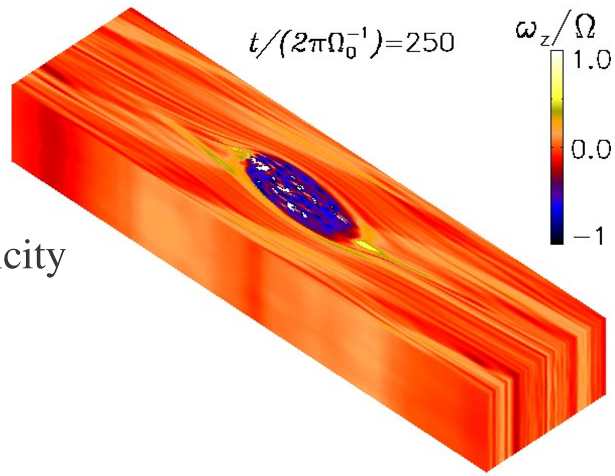
**Entropy equation**

$$\frac{\partial s}{\partial t} = -(\boldsymbol{u}\cdot\nabla)s + \frac{1}{\rho T}\left(\nabla\cdot(K\nabla T) - \rho c_V\frac{(T-T_0)}{\tau_c} + \eta\mu_0\boldsymbol{J}^2\right)$$

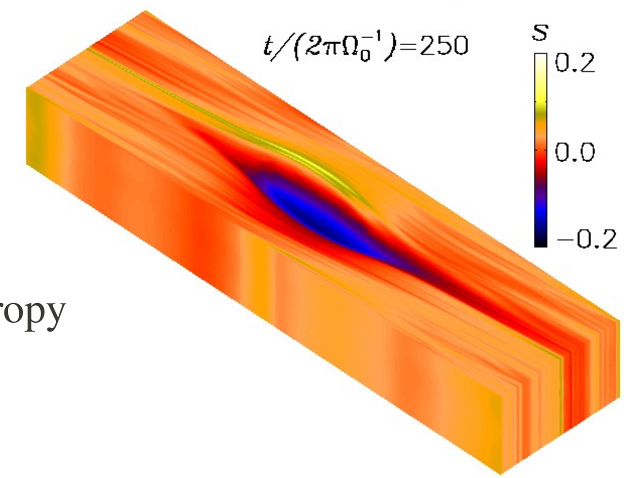
**Eq. of state (pressure)**

$$p = \rho c_s^2/\gamma$$

# ***Baroclinic Instability – Excitation and self-sustenance of vortices***



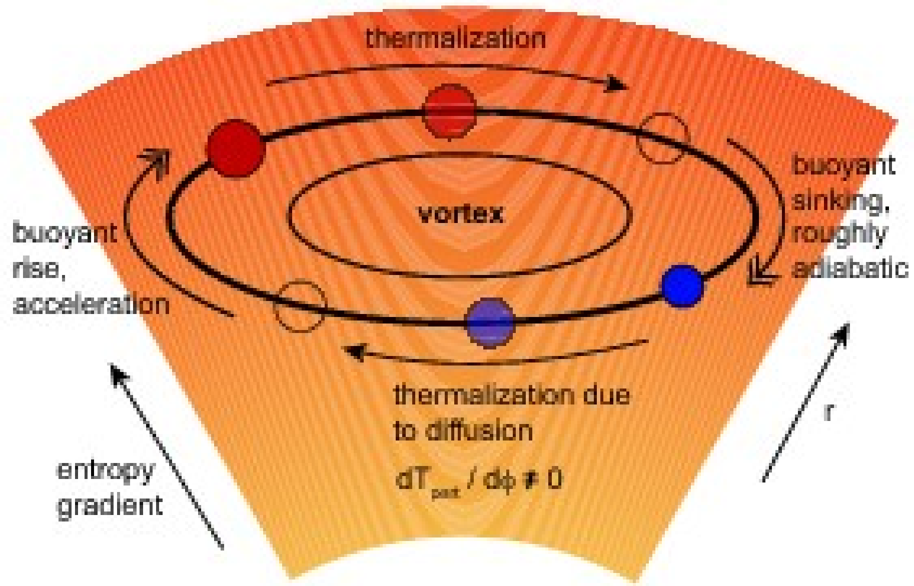

Vorticity




Entropy

Source: Lyra & Klahr (2010)

Sketch of the
Baroclinic Instability

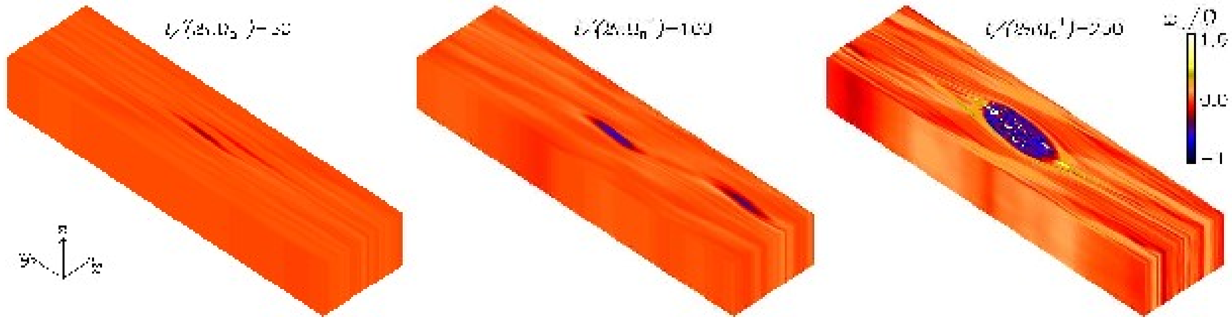



Source: Armitage (2010)

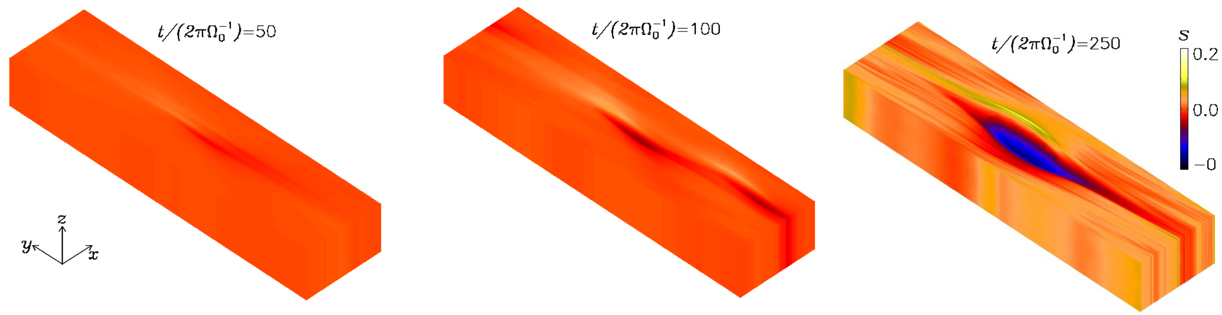
$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \underbrace{-(\boldsymbol{u}\cdot\nabla)\boldsymbol{\omega}}_{\textbf{advection}} \underbrace{-\boldsymbol{\omega}(\nabla\cdot\boldsymbol{u})}_{\textbf{compression}} + \underbrace{(\boldsymbol{\omega}\cdot\nabla)\boldsymbol{u}}_{\textbf{stretching}} + \underbrace{\frac{1}{\rho^2}\nabla\rho\times\nabla p}_{\textbf{baroclinicity}} + \underbrace{\nu\nabla^2\boldsymbol{\omega}}_{\textbf{dissipation}}$$

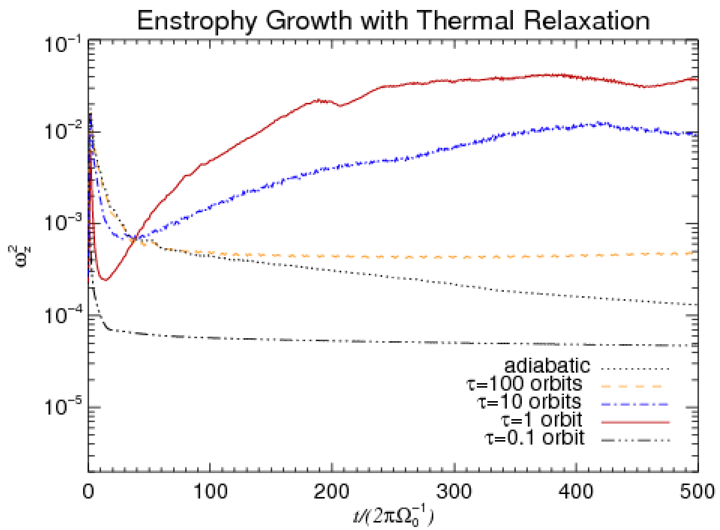
# Baroclinic Instability – Excitation and self-sustenance of vortices

Vorticity



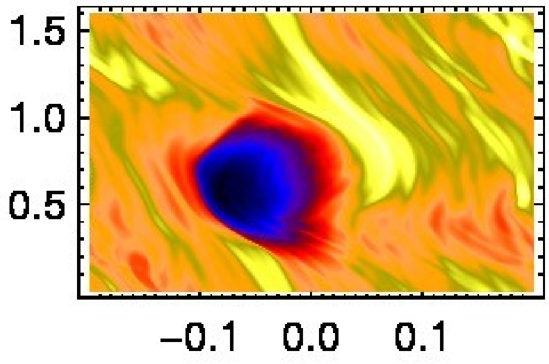
Entropy



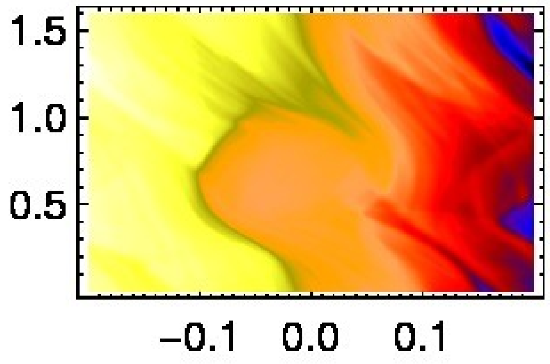




Source: Lyra & Klahr (2010)

# Flattening the Entropy Gradient




Residual Entropy




Residual Entropy
+
Background Entropy Gradient

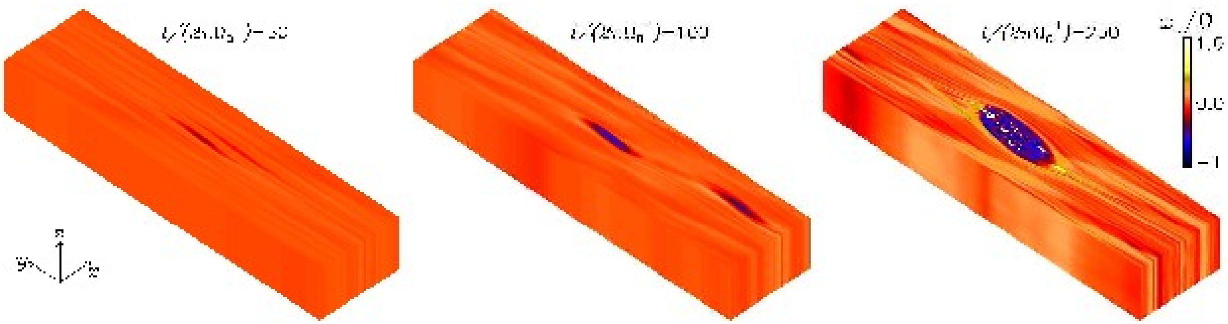
# The recipe for the Baroclinic Instability:

| | |
|---|---|
| • *Thermal diffusion or relaxation* | **Thermal diffusion** is needed to establish the azimuthal temperature gradients. |
| • *Finite amplitude perturbations* | **Finite amplitude** is needed because the perturbations have to be strong enough to not be sheared away by the Keplerian motion. The instability is ***non-linear***. |
| • *Long evolution times* | **Long evolution** times are needed for the same reason. The growth rate is proportional not to $h$ but to $h^2$. |
| • *High resolution* | A **resolution** requirement of at least 64 points per $H$ is found to capture the instability in numerical simulations with the **Pencil Code**. |

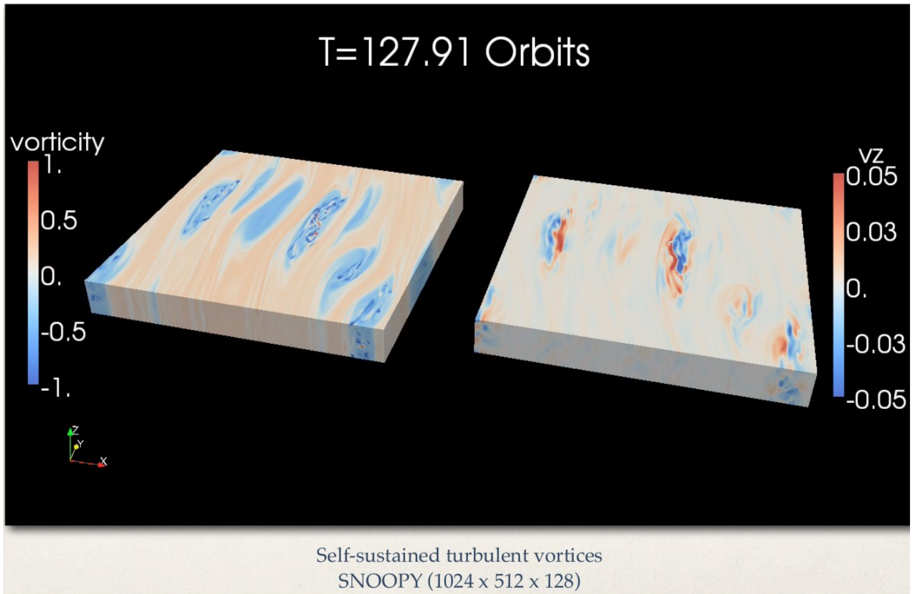
# Baroclinic Instability – Excitation and self-sustenance of vortices

The Baroclinic Instability in three dimensions

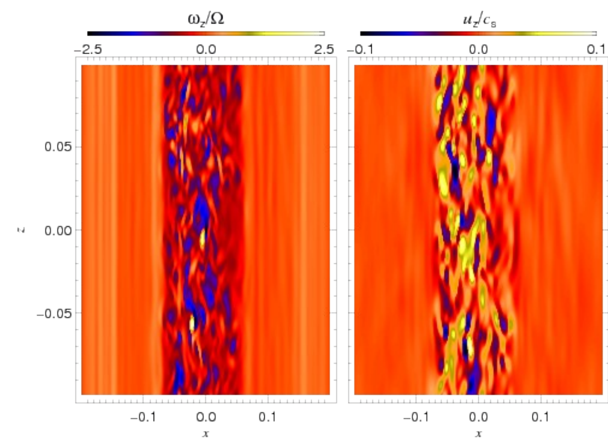
Vorticity



Source: Lyra & Klahr (2010)

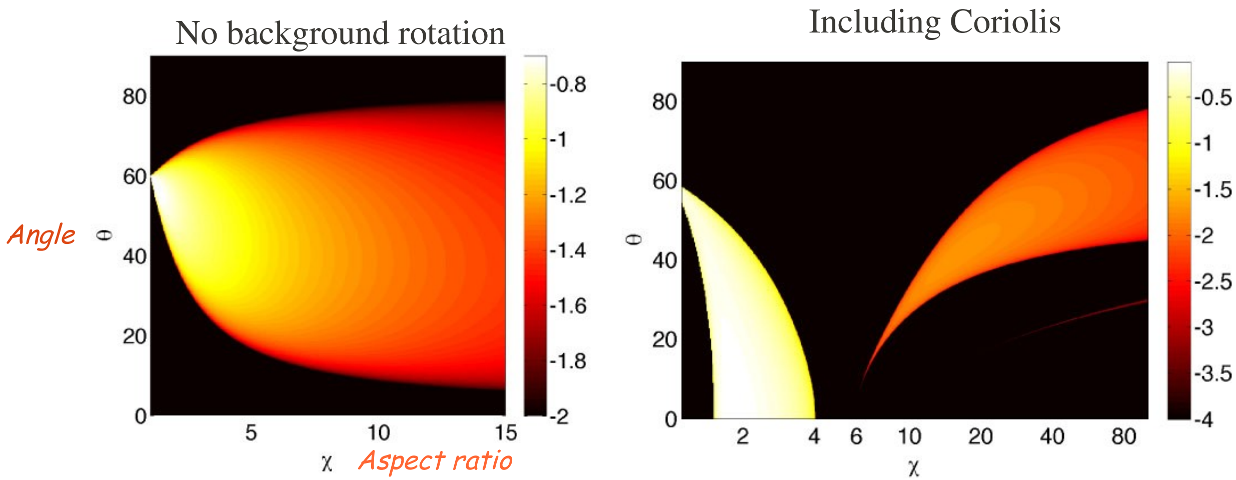



Source: Lesur & Papaloizou (2010)

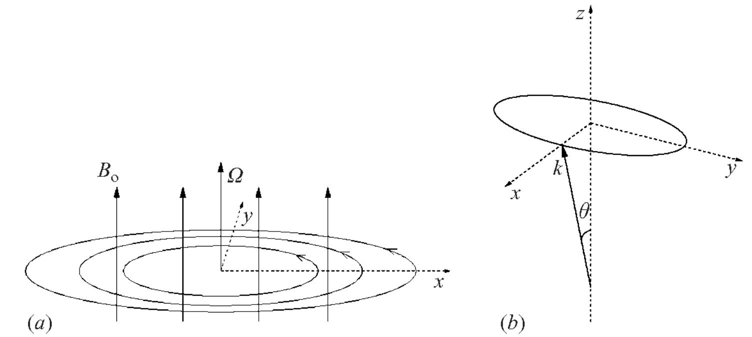



Vorticity Kin. Energy

# Elliptic Instability




Source: Lesur & Papaloizou (2009)




Source: Mizerski & Bajer (2009)

**Instability of elliptic streamlines**

* In the **non-rotating** case:

- **Resonance** between

  **Vortex turnover frequency** and **Inertial waves**

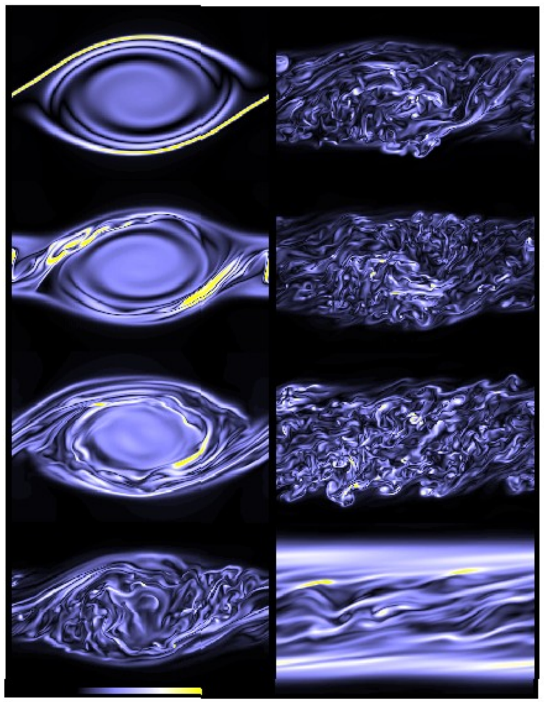
  *Rossby – Kelvin Instability ?*

* In the **rotating** case:

- Strong "horizontal" (theta=0) unstable mode:

  **Exponential growth of epicyclic disturbances**

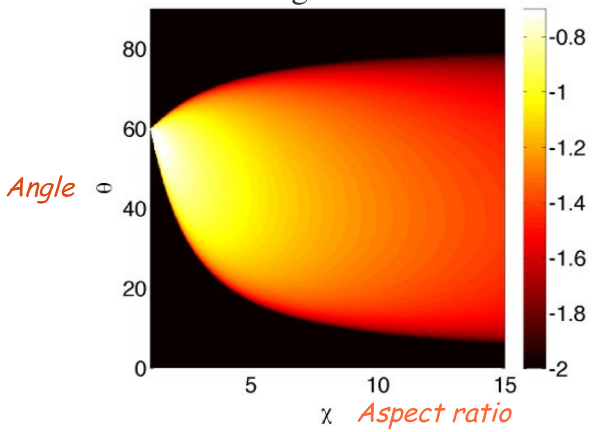
Vortex coherence is destroyed
Energy cascades forward and dissipates



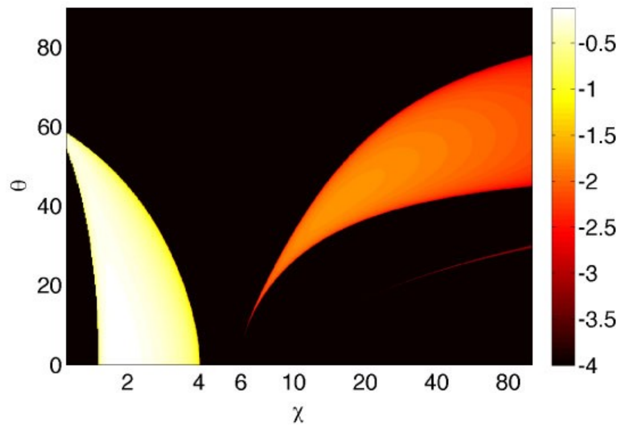
Source: McWilliams (2010)

# Elliptic Instability

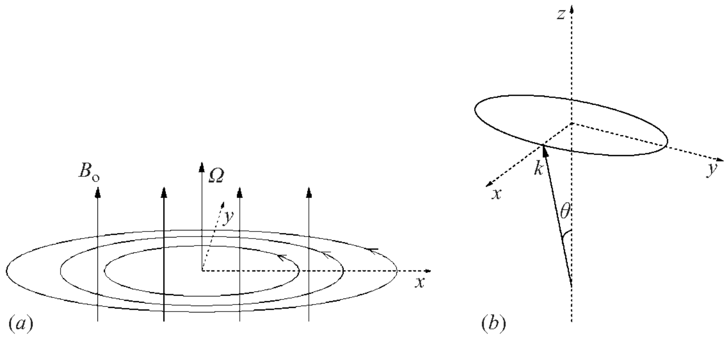
No background rotation




Including Coriolis




Source: Lesur & Papaloizou (2009)

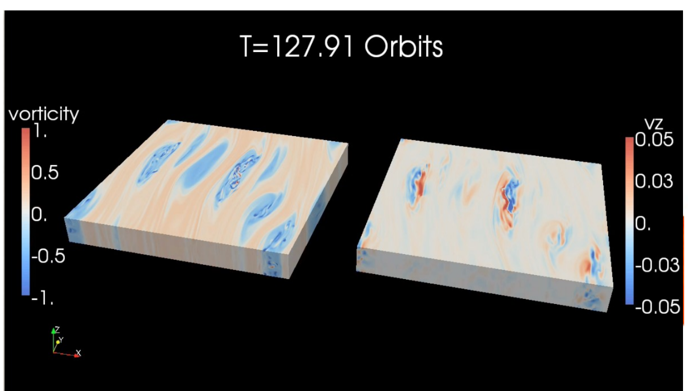



Source: Mizerski & Bajer (2009)

Despite the elliptical instability,

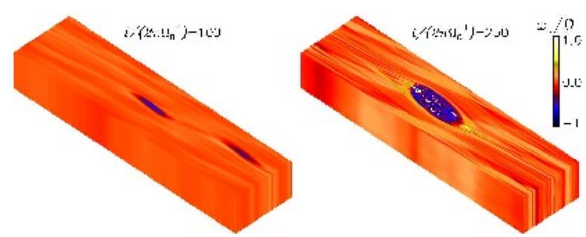
**baroclinity keeps the vortex coherent.**

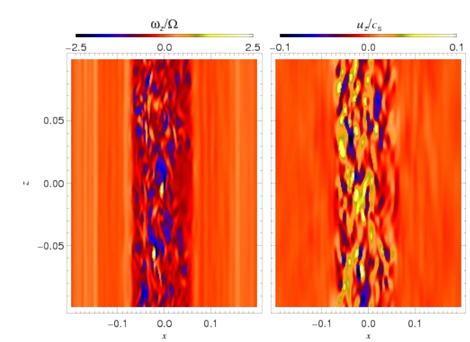
The result is "core turbulence" only




Self-sustained turbulent vortices
SNOOPY (1024 x 512 x 128)

Source: Lesur & Papaloizou (2010)

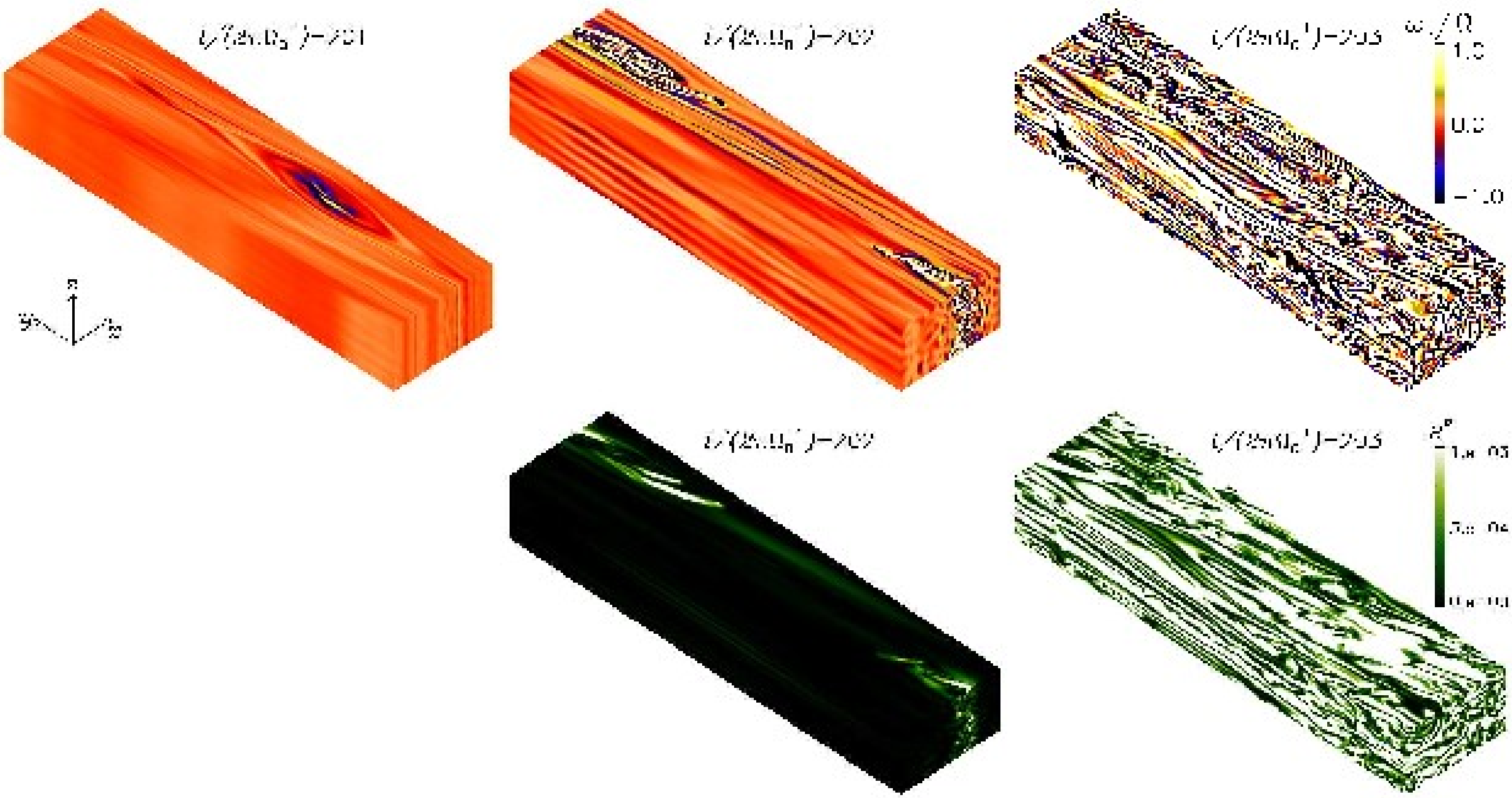





Source: Lyra & Klahr (2010)

# ***Interaction of Baroclinic and Magneto-Rotational Instabilities***

What happens when the vortex is magnetized?



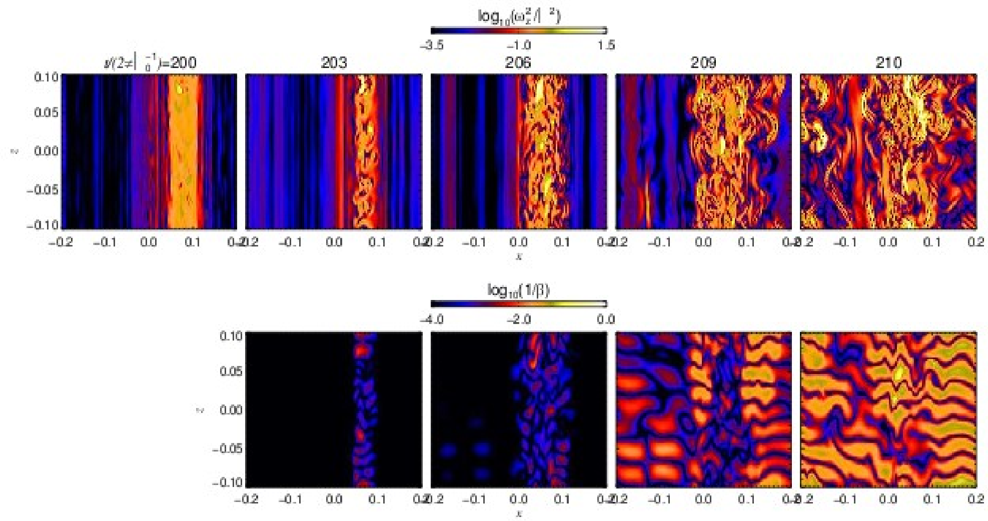
*Vorticity*

*Magnetic Energy*

Vortex gone!

# ***Vortex MHD instability?***

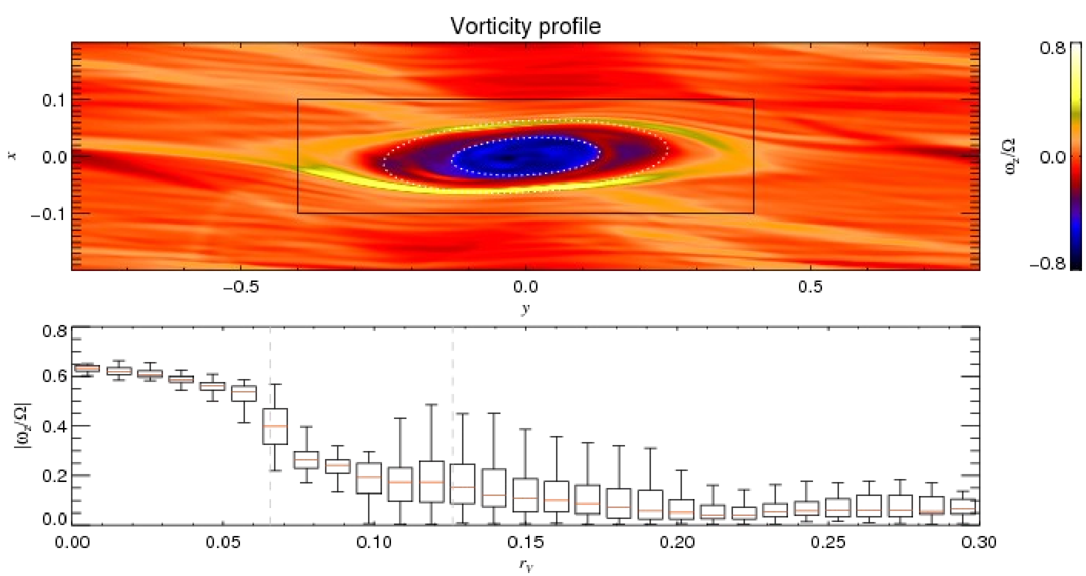
Notice that the vortex goes turbulent ***before*** the box.




*Enstrophy*

*Mag. Energy*

***Is this the MRI?***



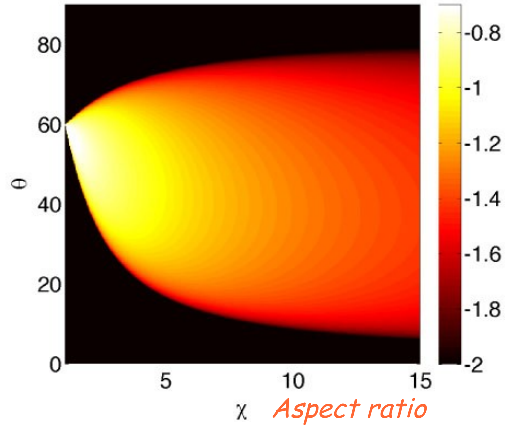

The MRI needs shear.
Yet, the core rotates close to rigid.

*So, **NO**, this is **not** the MRI.*

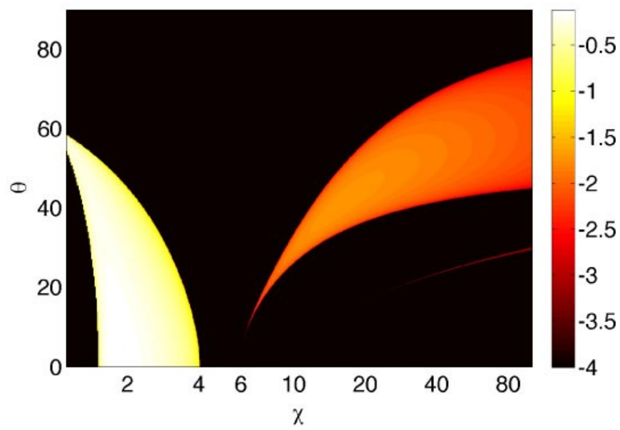
A magnetoelliptic mode?

# ***Magneto-Elliptic Instability***

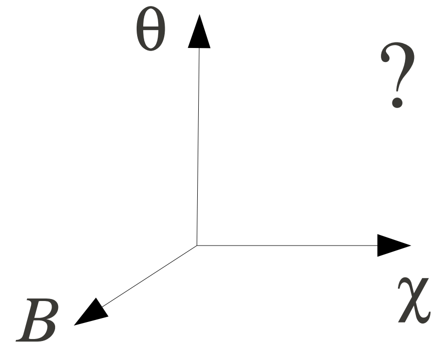
No background rotation




Including Coriolis




Including Lorentz

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doi:10.1017/S0022112009007307 Printed in the United Kingdom

## The magnetoelliptic instability of rotating systems

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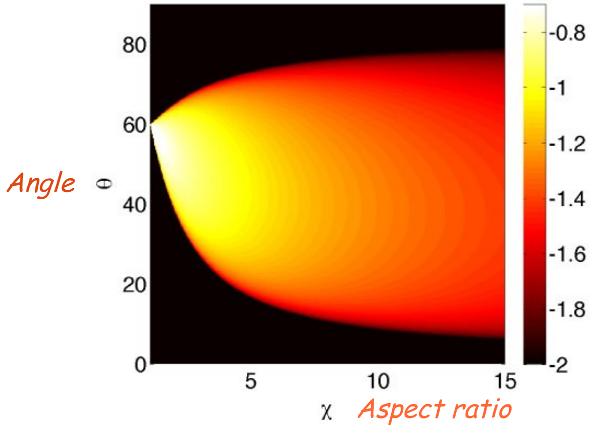
(Received 10 December 2008 and in revised form 12 March 2009)

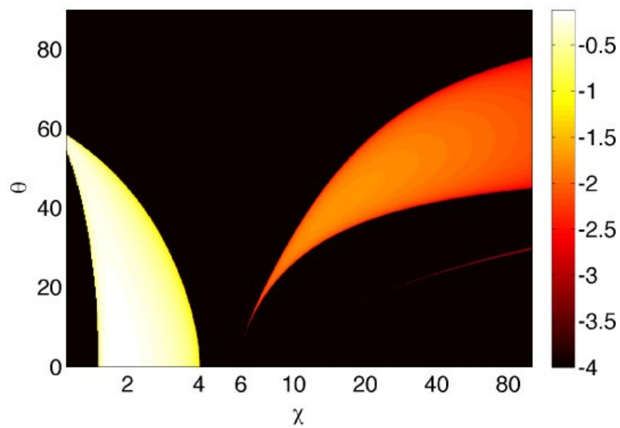
We address the question of stability of the Euler flow with elliptical streamlines in a rotating frame, interacting with uniform external magnetic field perpendicular to the plane of the flow. Our motivation for this study is of astrophysical nature, since many astrophysical objects, such as stars, planets and accretion discs, are tidally deformed through gravitational interaction with other bodies. Therefore, the ellipticity of the flow models the tidal deformations in the simplest way. The joint effect of the magnetic field and the Coriolis force is studied here numerically and analytically in the limit of small elliptical (tidal) deformations ($\zeta \ll 1$), using the analytical technique developed by Lebowitz & Zweibel (*Astrophys. J.*, vol. 609, 2004, pp. 301–312). We find that the effect of background rotation and external magnetic field is quite complex. Both factors are responsible for new destabilizing resonances as the vortex departs from axial symmetry ($\zeta \ll 1$); however, just like in the non-rotating case, there are three principal resonances causing instability in the leading order. The presence of the magnetic field is very likely to destabilize the system with respect to perturbations propagating in the direction of the magnetic field if the basic vorticity and the background rotation have opposite signs (i.e. for *anticyclonic* background rotation). We present the dependence of the growth rates of the modes on various parameters describing the system, such as the strength of the magnetic field ($h$), the inverse of the Rossby number ($\mathscr{R}_o$), the ellipticity of the basic flow ($\epsilon$) and the direction of

# Magneto-Elliptic Instability

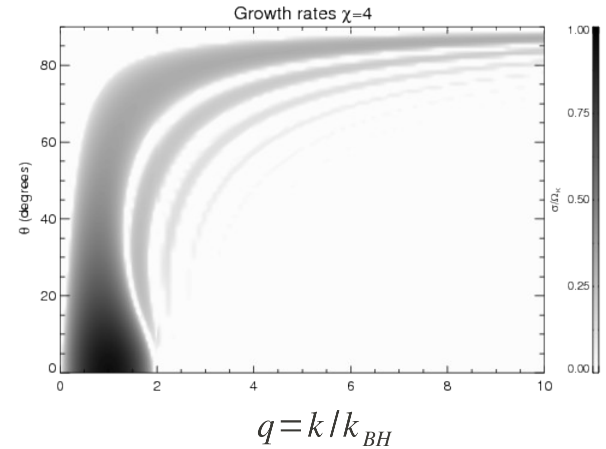
No background rotation




Including Coriolis




Including Coriolis and Lorentz




Lyra & Klahr 2011
(after Mizerski & Bajer 2009)

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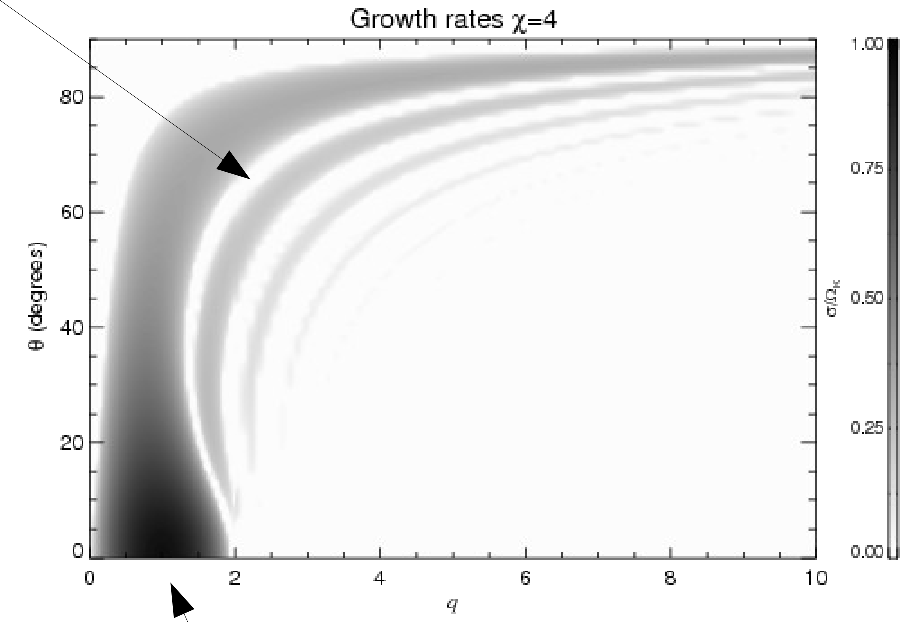
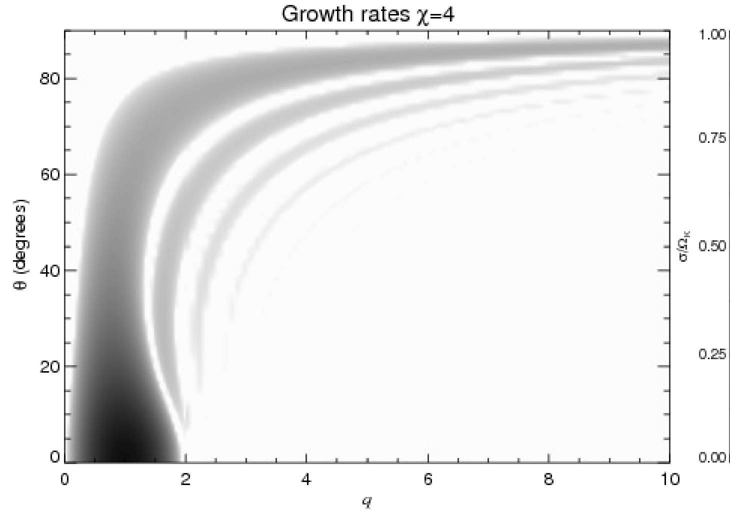
# ***Magneto-Elliptic Instability***

Bands: Resonances with
Alfvén waves
( *Rossby – Alfvén Instability* ? )




"Horizontal" magneto-elliptic instability

# ***Magneto-Elliptic Instability***




Lyra & Klahr 2011
(after Mizerski & Bajer 2009)

Mizerski & Bajer (2009, Journal of Fluid Mechanics)

"The presence of magnetic fields widens the range of existence of the horizontal instability to an unbounded interval of aspect ratios when

$$Ro^{-1} < -\frac{b^2}{4}$$

$$\boxed{0 < k/k_{BH} < 2|Ro|^{1/2}}$$

$$b = q/Ro$$

$$q = k/k_{BH}$$

$$Ro = \frac{\Omega_V \delta}{\Omega_K}$$

$$k_{BH} = \frac{\Omega_K}{v_A}$$

$$\delta = \frac{1}{2}(\chi + \chi^{-1})$$

# ***Magneto-Elliptic Instability***

$$Ro = \frac{\Omega_V \delta}{\Omega_K}$$

$$\delta = \frac{1}{2}(\chi + \chi^{-1})$$

$\chi$ Aspect ratio

$$\boxed{0 < k/k_{BH} < 2|Ro|^{1/2}}$$

Write the criterion in terms of vorticity instead of angular frequency:

Vortex flow

$$u_x = -\Omega_V\, y/\chi$$

$$u_y = \Omega_V\, x\,\chi$$

$$\xrightarrow{\nabla\times}$$

Vorticity

$$\omega_T = \Omega_V(\chi + \chi^{-1}) = 2\,\Omega_V\,\delta = 2\,Ro\,\Omega_K$$

$$= \omega_V + \omega_{box}$$

$$= \omega_V - 3/2\,\Omega_K$$

$$\boxed{Ro = \frac{\omega_V}{2\,\Omega_K} - \frac{3}{4}}$$

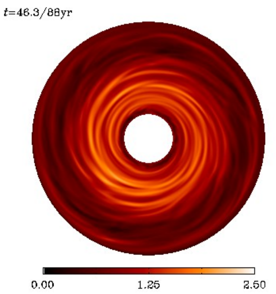
In the no-vortex limit ($\omega_v$=0) , Ro=-3/4

$$\boxed{0 < k/k_{BH} < \sqrt{3}}$$

## ***Magneto-Elliptic Instability – No vortex limit***

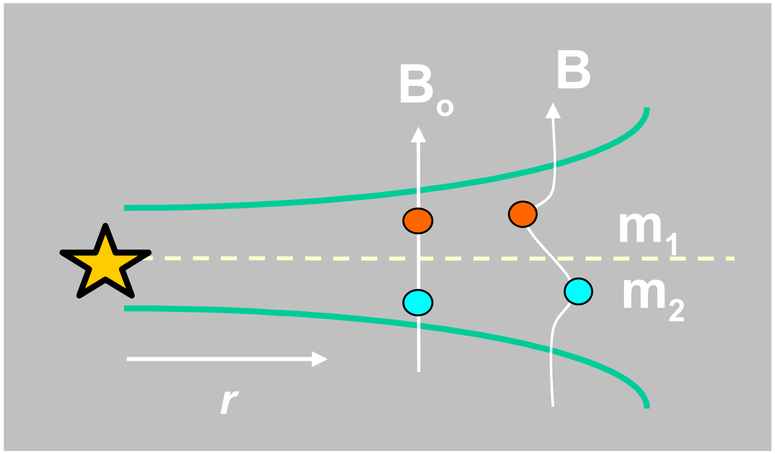
$$\boxed{0 < k / k_{BH} < \sqrt{3}}$$

# Accretion in disks occurs via turbulent viscosity



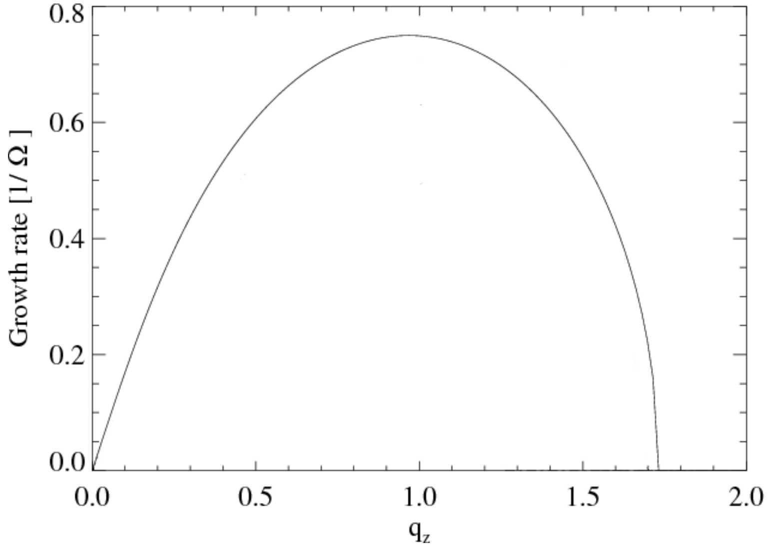

Turbulence in disks is enabled by
the Magneto-Rotational Instability

## Unstable Wavelength




$$k_{BH} = \frac{\Omega}{V_A}$$

## Dispersion Relation




$$q = k / k_{BH}$$

After: Balbus & Hawley (1991)
Hawley & Balbus (1992)

### *Properties*

| | |
|---|---|
| Unstable range: | $0 < q < \mathrm{sqrt}(3)$ |
| Most unstable wavelength: | $q \sim 1$ |
| Maximum growth rate: | $\sim 0.75\ \Omega$ |

## ***Magneto-Elliptic Instability – No vortex limit***

$$\boxed{0 < k/k_{BH} < \sqrt{3}}$$

**MRI**

# Note on consistency: The “no-vortex” limit is a pure shear flow

$$Ro=\frac{\Omega_V\,\delta}{\Omega_K}$$

$$\delta=\frac{1}{2}(\chi+\chi^{-1})$$

$\chi$ Aspect ratio

Kida solution

$$\Omega_V=-\frac{3\Omega_K}{2(\chi-1)}$$

$$Ro=-\frac{3}{4}\frac{\chi^2-1}{\chi(\chi-1)}$$

$$\Omega_V=0$$
$$\chi\rightarrow\infty$$

Remember:
In the no-vortex limit ($\omega_v$=0) , Ro=-3/4

$$\xrightarrow{\chi\rightarrow\infty}$$

$$lim_{\chi\rightarrow\infty}\,Ro=-\frac{3}{4}$$

*QED!*

Vortex flow

$$u_x=-\Omega_V\,y/\chi$$

$$u_y=\Omega_V\,x\,\chi$$

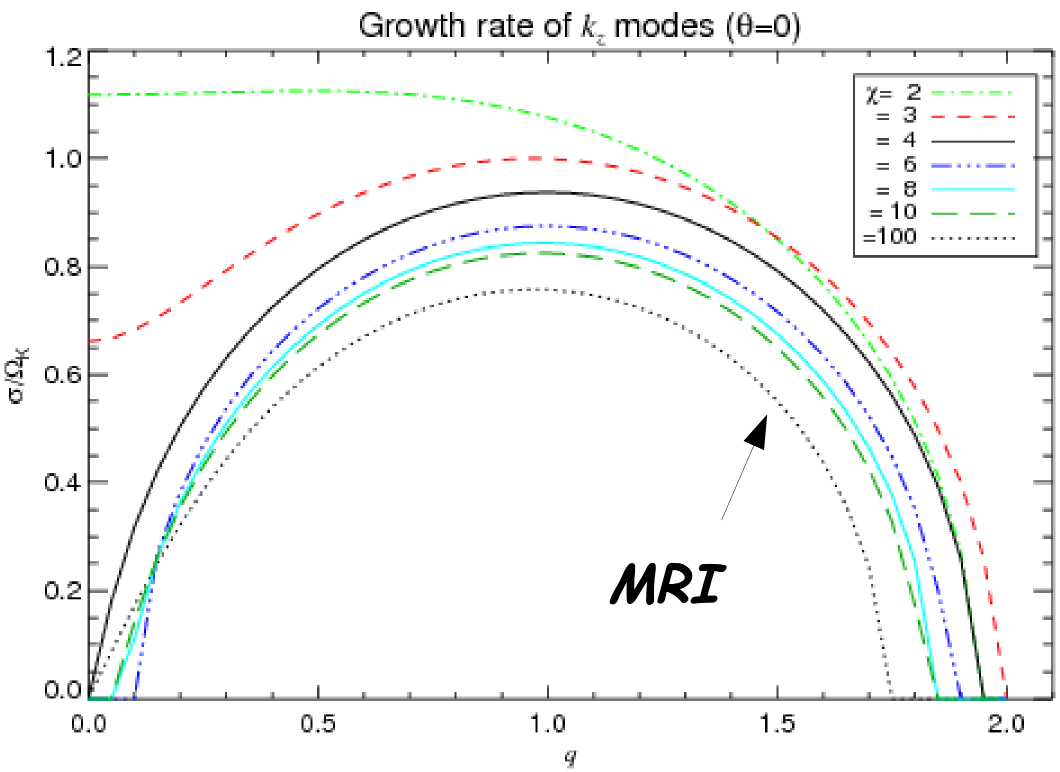
$$\xrightarrow{\chi\rightarrow\infty}$$

$$lim_{\chi\rightarrow\infty}\,u_x=0$$

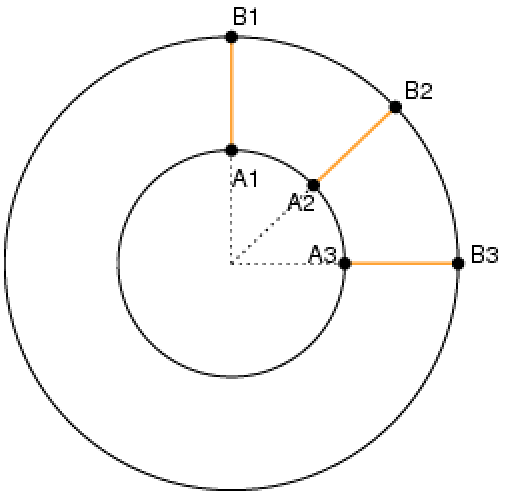
$$lim_{\chi\rightarrow\infty}\,u_y=-3/2\,\Omega_K\,x$$

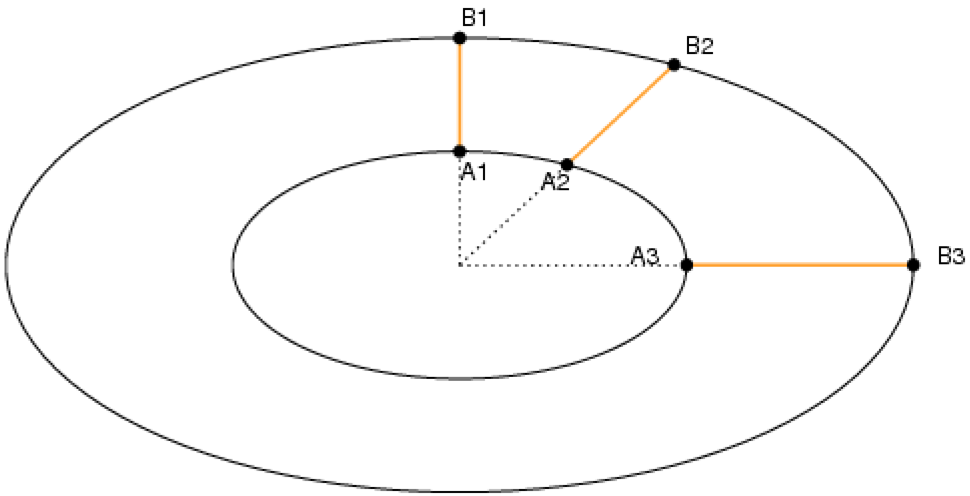
A vortex of infinite aspect ratio is equivalent to a shear flow

# ***Growth rates of the Horizontal MEI***



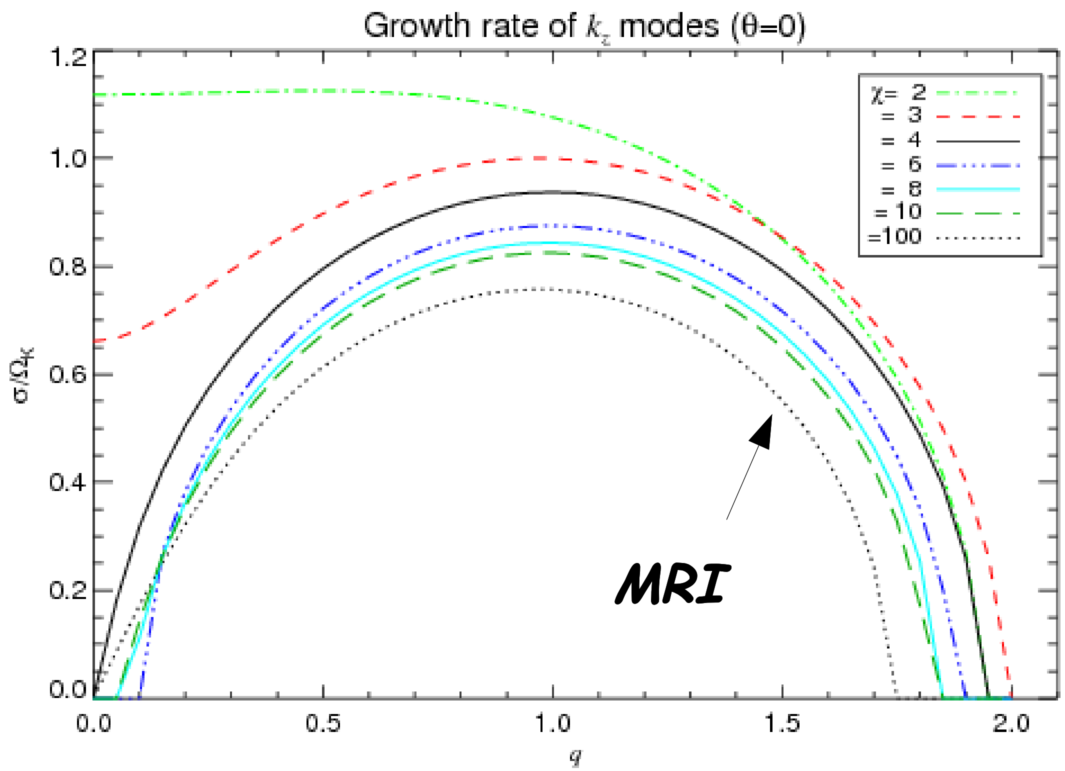
# ***Common ground between MRI and MEI***



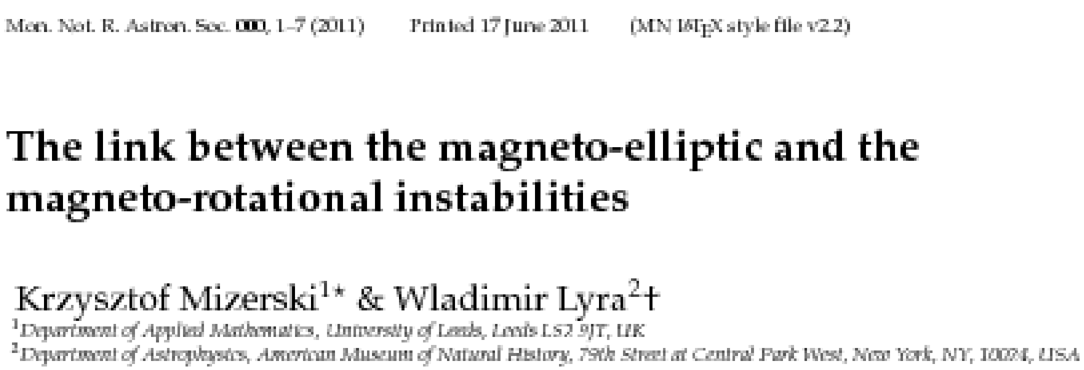

Elliptic streamlines have shear
even in uniform rotation.

# Growth rates of the Magneto-Elliptic-Rotational Instability




… we explain both
the MEI and the MRI
as different manifestations
of the same
***Magneto-Elliptic-Rotational Instability***

Mizerski & Lyra (in prep.)

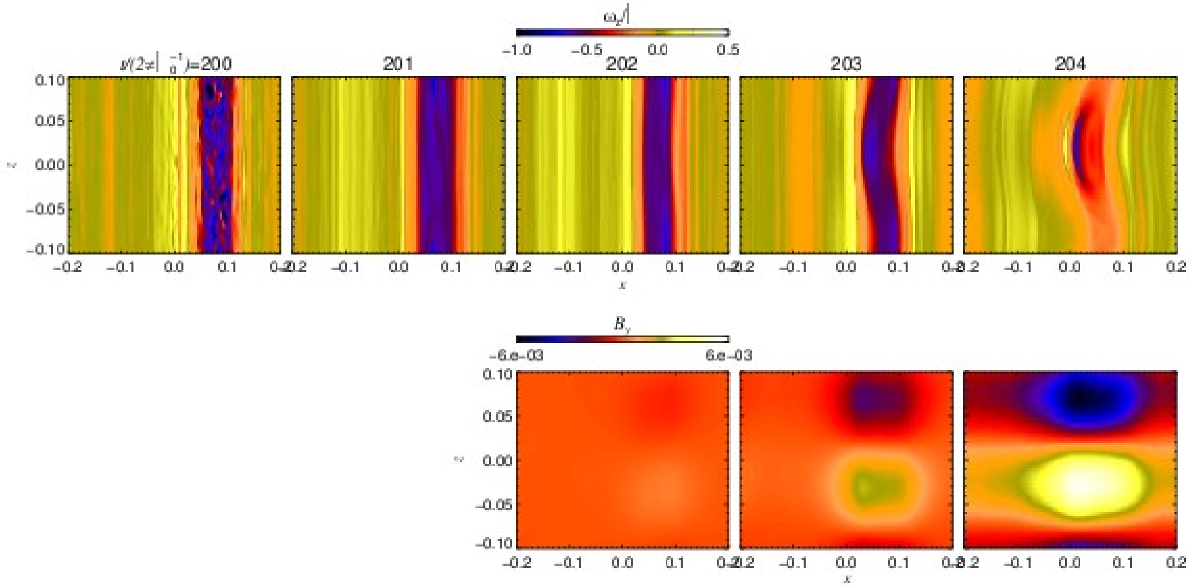


Mon. Not. R. Astron. Soc. 000, 1–7 (2011) Printed 17 June 2011 (MN LaTeX style file v2.2)

**The link between the magneto-elliptic and the magneto-rotational instabilities**

Krzysztof Mizerski[1]* & Wladimir Lyra[2]†
[1]Department of Applied Mathematics, University of Leeds, Leeds LS2 9JT, UK
[2]Department of Astrophysics, American Museum of Natural History, 79th Street at Central Park West, New York, NY, 10024, USA

# ***MEI-triggered channel flows***



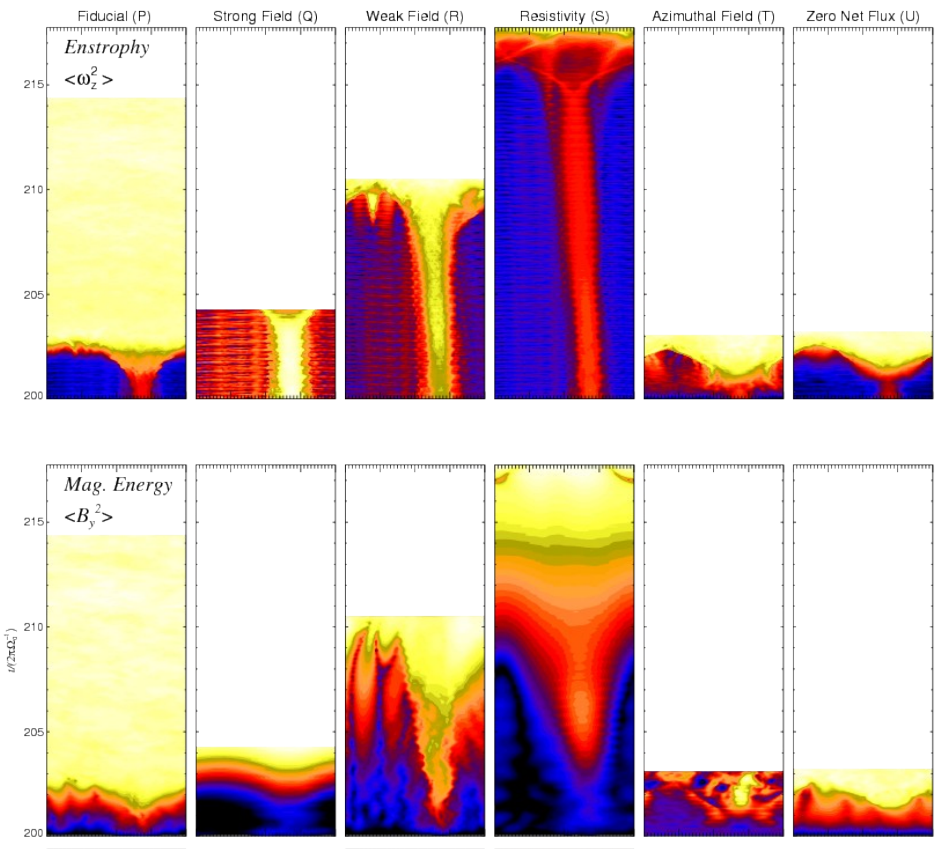
*Vorticity*

*Magnetic Field*

Channel flows!

Disruption of the vortex by channel currents.

# ***Other field configurations***

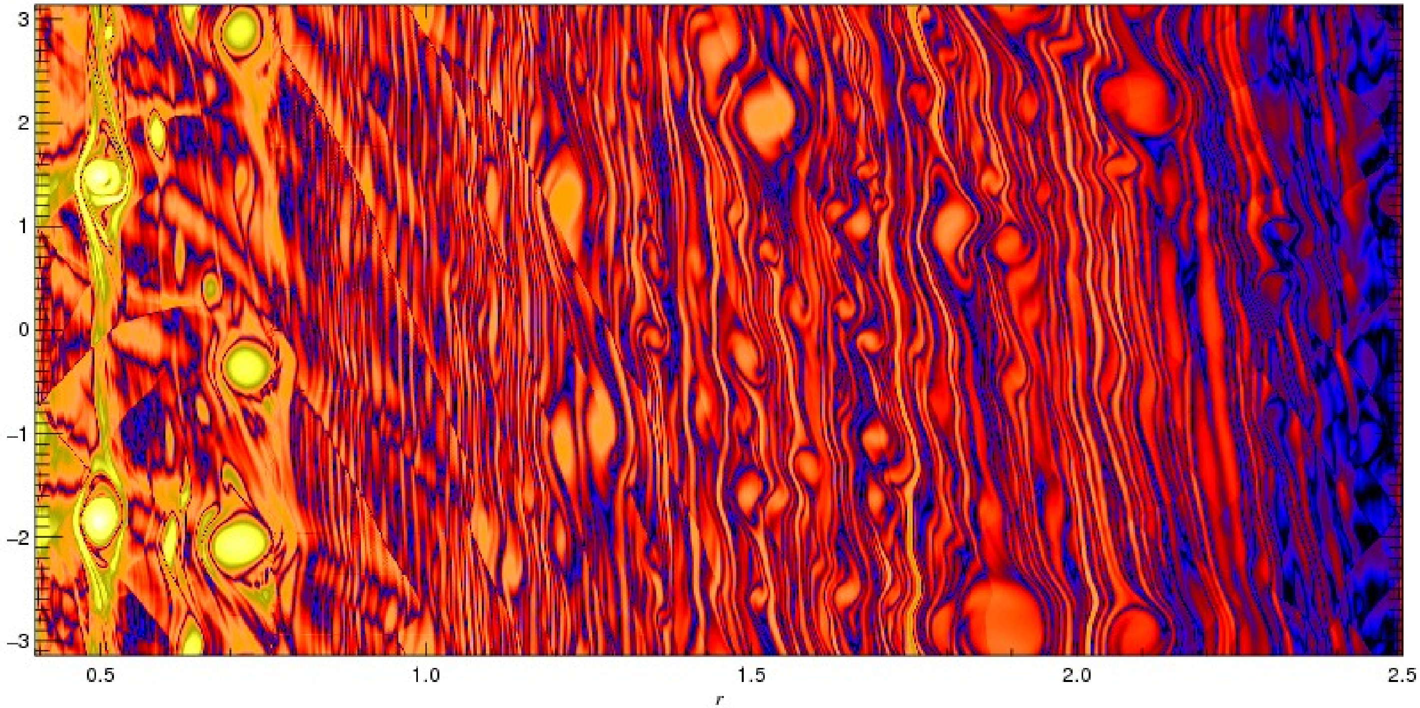



Azimuthal and zero-net flux fields also lead to growth, like the MRI.

Resistivity quenches growth below Reynolds number ~ 1, like the MRI

Notice that the vortices migrate radially (see also Paardekooper et al 2010)

## A global model of the Baroclinic Instability

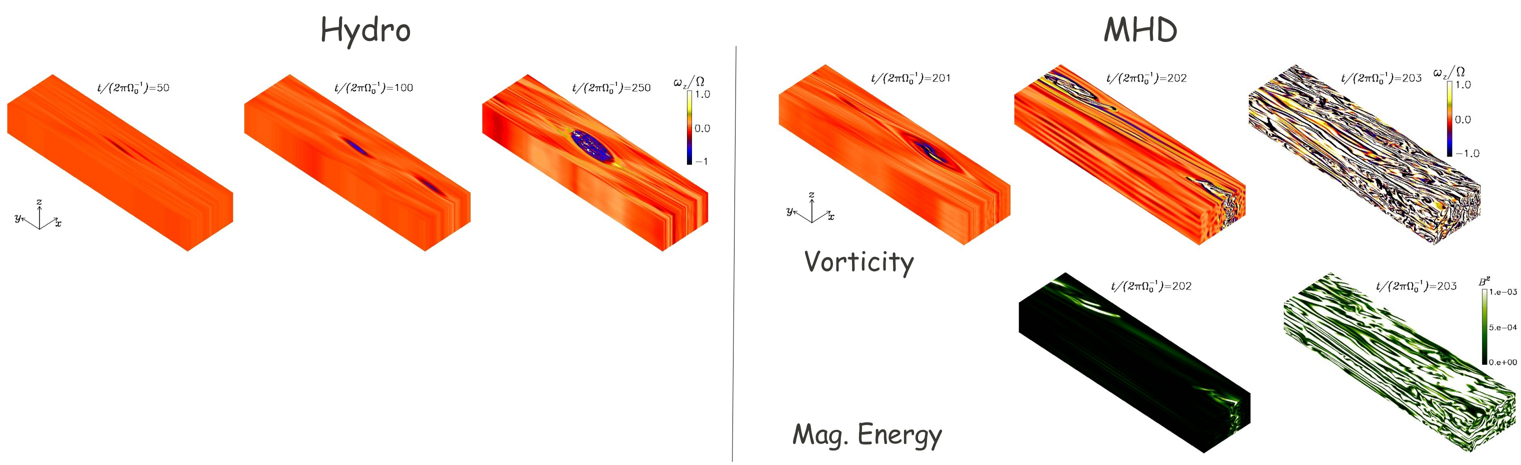


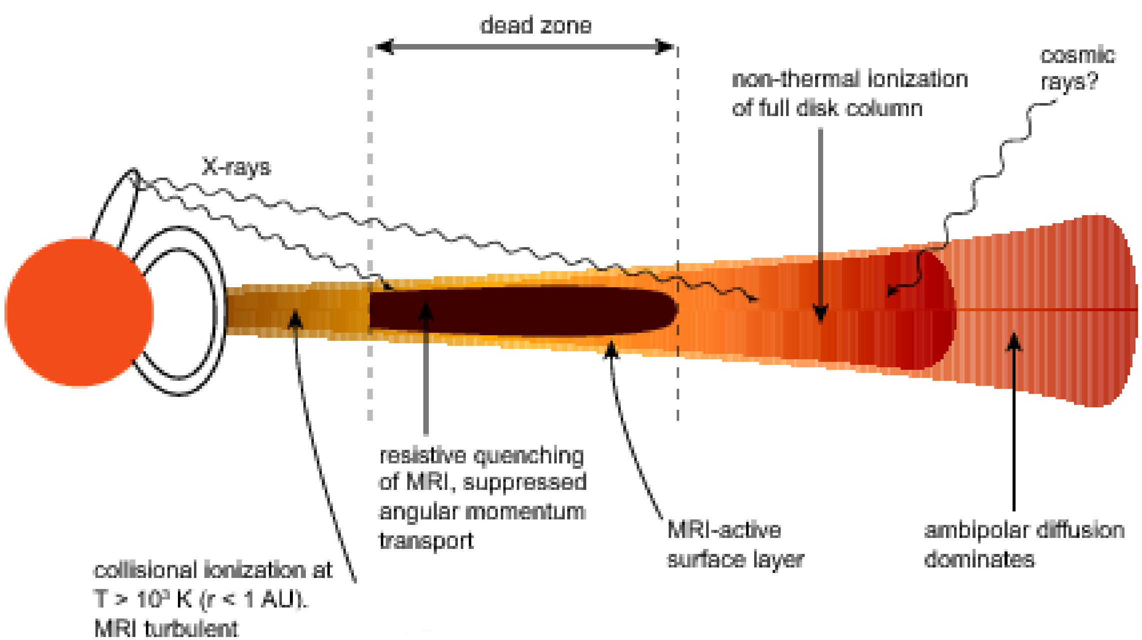

Vortices form and migrate (also seen by Paardekooper et al. 2010)

Once they reach the inner boundary, they are gone, and no further vortices form

Conclusion: the BI may need continuous forcing (from the active layers?)

# Suggested large-scale phenomenology



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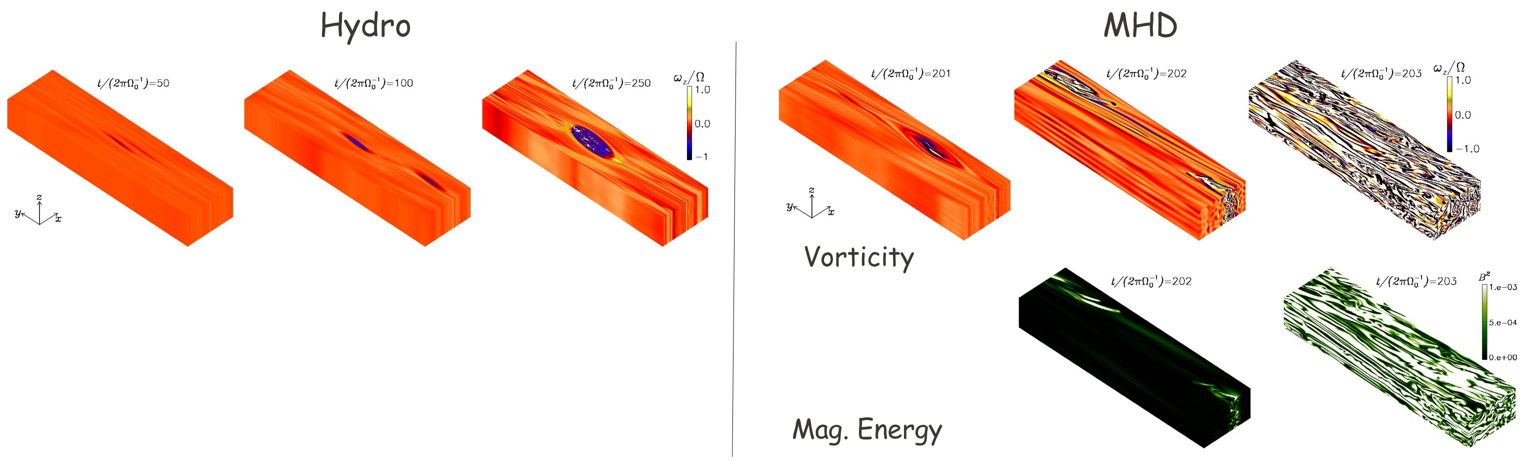
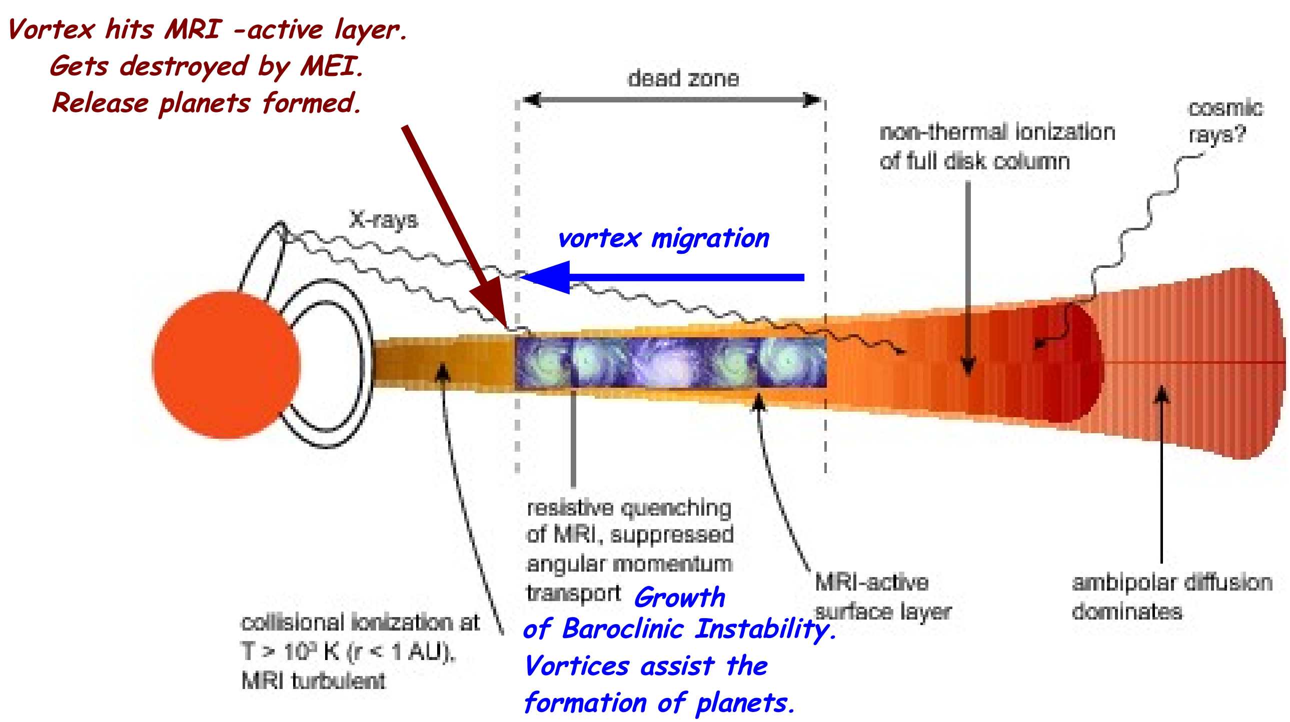
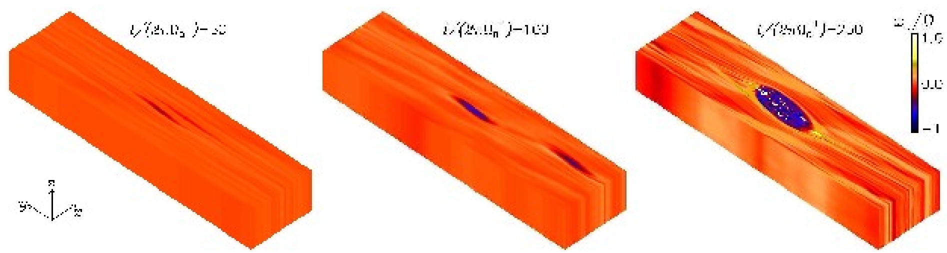


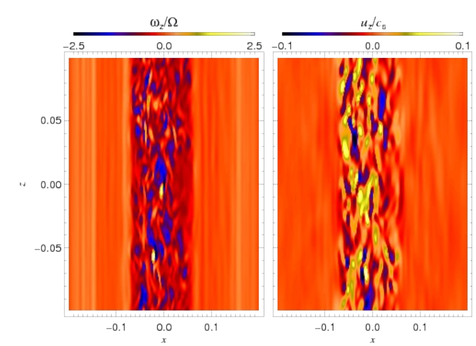
# Conclusions

- 3D non-magnetized vortices reach a steady state
    * Unstable yet coherent
    * Balance between baroclinicity (+) and stretching (-)
    * Subsonic core turbulence (10% of sound speed)

- Vortices do not survive the MRI
    * Channel flows
    * Violent core turbulence
        - Magneto-elliptic instability
        - MRI is a limit of the MEI



- Fits neatly in the layered accretion paradigm.
    * Active layers are unmodified
    * Dead zone only is endowed with vortices




**Open questions:**
- Vertical stratification
- Realistic entropy gradients and thermal diffusion
- *Particles??*

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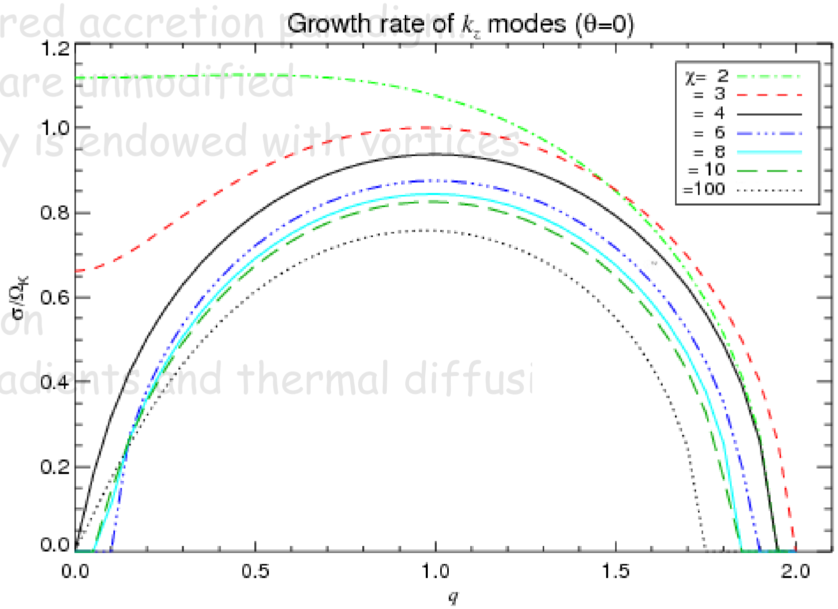


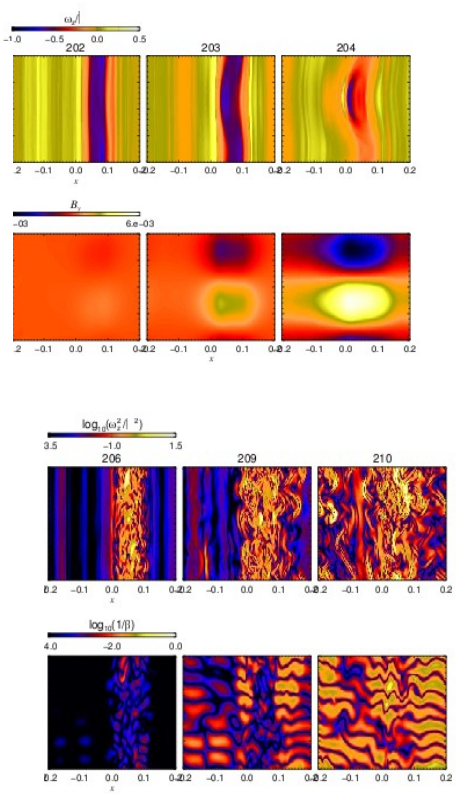

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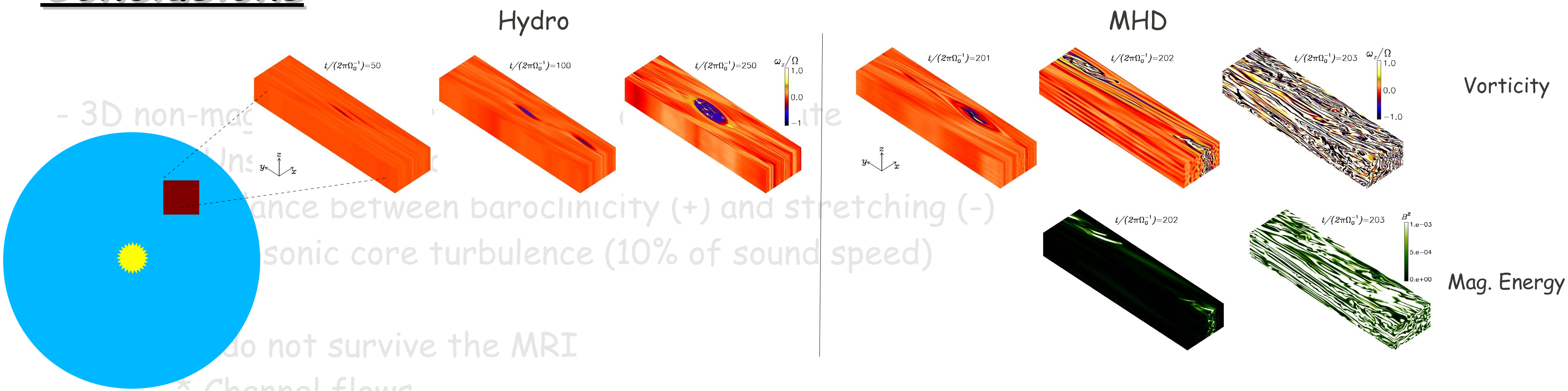


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- Particle
- Global large-scale phenomenology

# Thanks for your attention

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Astronomy & Astrophysics

## The baroclinic instability in the context of layered accretion

### Self-sustained vortices and their magnetic stability in local compressible unstratified models of protoplanetary disks

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**ABSTRACT**

*Context.* Turbulence and angular momentum transport in accretion disks remains a topic of debate. With the realization that dead zones are robust features of protoplanetary disks, the search for hydrodynamical sources of turbulence continues. A possible source is the baroclinic instability (BI), which has been shown to exist in unmagnetized non-barotropic disks.
*Aims.* We aim to verify the existence of the baroclinic instability in 3D magnetized disks, as well as its interplay with other instabilities, namely the magneto-rotational instability (MRI) and the magneto-elliptical instability.
*Methods.* We performed local simulations of non-isothermal accretion disks with the Pencil Code. The entropy gradient that generates the baroclinic instability is linearized and included in the momentum and energy equations in the shearing box approximation. The model is compressible, so excitation of spiral density waves is allowed and angular momentum transport can be measured.
*Results.* We find that the vortices generated and sustained by the baroclinic instability in the purely hydrodynamical regime do not survive when magnetic fields are included. The MRI by far supersedes the BI in growth rate and strength at saturation. The resulting turbulence is virtually identical to an MRI-only scenario. We measured the intrinsic vorticity profile of the vortex, finding little radial variation in the vortex core. Nevertheless, the core is disrupted by an MHD instability, which we identify with the magneto-elliptic instability. This instability has nearly the same range of unstable wavelengths as the MRI, but has higher growth rates. In fact, we identify the MRI as a limiting case of the magneto-elliptic instability, when the vortex aspect ratio tends to infinity (pure shear flow). We isolated its effect on the vortex, finding that a strong but unstable vertical magnetic field leads to channel flows inside the vortex, which stretch it apart. When the field is decreased or resistivity is used, we find that the vortex survives until the MRI develops in the box. The vortex is then destroyed by the strain of the surrounding turbulence. Constant azimuthal fields and zero net flux fields also lead to vortex destruction. Resistivity quenches both instabilities when the magnetic Reynolds number of the longest vertical wavelength of the box is near unity.
*Conclusions.* We conclude that vortex excitation and self-sustenance by the baroclinic instability in protoplanetary disks is viable only in low ionization, i.e., the dead zone. Our results are thus in accordance with the layered accretion paradigm. A baroclinicly unstable dead zone should be characterized by the presence of large-scale vortices whose cores are elliptically unstable, yet sustained by the baroclinic feedback. Since magnetic fields destroy the vortices and the MRI outweighs the BI, the active layers are unmodified.

**Key words.** accretion, accretion disks – hydrodynamics – instabilities – magnetohydrodynamics (MHD)– turbulence – methods: numerical