Vortex theory meets observation:
Is ALMA seeing vortices in transitional disks?

Wladimir (Wlad) Lyra

Sagan Fellow

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Collaborators:
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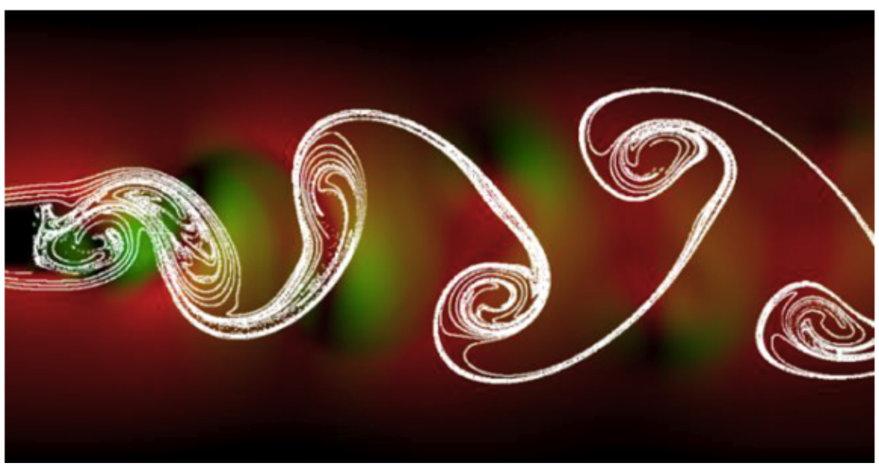
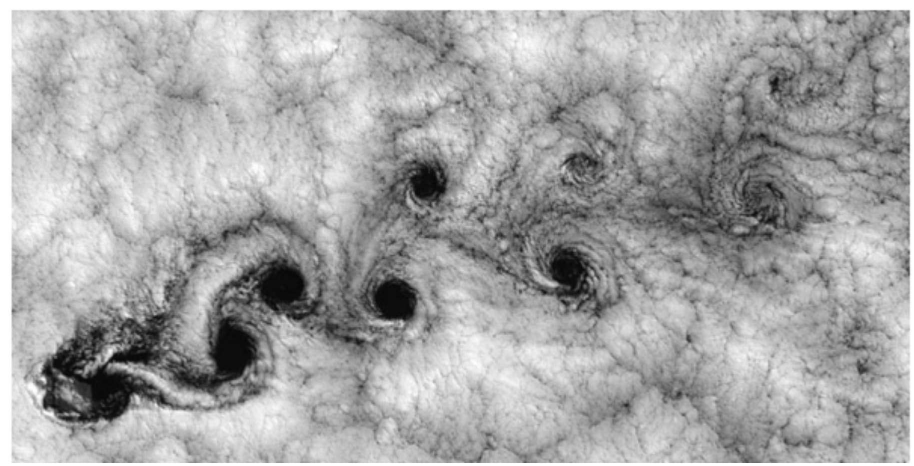
Min-Kai Lin (CITA), Mordecai-Mark Mac Low (AMNH)
Nikolai Piskunov (Uppsala), Natalie Raettig (MPIA), Andras Zsom (MPIA)

Vortices – An ubiquitous fluid mechanics phenomenon

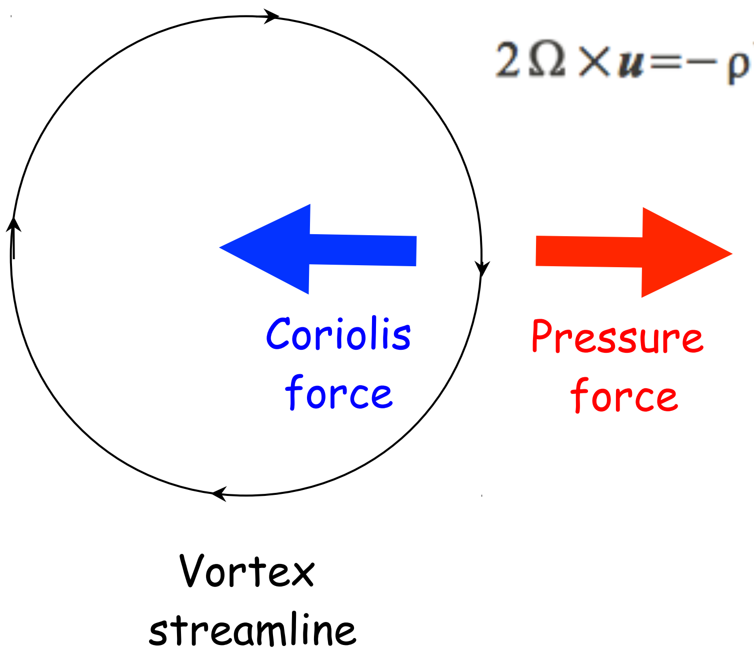


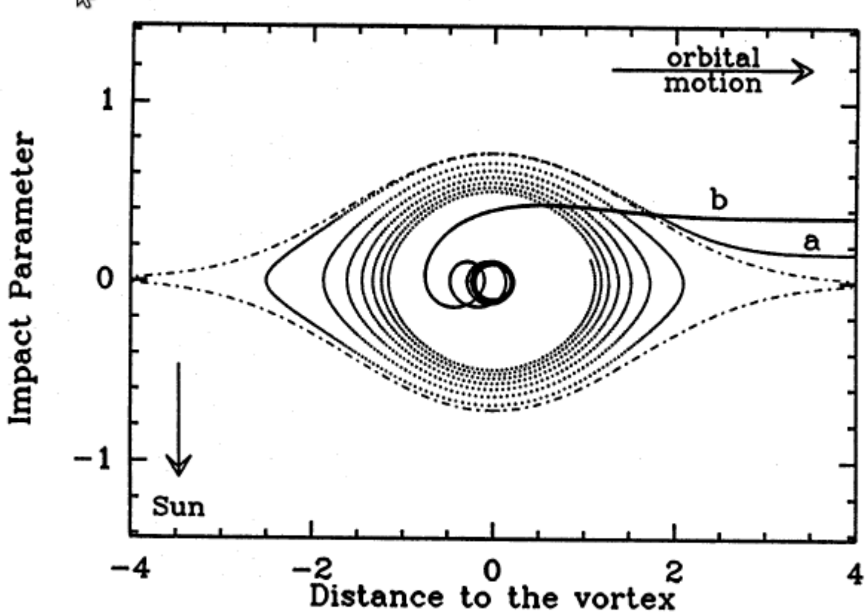


Vortices and Planet Formation

Geostrophic balance:

$$2\Omega \times \boldsymbol{u} = -\rho^{-1} \nabla p$$



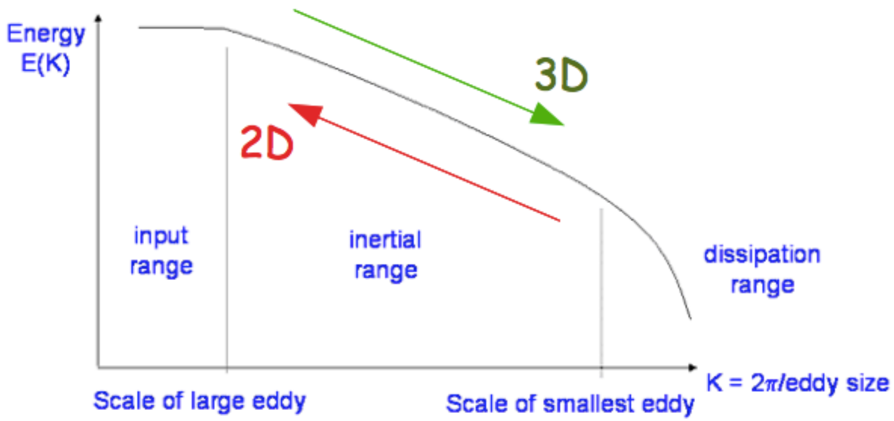


Barge & Sommeria (1995)

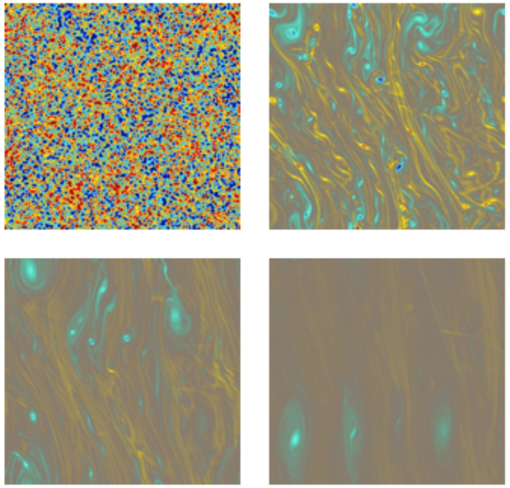
Particles do not feel the pressure gradient.
They sink towards the center, where they accumulate.

Also independently suggested by:
Tanga et al. 1996
Adams & Watkins 1996

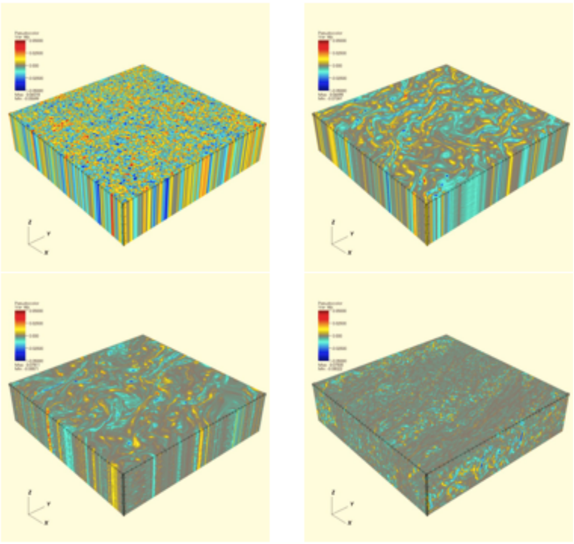
The energy cascade



Shen et al. (2006).
See also Batchelor (1967)



2D



3D

Inverse Cascade

No 3D instability
Eddies merge

Direct Cascade

Destruction occurs
faster than merging

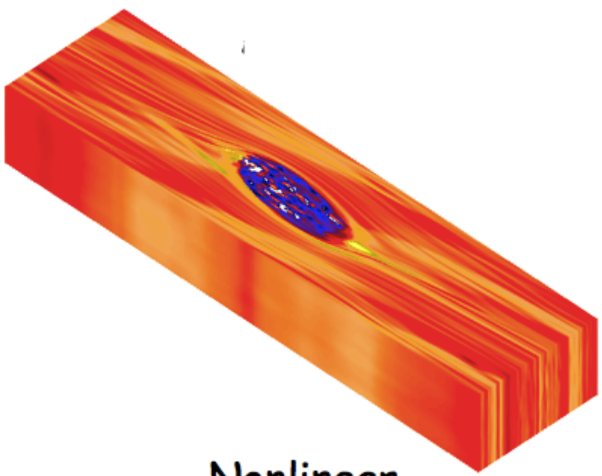
Sustaning vortices

Mechanisms to

inject vorticity

to counteract the vorticity lost in the direct cascade

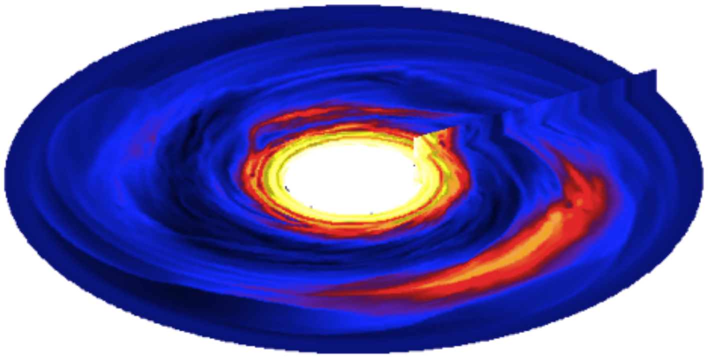
Baroclinic Instability



Nonlinear

Powered by:
buoyancy, thermal diffusion
(baroclinic source term)

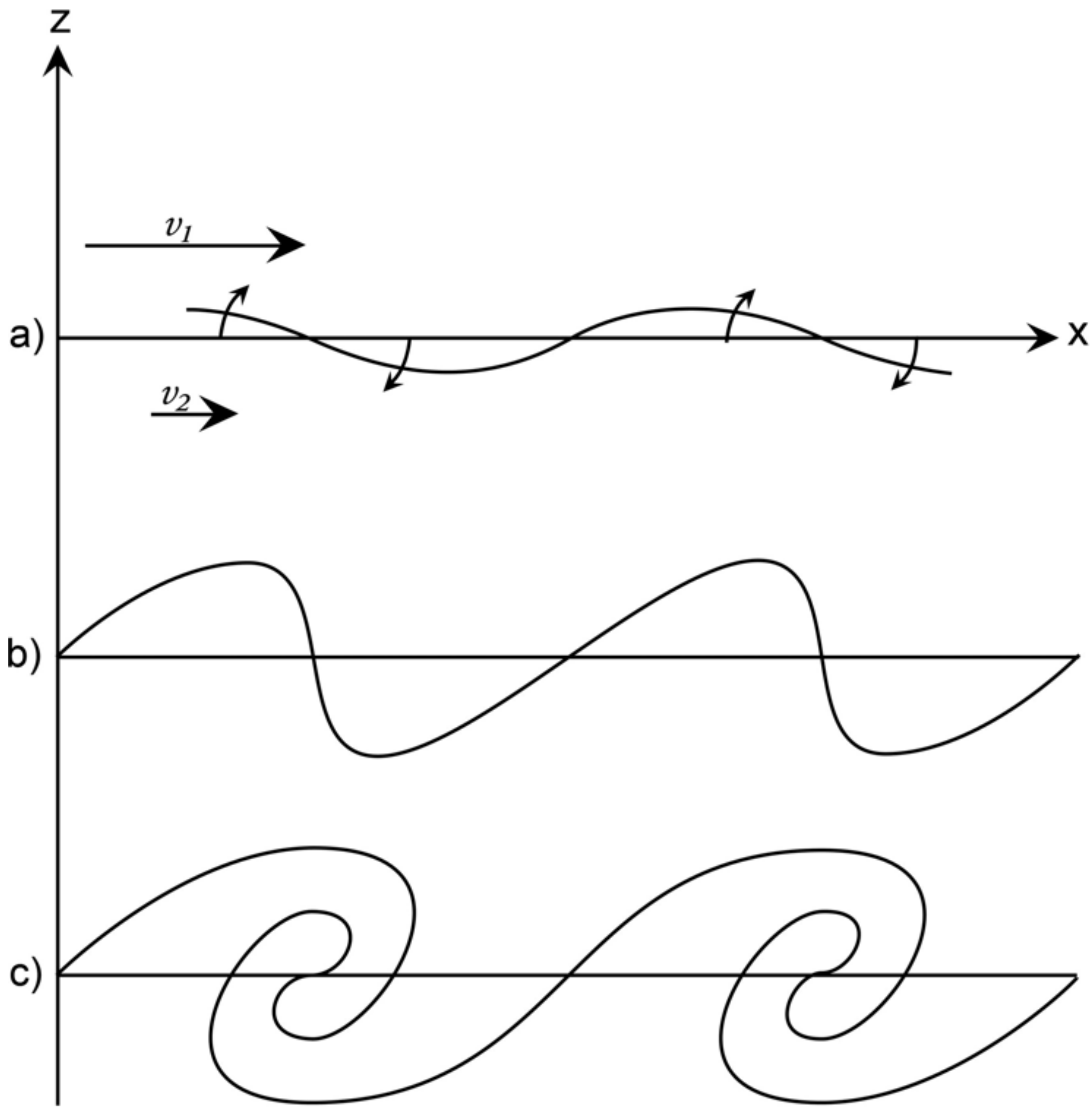
Rossby Wave Instability



Linear

Powered by:
Modification of shear profile
(external vortensity reservoir)

Rossby Wave Instability
(or.... **Kelvin-Helmholtz** in rotating disks)



(Pre-)History of Rossby Wave Instability

Lovelace & Hohlfeld 1978

NEGATIVE MASS INSTABILITY OF FLAT GALAXIES

R. V. E. Lovelace and R. G. Hohlfeld
Center for Radiophysics and Space Research, and Department of Applied Physics, Cornell University
Received 1977 May 4; accepted 1977 September 15

ABSTRACT

A study is made of the linear initial value problem of a flat, low-"temperature," self-gravitating disk for perturbations which are radially localized with $|\omega - n\Omega|^2 \ll \Omega^2$, where ω is the angular frequency and n the azimuthal mode number ($\neq 0$) of the perturbation, and where $\Omega(r)$ is the angular velocity of the differentially rotating disk matter at a radial distance r. We find that instability is possible in situations where the distribution function for angular momentum, $f(r) \equiv \sigma\Omega\kappa^{-2}$, has a maximum or minimum as a function of r and $(d/dr)\Omega \neq 0$ at the extremum of f, where $\sigma(r)$ is the surface mass-density of the disk, and $\kappa(r)$ is the epicyclic frequency. Approximate growth rates are derived. The mechanism of the instability is related to that of the negative mass instability of charged-particle rings. We propose that the instability may drive a disk toward a state in which $f(r)$ is approximately constant.

Values of $\Omega(r)$, $\kappa(r)$, and $\sigma(r)$ derived from observations are used to calculate $f(r)$ for two cases: For our Galaxy we find $3.8 < f(r) < 4.5$ for $0.3 \leq r \leq 10$ kpc, with f in units of $M_\odot$ pc^{-2} (km s^{-1} kpc^{-1})$^{-1}$. For M31, $3.2 < f(r) < 5.6$ for $3 \leq r \leq 30$ kpc.

Subject headings: galaxies: internal motions — galaxies: structure — stars: stellar dynamics

Vorticity criterion already derived back then

(Pre-)History of Rossby Wave Instability

WHAT AMPLIFIES THE SPIRALS?

Alar Toomre

Massachusetts Institute of Technology

Toomre 1981'

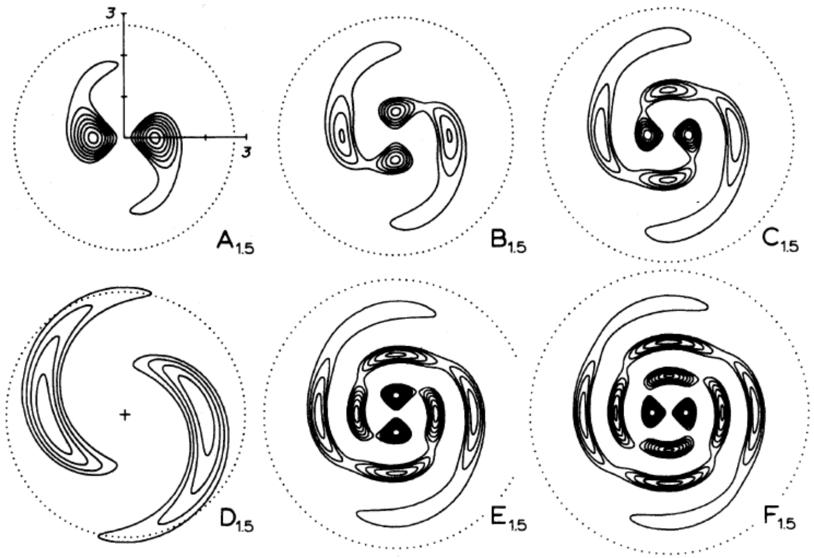


Fig. 12 Comparison of modes A-F for that Gaussian disk in which only 2/3 of the density remains "active". Their eigenfrequencies were reported at location 1.5 in Fig. 11. The corotation circles are again shown dotted; they have expanded markedly from Fig. 10.

One black sheep still needs to be dealt with. I am referring, of course, to the mode marked D in Figs. 10-12. As luck has it, the pattern speed (and even the growth rate) of this mode lands it smack amidst the swing-amplified modes in the full-mass Fig. 10. And it is there somewhat contaminated by the latter — as if only to confuse us! **That mode D is a wolf in sheep's clothing becomes clear,** however, once we weaken those rival modes in Figs. 11-12 by reducing the active disk mass. Its shape and hefty growth rate then point firmly to a different kind of animal.

What is mode D? It seems genuinely to be an edge mode which (a) arises only if the disk density drops off abruptly enough with radius, and yet (b) does not require any wave transport into or through the central regions. Kalnajs and I can support claim (a) with some experimental findings that any analogue of mode D occurs at most very weakly in the yet more soft-edged exponential disk — and it is altogether absent from Zang's $V = \text{const}$ disk — whereas it can be aroused to fresh fury by artificially truncating either of those disks in a smooth but sudden enough manner. We can also vouch for claim (b) with the little discovery that any "freezing" of our Gaussian disk inward of (say) $r = 1$ hardly alters the eigen-

(Pre-)History of Rossby Wave Instability

Papaloizou-Pringle Instability (1984ab)

(Goldreich & Narayan 1985, Blaes 1985, Blaes & Glatzel 1986, Hawley 1987, Narayan et al. 1987, Goldreich et al. 1987, 1988)

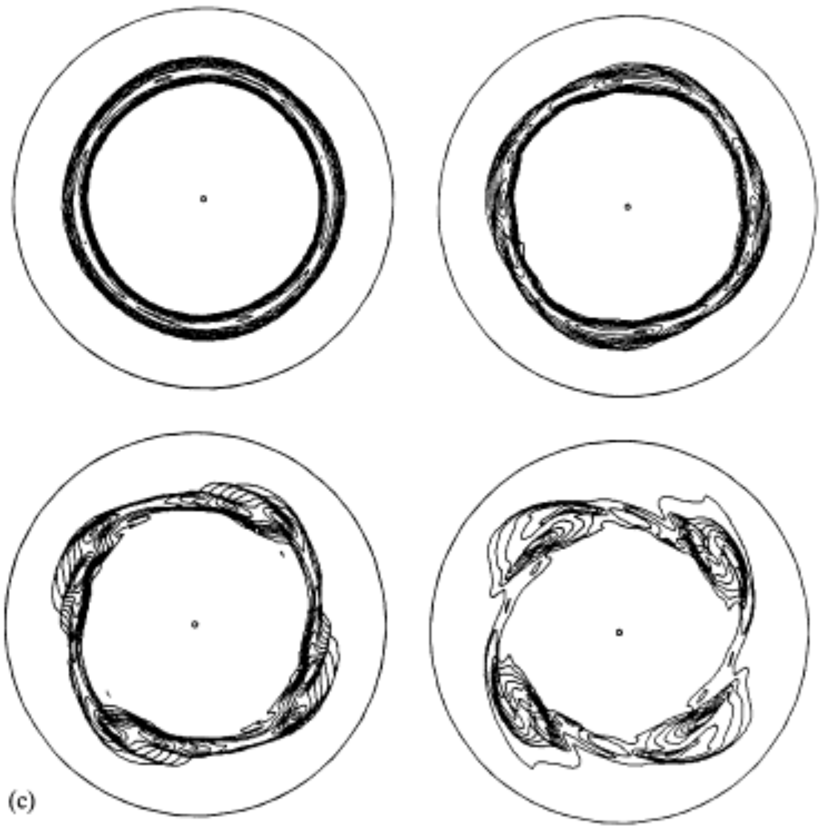


Figure 5–*continued*

Numerical model by Hawley 1987

The dynamical stability of differentially rotating discs with constant specific angular momentum

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J. E. Pringle *Institute of Astronomy, Madingley Road, Cambridge CB3 0HA*

Received 1983 August 10

The dynamical stability of differentially rotating discs – II

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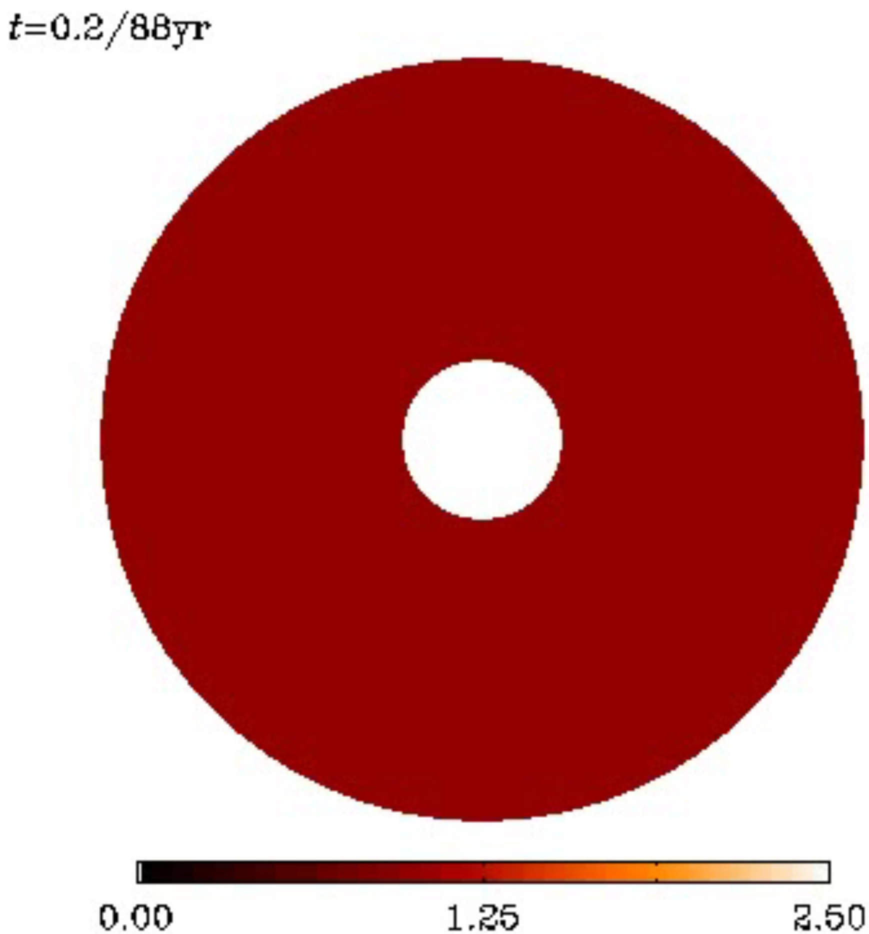
Accepted 1984 November 26. Received 1984 November 23; in original form 1984 October 1

1991: The MRI Revolution

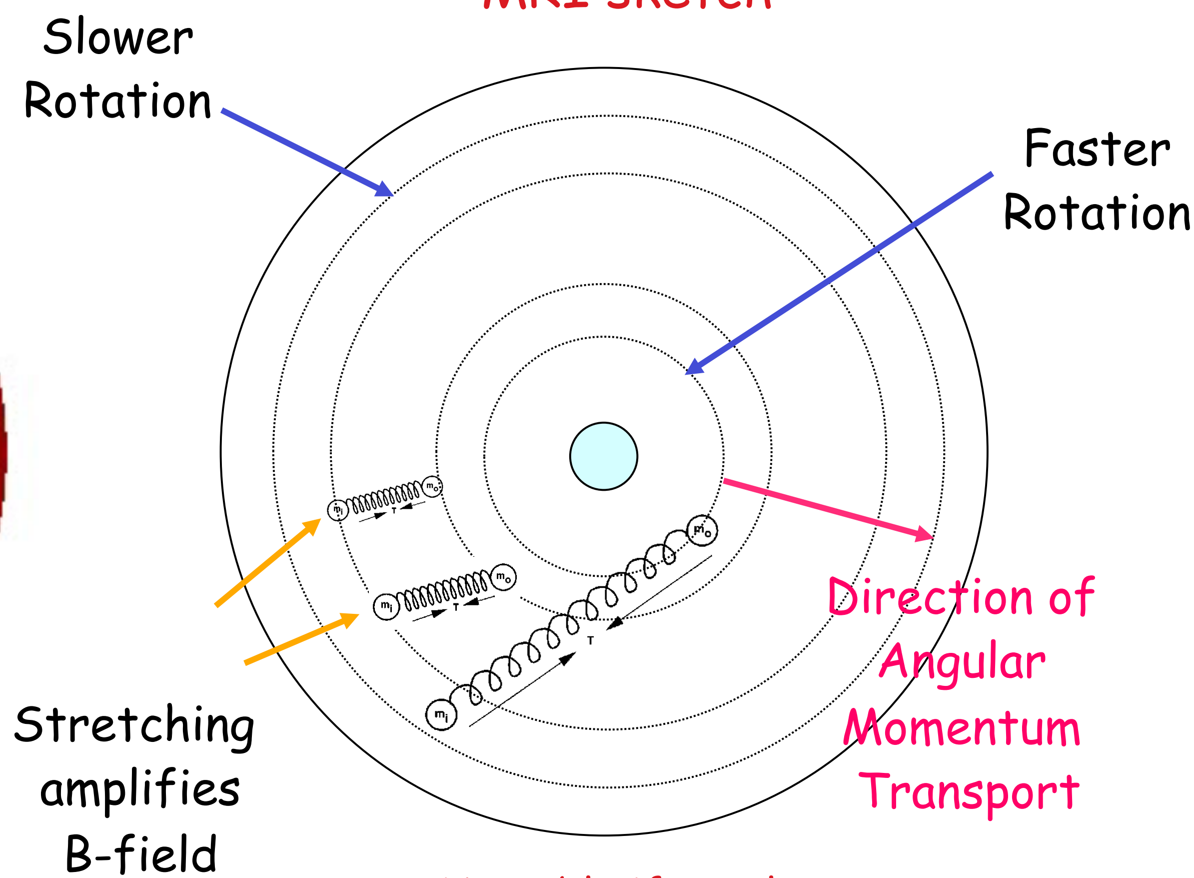
Rediscovery of the Magneto-Rotational Instability



Magnetized disk

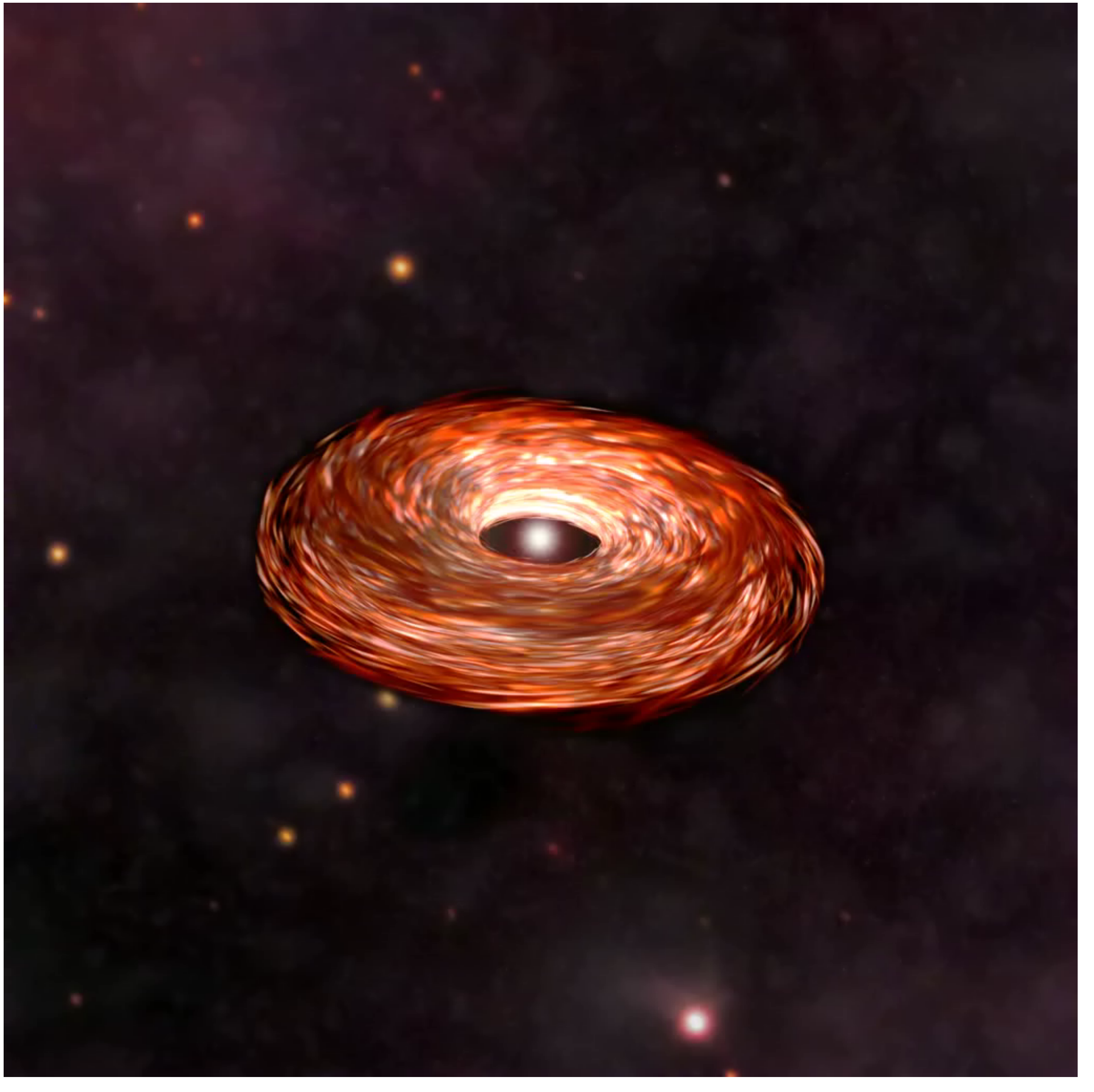
Lyra et al. (2008a)

MRI sketch



Unstable if angular velocity decreases outward

1991: The MRI Revolution



Video credit: Mario Flock (MPIA/CEA)

History of Rossby Wave Instability

Lovelace et al. (1999) resurrect the process;
call it "Rossby Wave" Instability

ROSSBY WAVE INSTABILITY OF KEPLERIAN ACCRETION DISKS

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Received 1998 March 19; accepted 1998 October 15

disk quantities, such as surface density and entropy have steep radial gradients. The conditions we consider are in general nonbarotropic which distinguish our work from that of Papaloizou and Pringle (1984, 1985; Goldreich, Goodman, & Narayan 1986; Narayan, Goldreich, & Goodman 1987). Also, in contrast with the work of Papaloizou and Pringle, the modes we consider are trapped at least initially in a narrow range of radii and therefore do not depend on reflections from inner and outer radii of the disk (or tori).

Peggy Varnière & Michel Tagger
RWI at dead zone boundary!

Reviving Dead Zones in Accretion Disks by Rossby Vortices at their Boundaries

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[2] LAOG, Université J. Fourier UMR 5571 France, e-mail: pvarni@pas.rochester.edu
[3] Service d'Astrophysique (UMR Astroparticules et cosmologie) CEA/Saclay F-91191 Gif sur Yvette e-mail: tagger@cea.fr

the date of receipt and acceptance should be inserted later

Abstract. Models of the accretion disks of Young Stellar Objects show that they should not be ionized at a few AU from the star, and thus not subject to the MHD turbulence believed to cause accretion. This has been suggested to create a 'Dead Zone' where accretion remains unexplained. Here we show that the existence of the Dead Zone self-consistently creates a density profile favorable to the Rossby Wave Instability of Lovelace et al. (1999). This instability will create and sustain Rossby vortices in the disk which could lead to enhanced planet formation.

Key words. accretion disks; Instabilities; planetary systems: formation

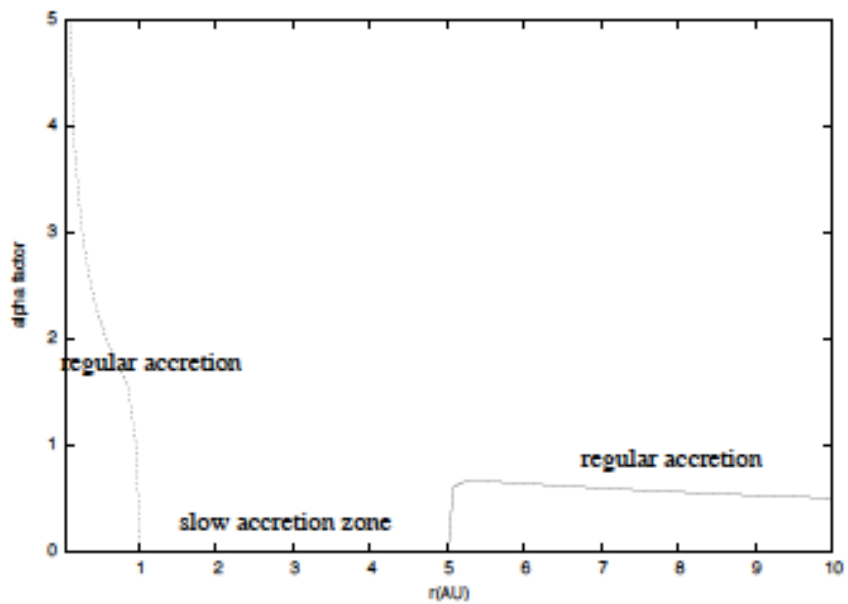


Fig. 1. Profile of the α-viscosity implemented to represent a Dead Zone between 1 and 5 AU with $(\epsilon, \delta_r) = (10^{-5}, 50)$.

Varnière & Tagger (2006)

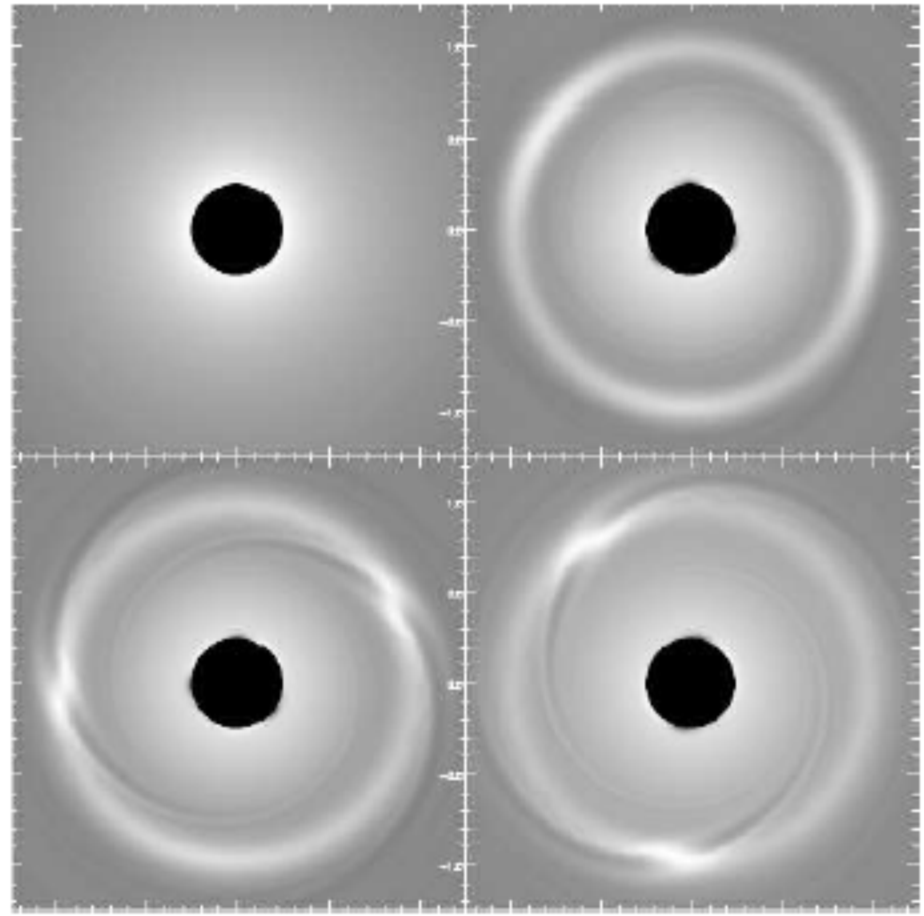
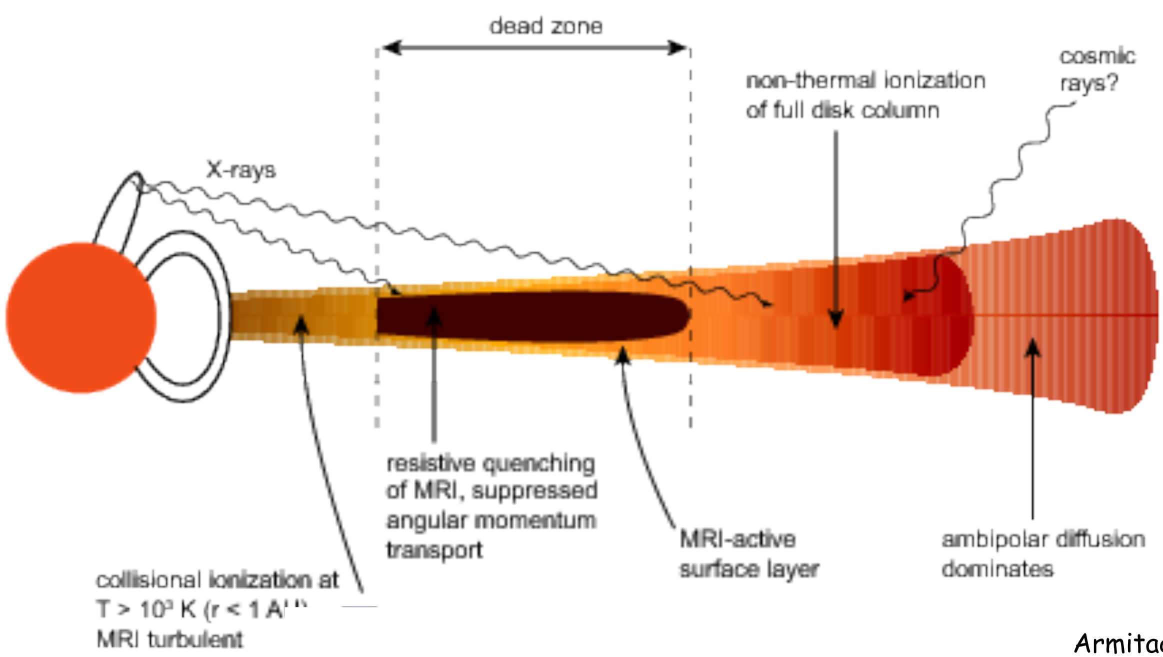


Fig. 3. Zoom of the first 2 inner AU of the simulation at $t = 0, 100, 200, 300$ years, showing the density. One sees three vortices forming, later evolving to two vortices, near the outer edge of the Dead Zone.

Dead zones are robust features of accretion disks



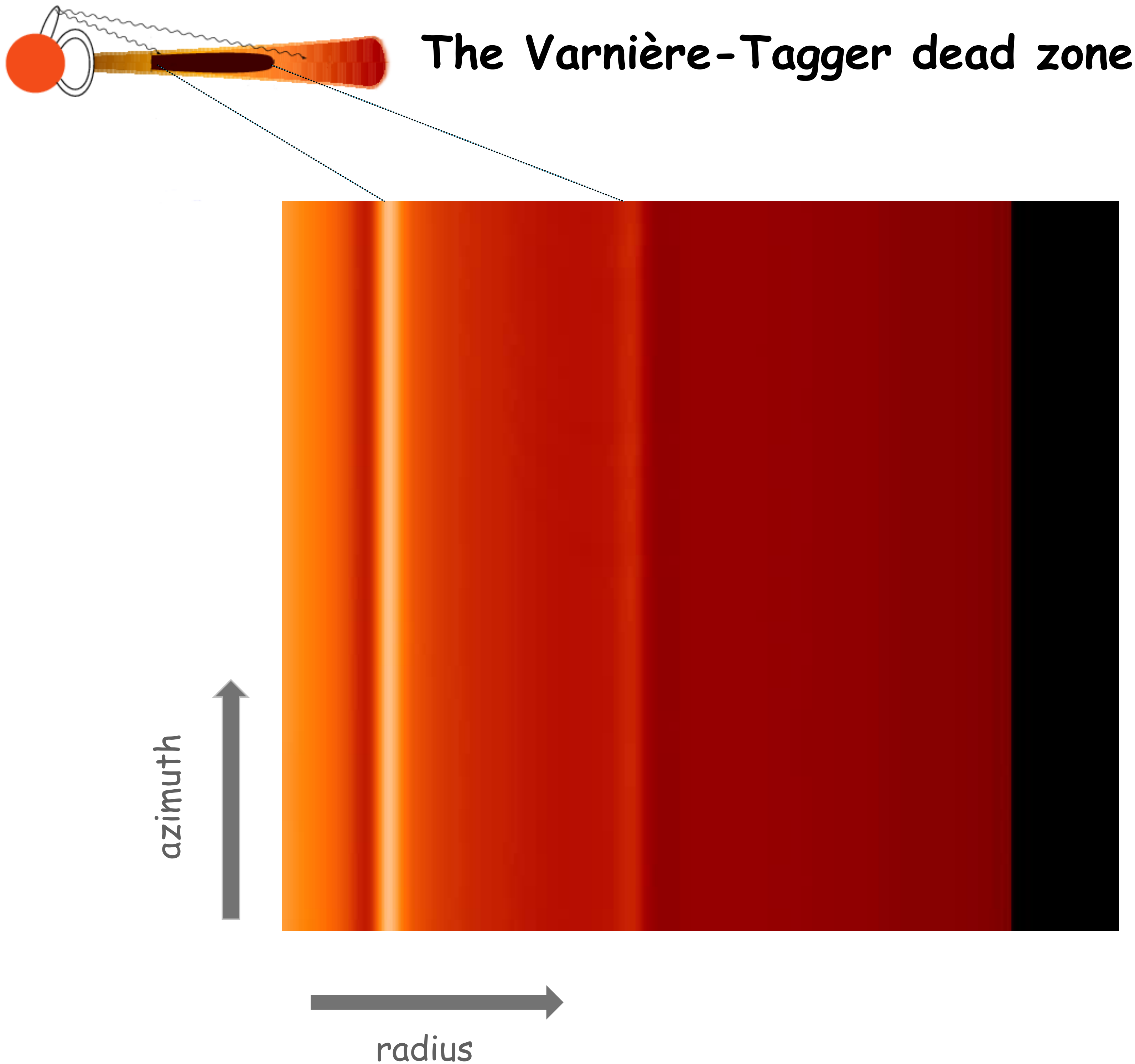
Armitage (2010)

Disks are cold and thus poorly ionized
(Blaes & Balbus 1994)

Therefore, accretion is **layered** (Gammie 1996)

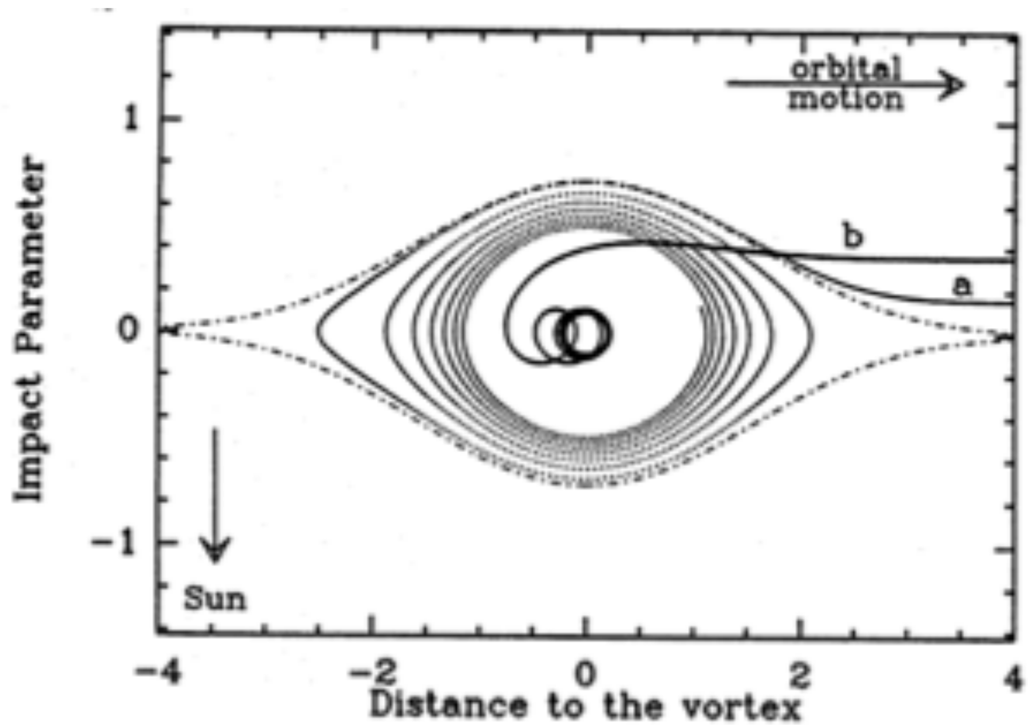
There should be a **magnetized, active zone**,
and a **non-magnetic, dead zone.**

The Varnière-Tagger dead zone model



Lyra et al. (2008b)

Tea-leaf effect



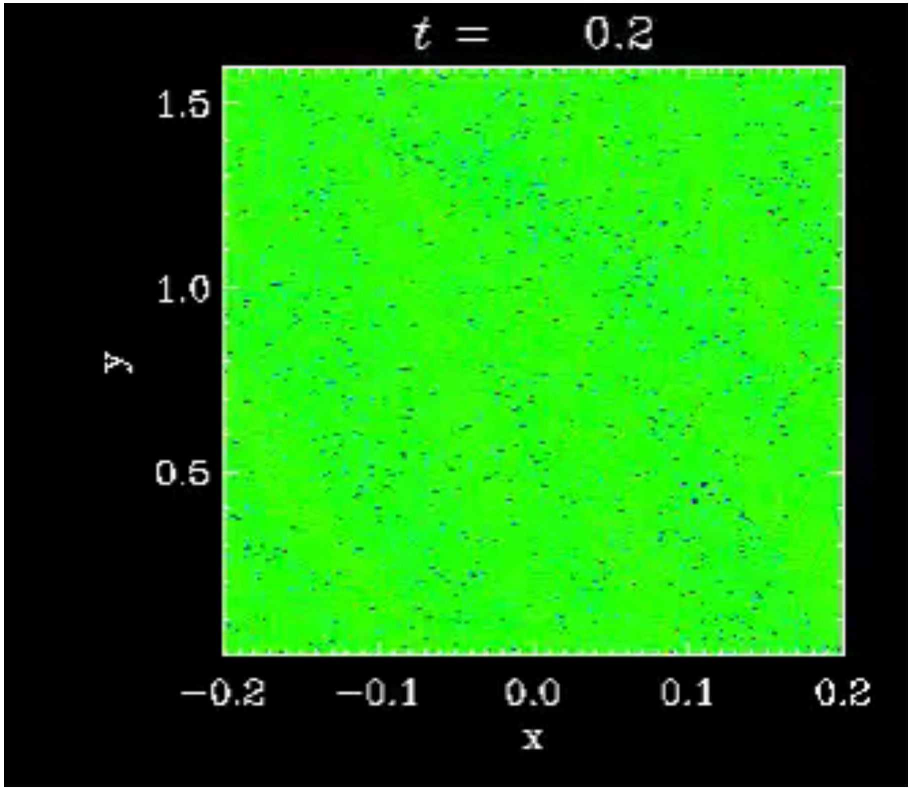
Barge & Sommeria (1995)

Particles sink to the center

Aid to planet formation (Barge & Sommeria 1995)

Speed up planet formation enormously
(Lyra et al. 2008b, 2009a, 2009b, Raettig, Lyra & Klahr 2012)

Tea-leaf effect



Credit: Natalie Raettig

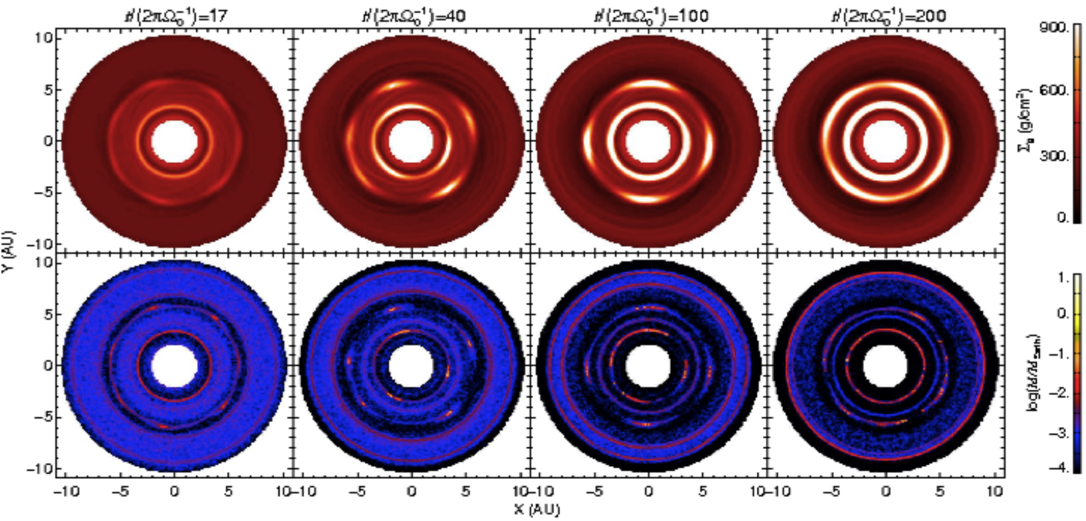
Particles sink to the center

Aid to planet formation (Barge & Sommeria 1995)

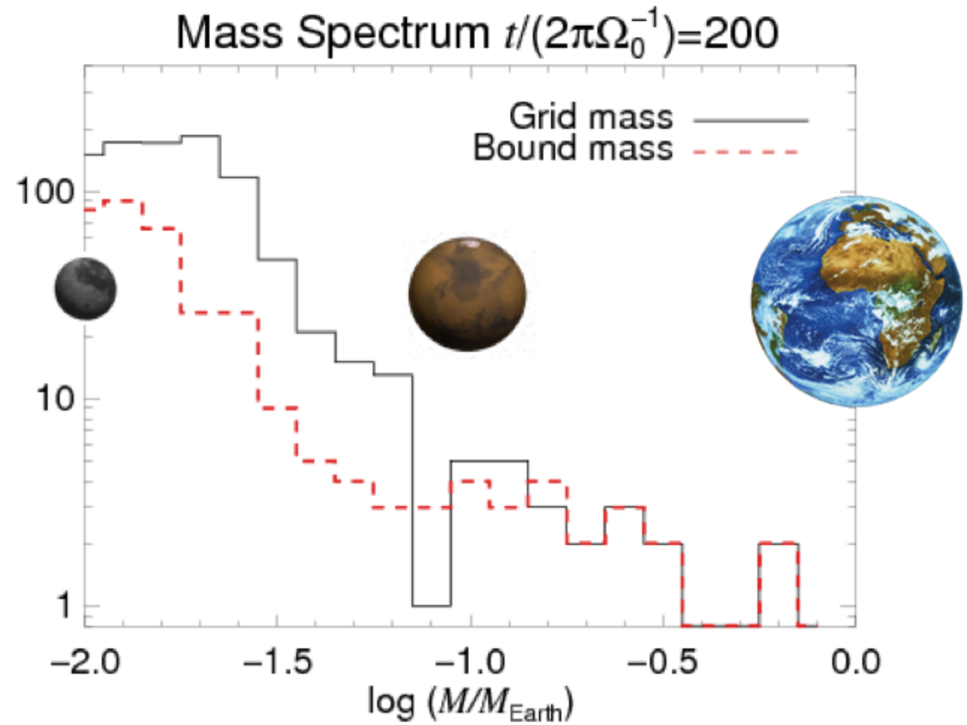
Speed up planet formation enormously
(Lyra et al. 2008b, 2009a, 2009b, Raettig, Lyra & Klahr 2012)

Vortices and Planet Formation

Gas



Particles



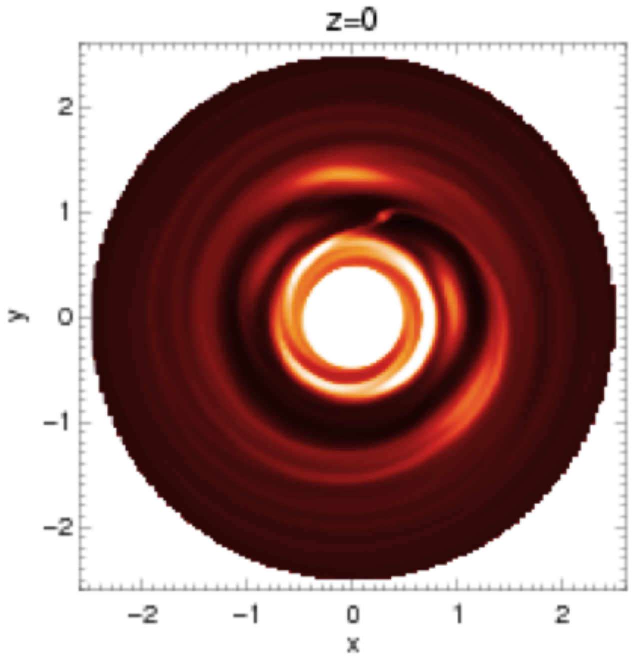
Collapse into Mars mass objects

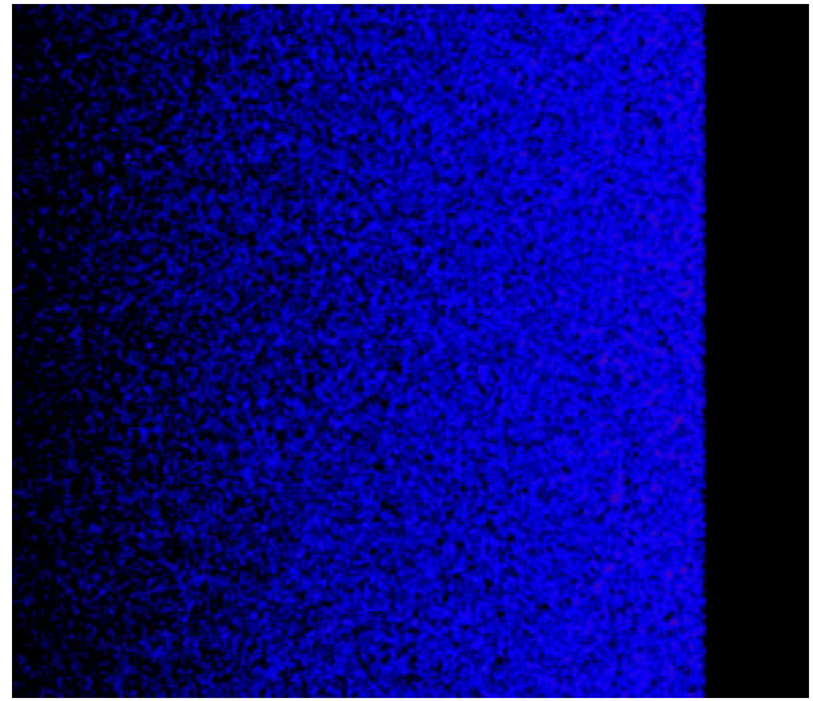
(Lyra et al. 2008b, 2009a, 2009b,

see also Lambrets & Johansen 2012)

Another way of exciting the RWI:

Lyra et al. (2009b),
see also de Val-Borro et al. (2007)

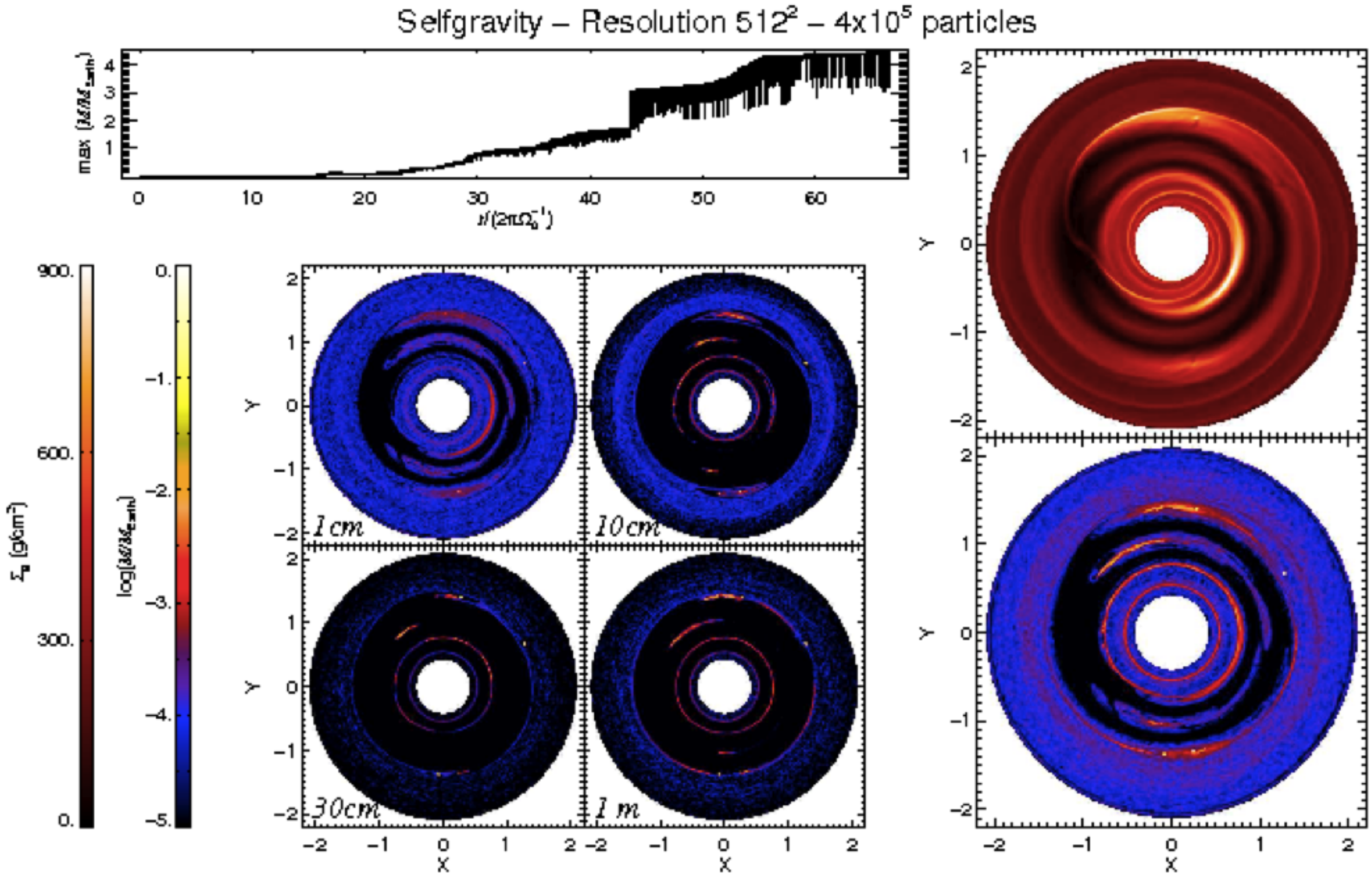




The edges of a planet-carved gap are also prone to vortex excitation.

What happens when particles are introduced?

Vortex trapping

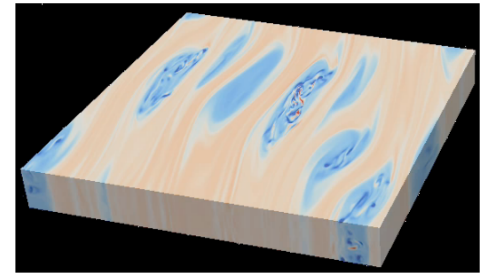


3 Super-Earths formed + Mars mass Trojans

Lyra et al. (2009b)

What have we learned since?

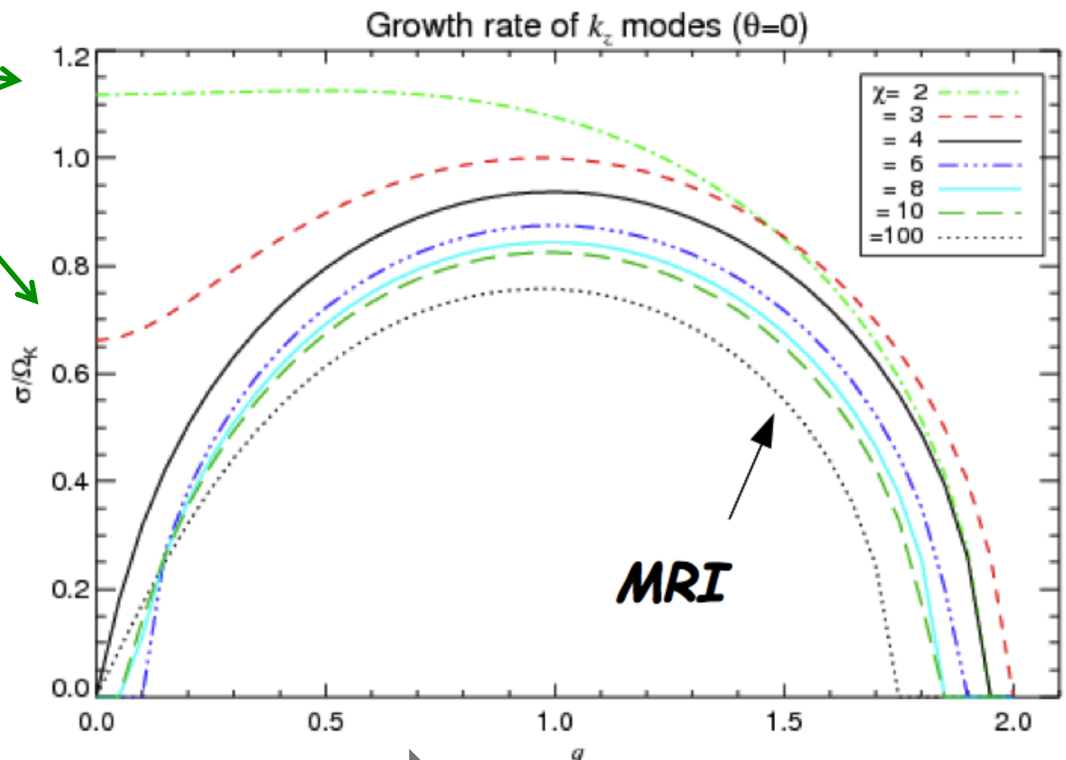
“Elliptic” Instability



Lesur & Papaloizou (2010)

See also
Pierrehumbert 1986
Bayly 1986
Kerswell 2002
Lesur & Papaloizoy 2009
Lesur & Papaloizou 2010
Lyra & Klahr 2011
Lyra 2013

Hydrodynamic instability (B=0)



Growth rate

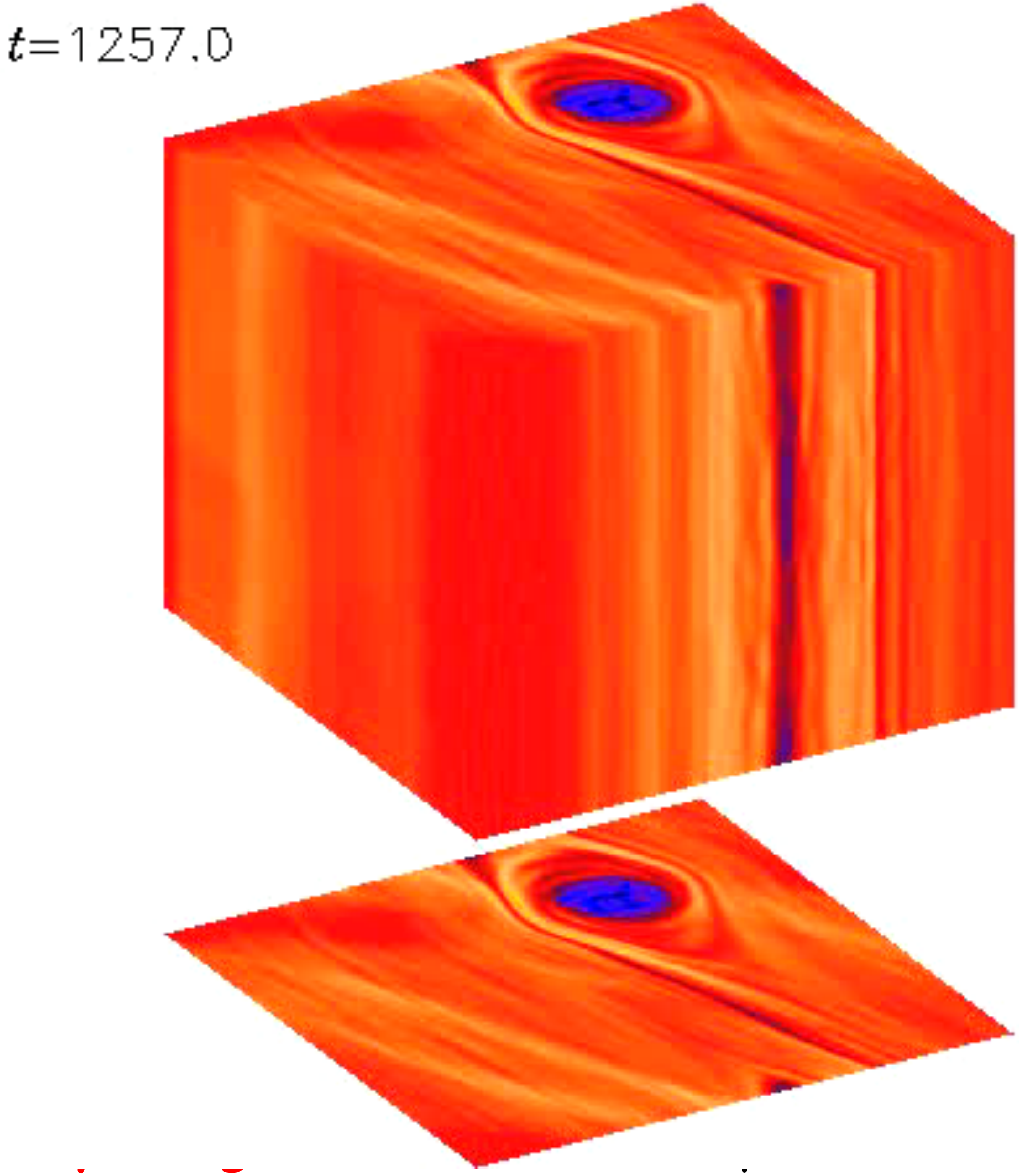
Magnetic Field

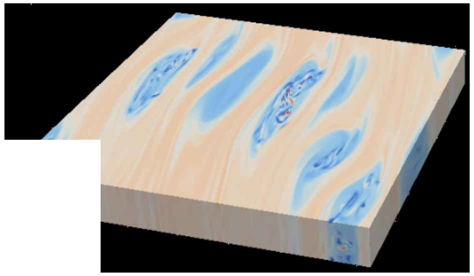
Lyra & Klahr (2011)

Infinitely elongated vortices are equivalent to **shear flows**.

They are subject to an MRI-like instability when magnetized.

What have we learned since?





sur & Papaloizou (2010)

Hydrodynamic
instability (B=0

Growth rate

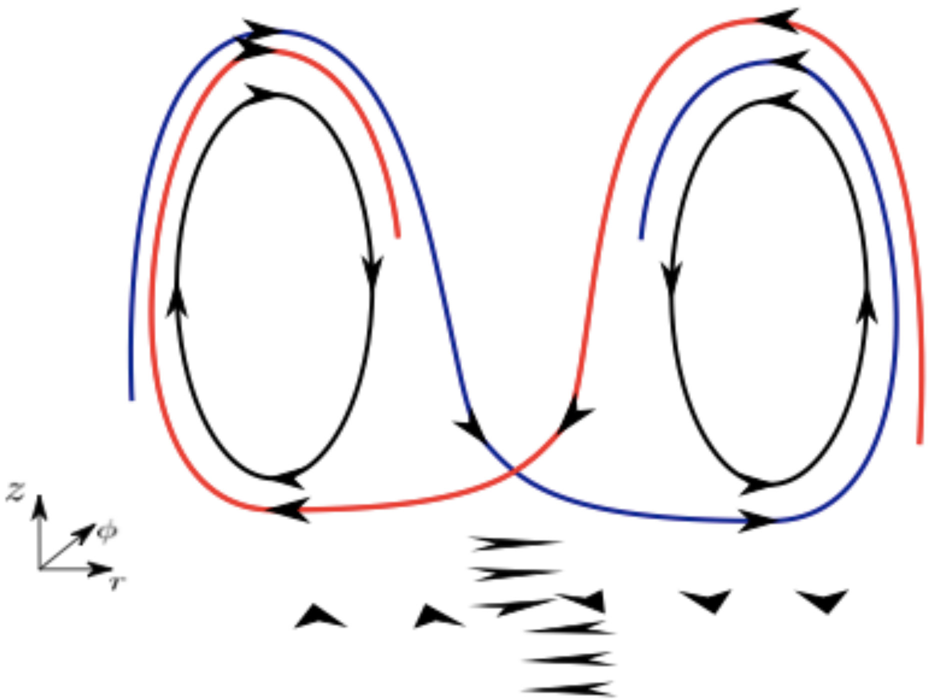
See also
Pierrehumbert 1986
Bayly 1986
Kerswell 2002
sur & Papaloizoy 2009
sur & Papaloizou 2010
Lyra & Klahr 2011
Lyra 2013

In ws.

They are subject to an MRI-like instability when magnetized.

What have we learned since?

Vertical Circulation

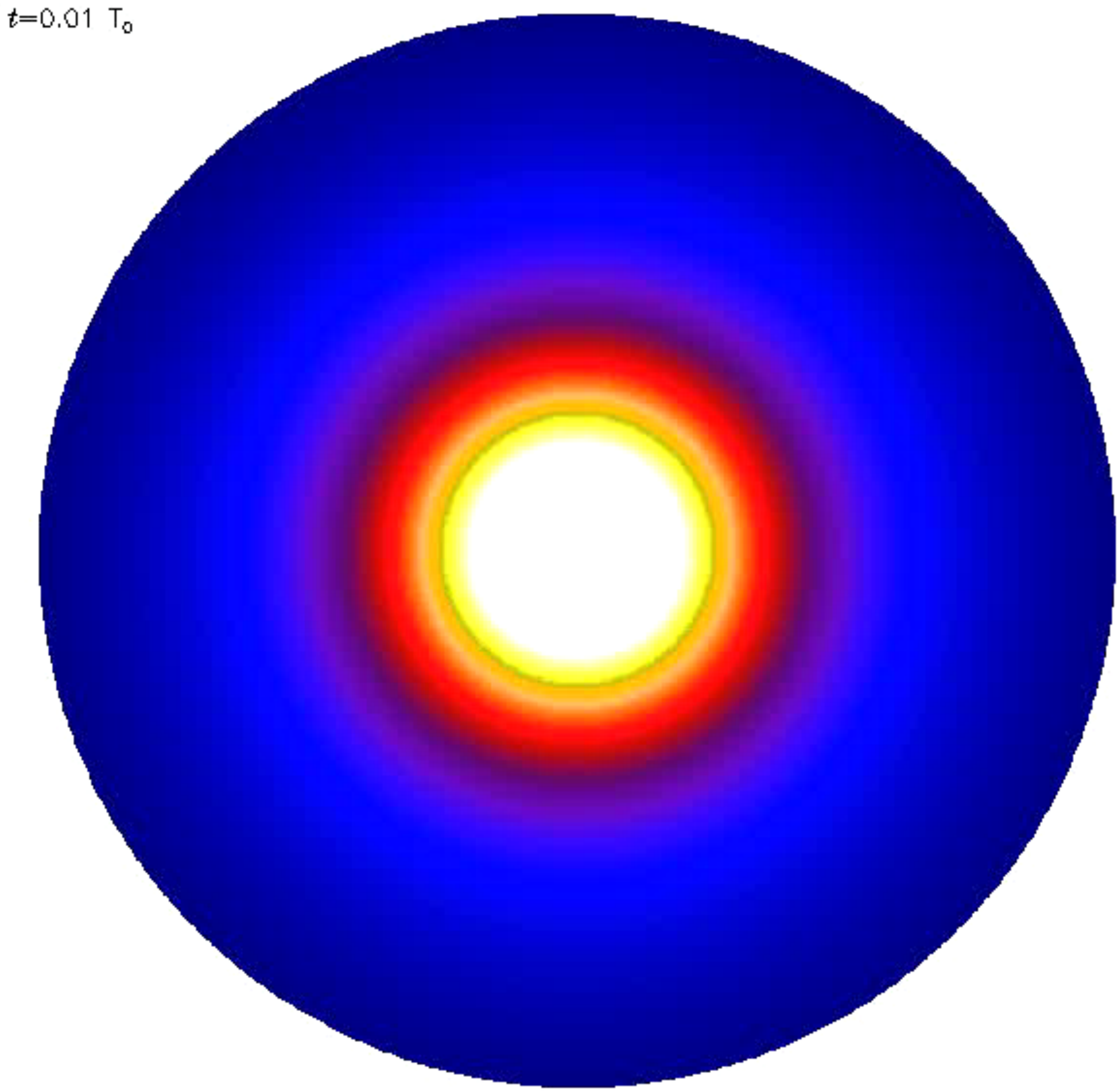


Meheut et al. (2010)
See also Meheut et al. 2012abc, Meheut et al. 2013

With **stratification**, k_z=2 and k_z=0 are **coupled**.
Vertical circulation ensues.

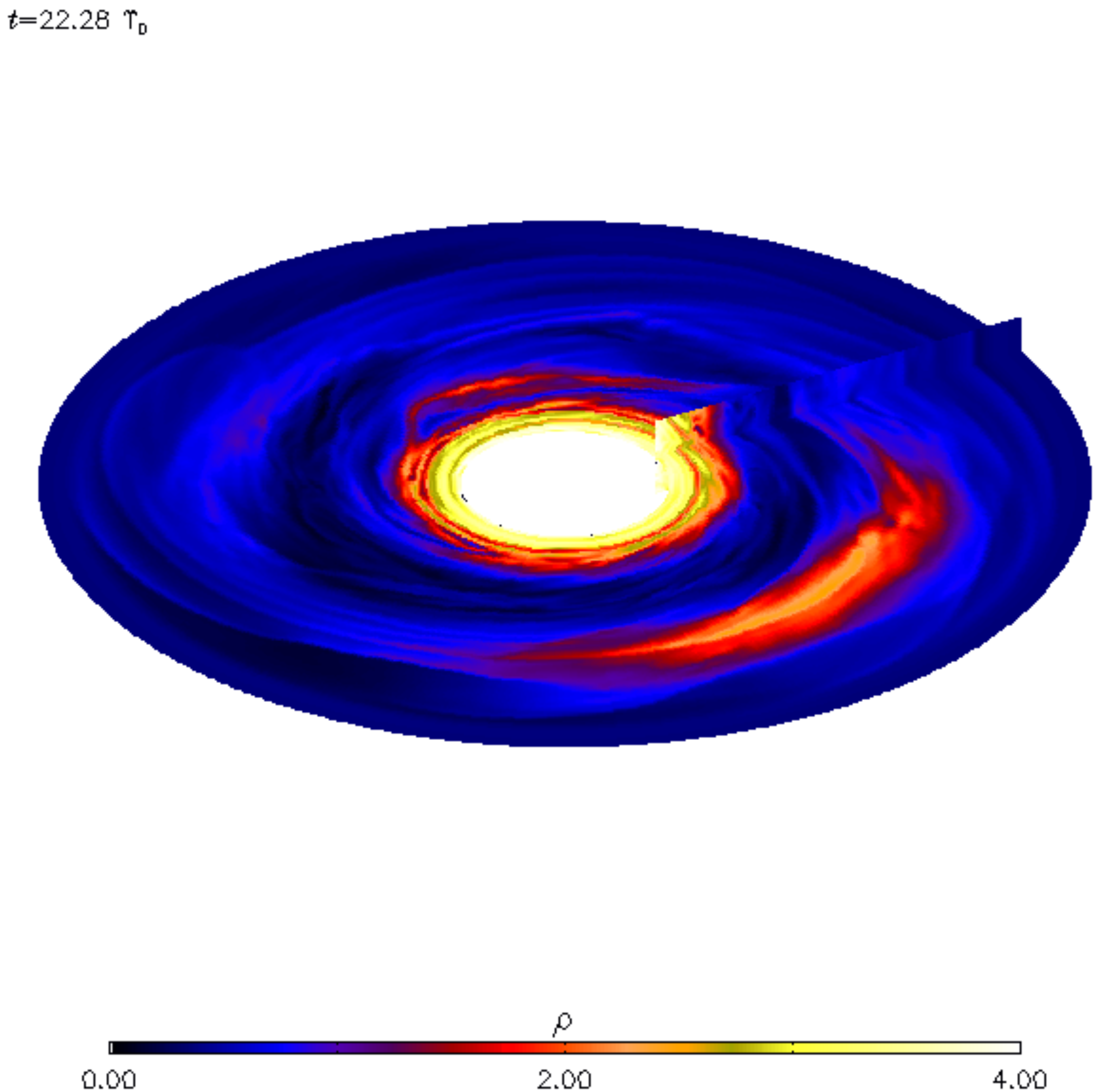
No closed elliptical streamlines, no elliptic instability.

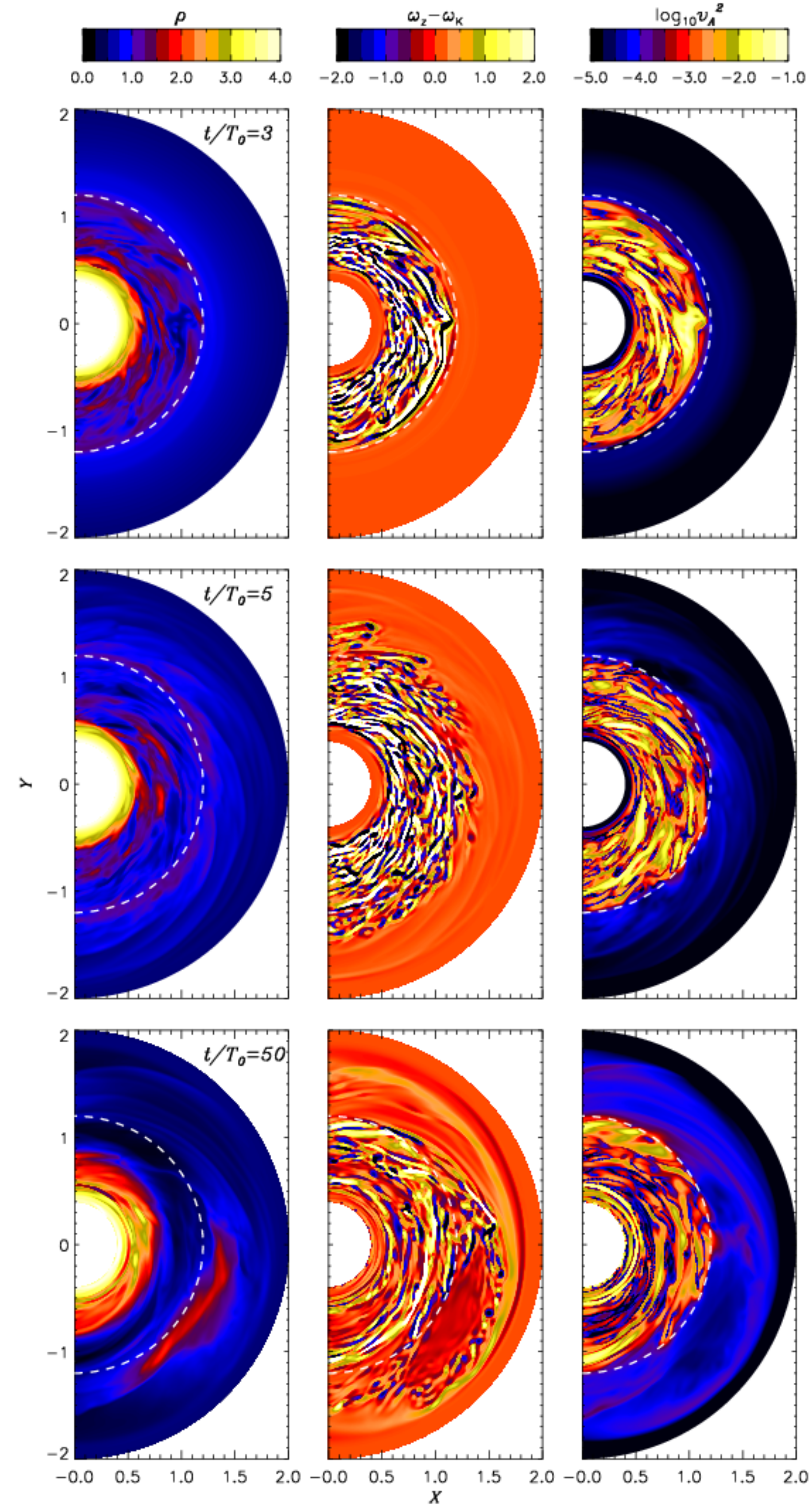
Realistic Active/Dead zone boundary



Lyra & Mac Low (2012)

Active/dead zone boundary





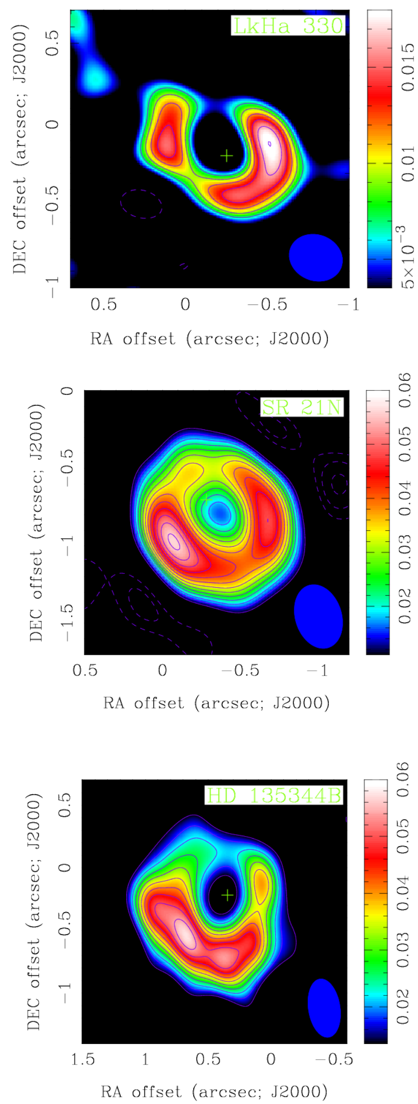
Magnetized inner disk + resistive outer disk

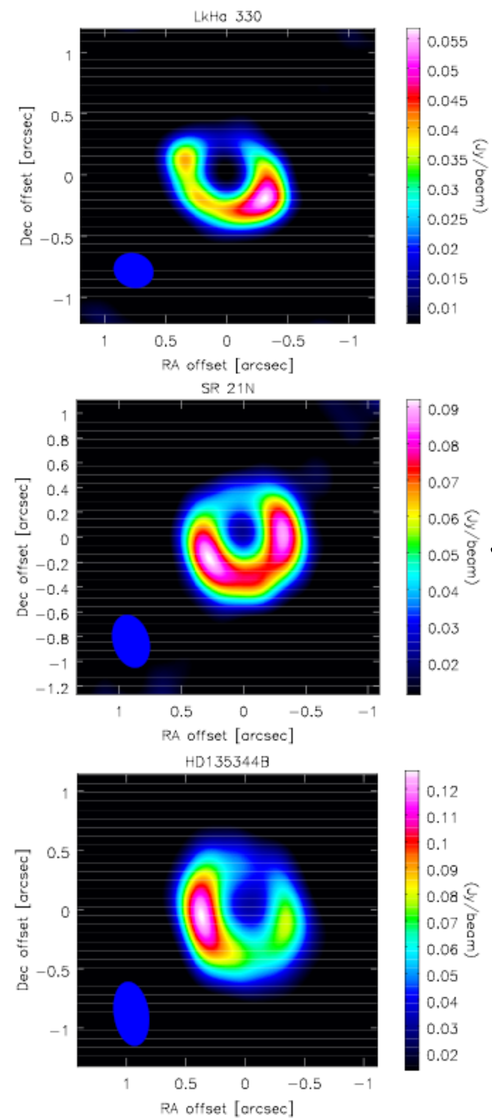
Lyra & Mac Low (2012)

A possible detection of vortices in disks?

Observations

Brown et al. (2009)





Models

Simulated observations of Rossby vortices

Regaly et al. (2012)

And out of Science embargo on June 6th....

A Major Asymmetric Dust Trap in a Transition Disk

Nienke van der Marel,[1*] Ewine F. van Dishoeck,[1,2] Simon Bruderer,[2] Til Birnstiel,[3] Paola Pinilla,[4] Cornelis P. Dullemond,[4] Tim A. van Kempen,[1,5] Markus Schmalzl,[1] Joanna M. Brown,[3] Gregory J. Herczeg,[6] Geoffrey S. Mathews,[1] Vincent Geers[7]

The statistics of discovered exoplanets suggest that planets form efficiently. However, there are fundamental unsolved problems, such as excessive inward drift of particles in protoplanetary disks during planet formation. Recent theories invoke dust traps to overcome this problem. We report the detection of a dust trap in the disk around the star Oph IRS 48 using observations from the Atacama Large Millimeter/submillimeter Array (ALMA). The 0.44-millimeter–wavelength continuum map shows high-contrast crescent-shaped emission on one side of the star, originating from millimeter-sized grains, whereas both the mid-infrared image (micrometer-sized dust) and the gas traced by the carbon monoxide 6-5 rotational line suggest rings centered on the star. The difference in distribution of big grains versus small grains/gas can be modeled with a vortex-shaped dust trap triggered by a companion.

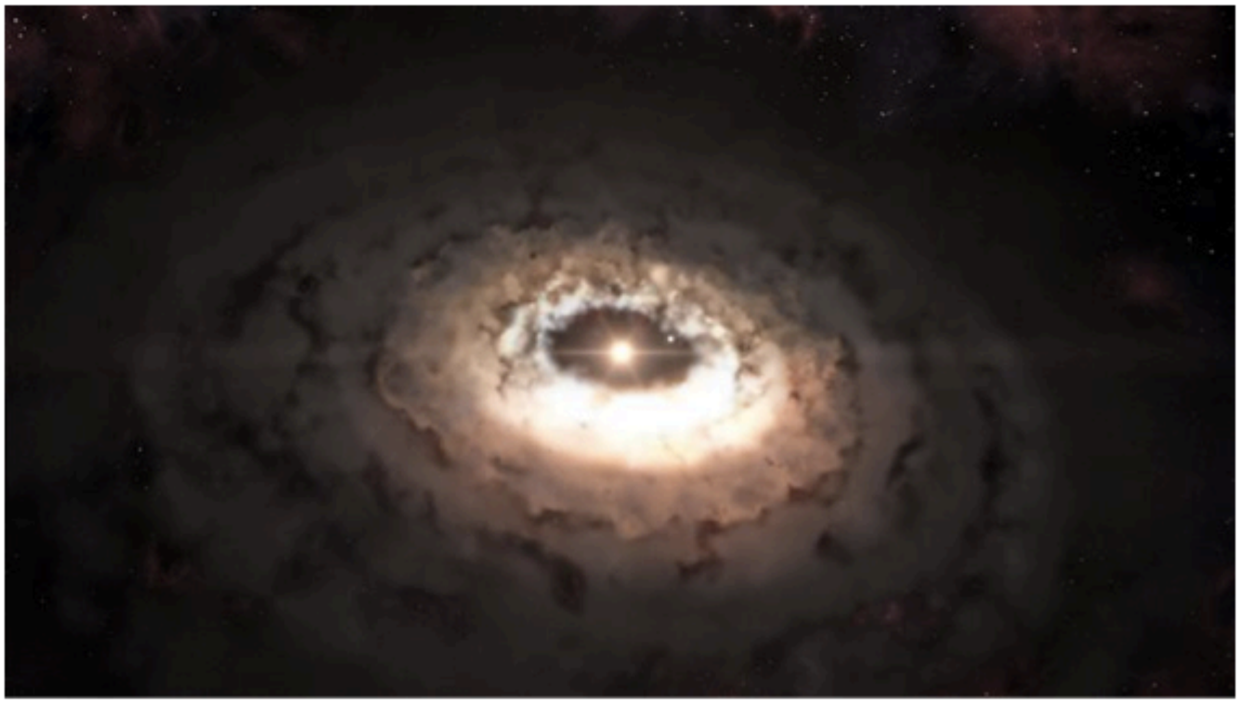
Although the ubiquity of planets is confirmed almost daily by detections of new exoplanets (*1*), the exact formation mechanism of planetary systems in disks of gas and dust around young stars remains a long-standing problem in astrophysics (*2*). In

van der Marel et al. 2013

A possible huge vortex observed with ALMA

Un piège à poussière formateur de planètes

Mise à jour le vendredi 7 juin 2013 à 15 h 34 HAE

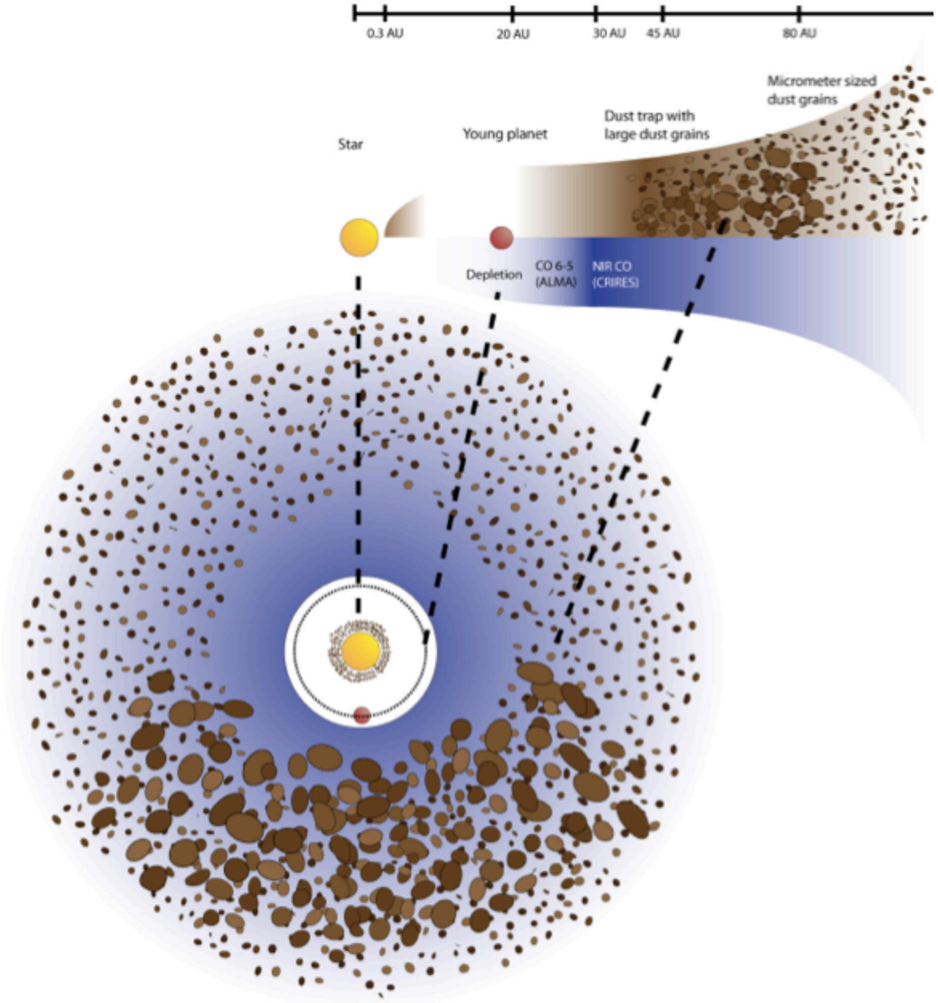


Vue d'artiste de la fabrique de comètes découverte par ALMA *Photo : ESO*

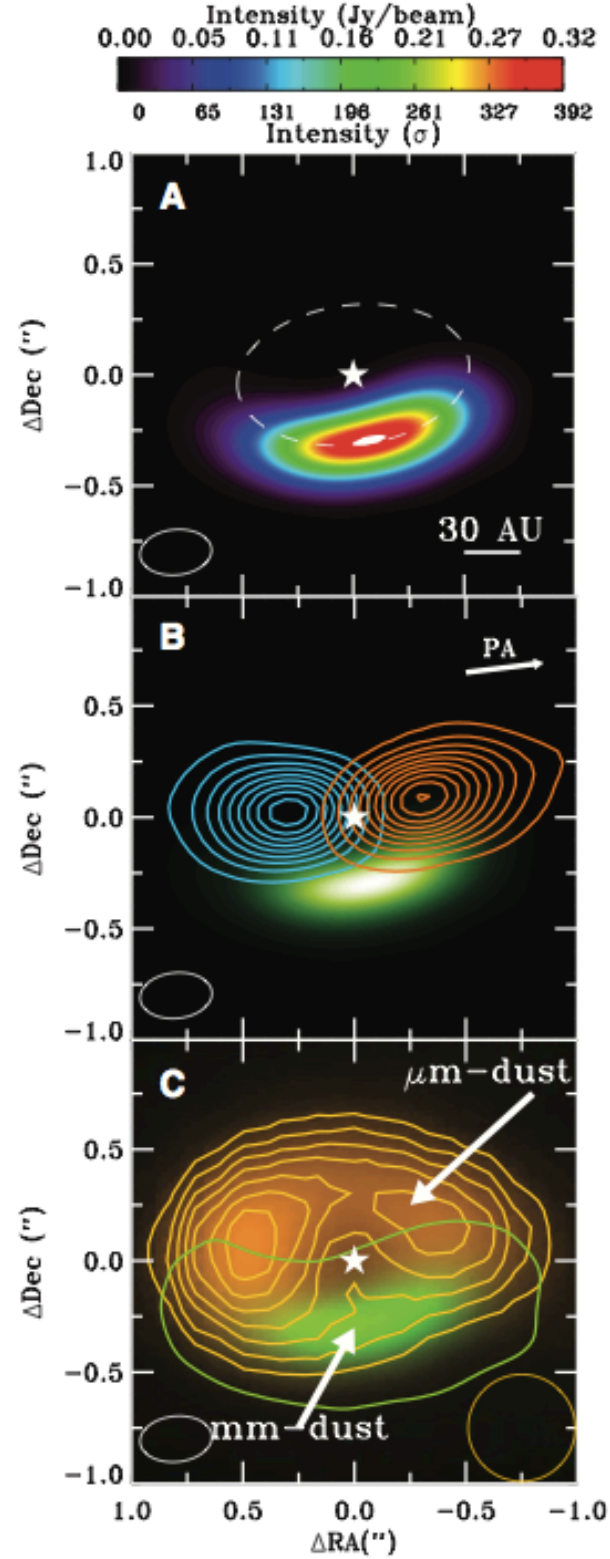
Une zone située autour d'une étoile jeune, au sein de laquelle les particules de poussière peuvent grossir par agglomération et former des corps rocheux, a été observée par des astrophysiciens européens à l'aide du nouveau réseau d'antennes submillimétrique ALMA.

La doctorante Nienke van der Marel et ses collègues de l'Observatoire de Leiden aux Pays-Bas affirment que c'est la première fois qu'un tel piège à poussière est clairement observé et modélisé.

The ALMA "dust trap"



van der Marel et al. (2013)



asymmetric mm dust at 63 AU

Gas detection: Keplerian rotation

Micron-sized dust follows gas

The ALMA "dust trap": Properties

Aspect ratio: 3.1

Temperature: 60K

Trapped mass: 9 M_{Earth}

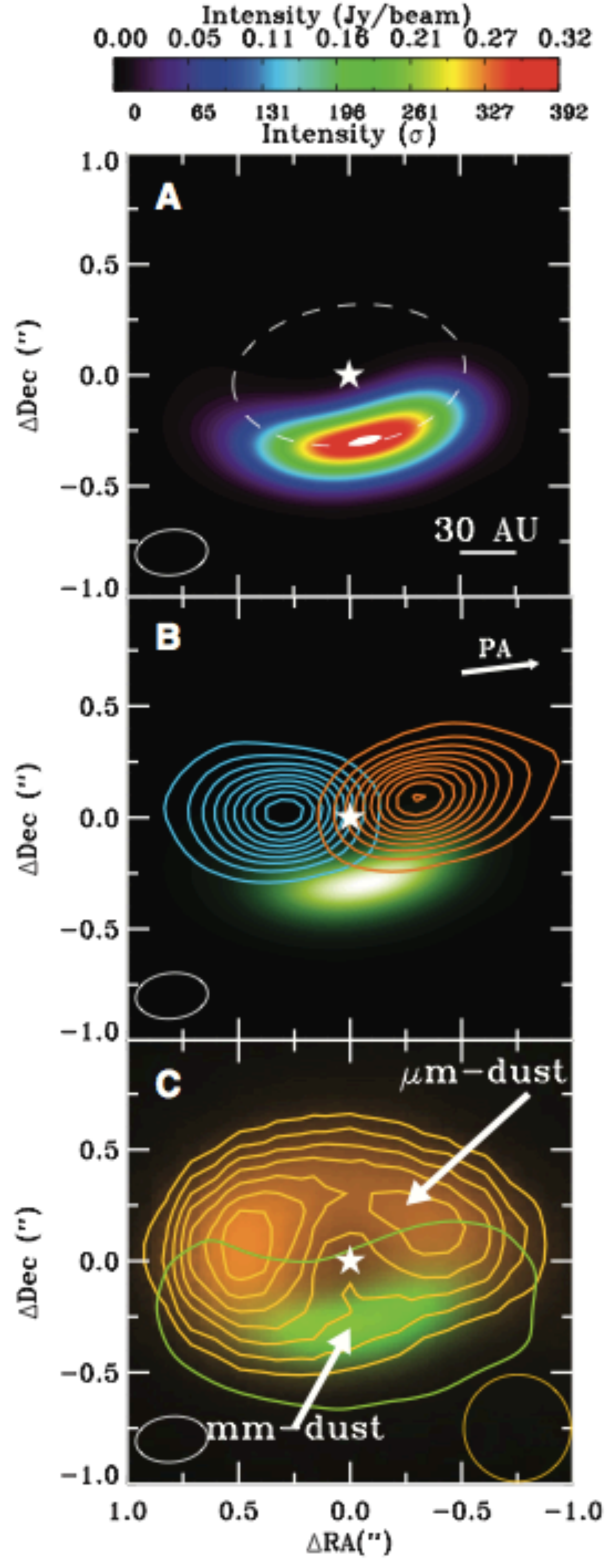
Center: 63 AU

Extent: 45-80 AU

Gas cavity at 25AU

Aspect ratio: 3
should be unstable to elliptic instability

Temperature: 60K
perhaps not yet at isothermal radii,
may not be Rossby, but baroclinic

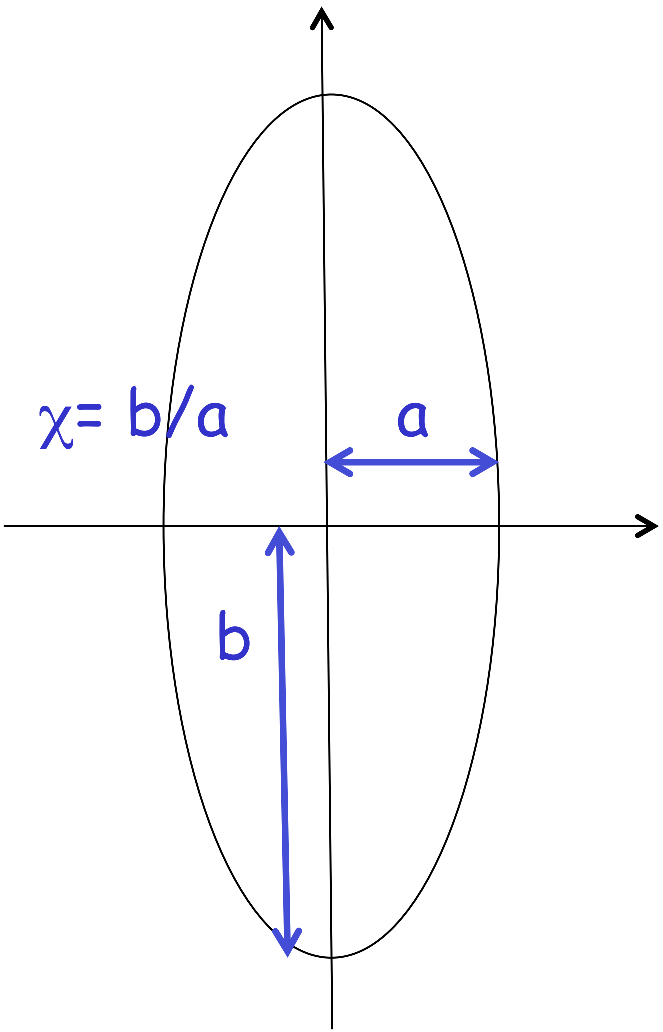


asymmetric
mm dust
at 63 AU

Gas detection:
Keplerian rotation

Micron-sized
dust follows gas

A fairly general analytical solution for vortex trapping



$$u_x = \Omega_V y/\chi \qquad u_y = -\Omega_V x\chi,$$

Dust continuity equation

$$\frac{\partial \rho_d}{\partial t} = -(\boldsymbol{v}\cdot\boldsymbol{\nabla})\rho_d - \rho_d \boldsymbol{\nabla}\cdot\boldsymbol{v} + D\nabla^2\rho_d,$$

$$\boldsymbol{v} = \boldsymbol{u} + \tau\boldsymbol{\nabla}h,$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{v} = \tau\nabla^2 h,$$

Equilibrium between diffusion and drag

$$\left(D\nabla^2 - \boldsymbol{v}\cdot\boldsymbol{\nabla} + C\right)\rho_d = 0.$$

Lyra & Lin (2013, submitted)

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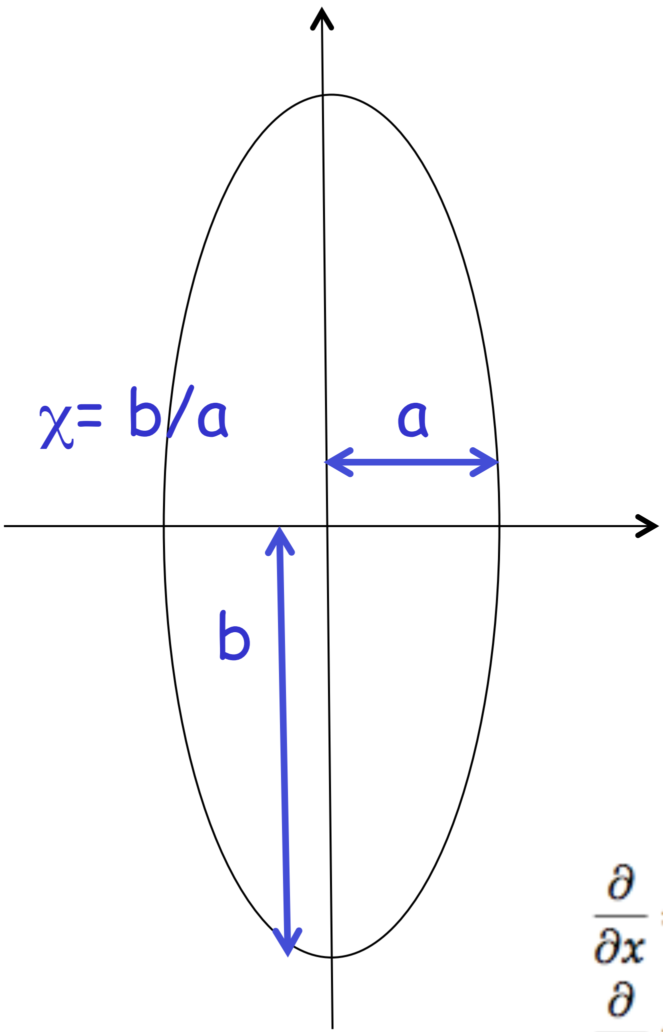

Equilibrium between diffusion and drag

$$\left(D\nabla^2 - v\cdot\boldsymbol{\nabla} + C\right)\rho_d = 0.$$

Transformation of Chang & Oishi (2010)

$$x = a\cos\nu,$$
$$y = a\chi\sin\nu.$$

Derivatives

$$\frac{\partial}{\partial x} = \cos\nu\frac{\partial}{\partial a} - \frac{\sin\nu}{a}\frac{\partial}{\partial \nu},$$
$$\frac{\partial}{\partial y} = \frac{1}{\chi}\left(\sin\nu\frac{\partial}{\partial a} + \frac{\cos\nu}{a}\frac{\partial}{\partial \nu}\right),$$

Laplacian

$$\begin{aligned}\nabla^2 = &\frac{1}{2}\left[\varepsilon_-\cos 2\nu + \varepsilon_+\right]\partial_a^2\\ &+\frac{1}{2a^2}\left[\varepsilon_+ - \varepsilon_-\cos 2\nu\right]\partial_\nu^2\\ &-\frac{\sin 2\nu}{a}\varepsilon_-\partial_{a\nu}^2\\ &+\frac{1}{2a}\left[\varepsilon_+ - \varepsilon_-\cos 2\nu\right]\partial_a\\ &+\frac{\sin 2\nu}{a^2}\varepsilon_-\partial_\nu,\end{aligned}$$

Lyra & Lin (2013, submitted)

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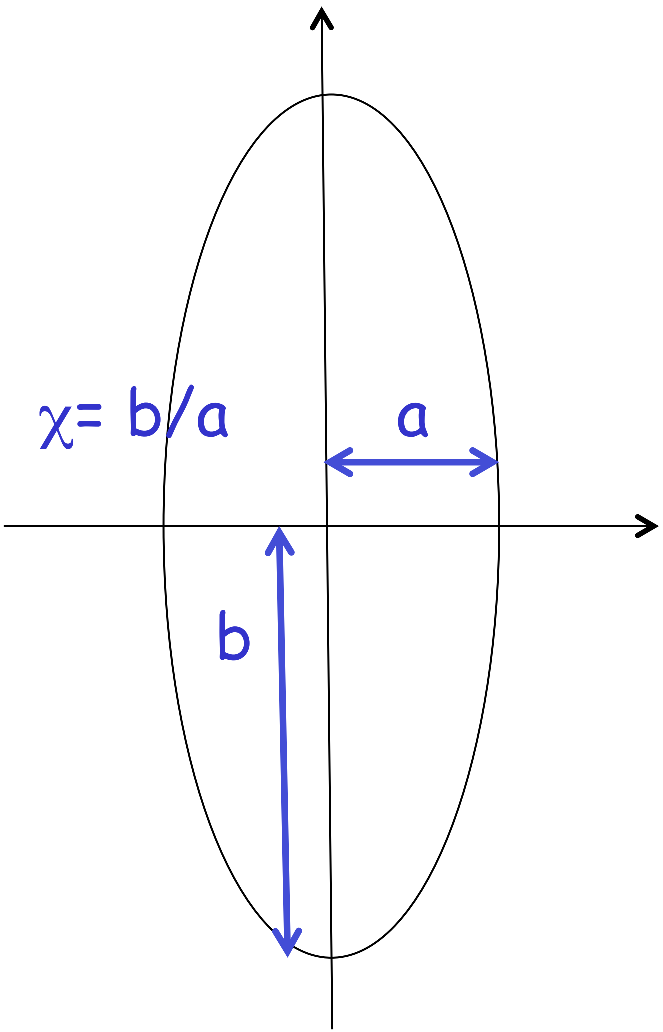


$$\left(D\nabla^2 - \boldsymbol{v}\cdot\boldsymbol{\nabla} + C\right)\rho_d = 0.$$

$$x = a\cos\nu,$$
$$y = a\chi\sin\nu.$$

"Axis-symmetric" equation

$$\left[\partial_a^2 + \left(\frac{1}{a} + \frac{k^2}{2}a\right)\partial_a + k^2\right]\rho_d = 0,$$

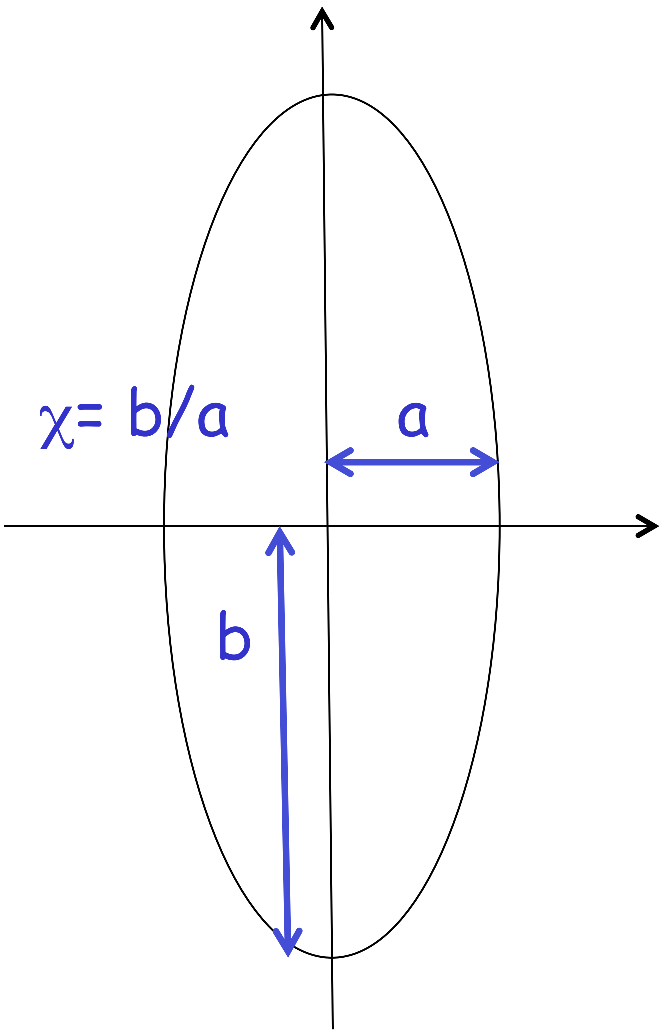
Solution

$$\rho_d(a) = \rho_{d0}\exp\left(-\frac{a^2}{2H_V^2}\right),$$

$$H_V = \frac{H}{f(\chi)}\sqrt{\frac{\delta}{\mathrm{St}}}.$$

Lyra & Lin (2013, submitted)

Axis-symmetric dust trapping solution



Solution

$$\rho_d(a) = \rho_{d0} \exp\left(-\frac{a^2}{2H_V^2}\right),$$

$$H_V = \frac{H}{f(\chi)}\sqrt{\frac{\delta}{\mathrm{St}}}.$$

Depends on H, χ, δ, and St

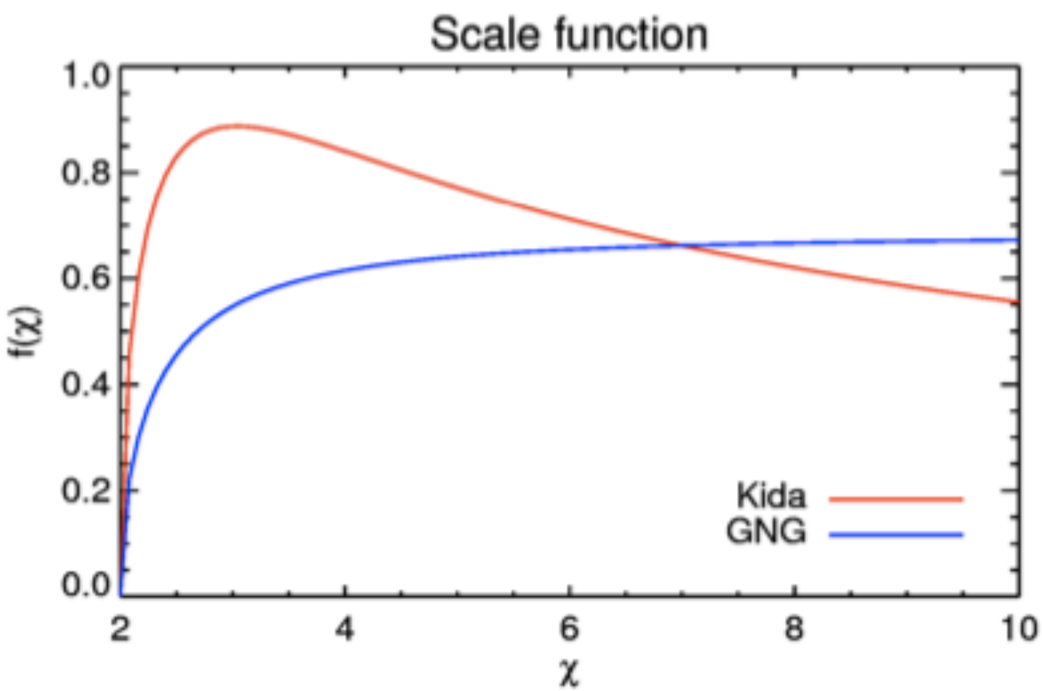
In principle, these are ALL observable.

H = disk scale height, sonic scale, temperature
χ = vortex aspect ratio
δ = diffusion parameter ($D = \delta\, c_s H$)
St = Stokes number (particle size)
$f(\chi)$ = model-dependent scale function

Lyra & Lin (2013, submitted)

Scale function

$$f^2(\chi) = \varepsilon_+^{-1}\left[2\omega_V\left(\frac{\chi^2+1}{\chi}\right) - (2\omega_V^2+3)\right]$$
$$= 2\omega_V\chi - \varepsilon_+^{-1}(2\omega_V^2+3),$$



Kida solution

$$\Omega_V = \frac{3\Omega}{2(\chi-1)},$$

GNG solution

$$\Omega_V = \Omega\sqrt{3/(\chi^2-1)}.$$

Lyra & Lin (2013, submitted)

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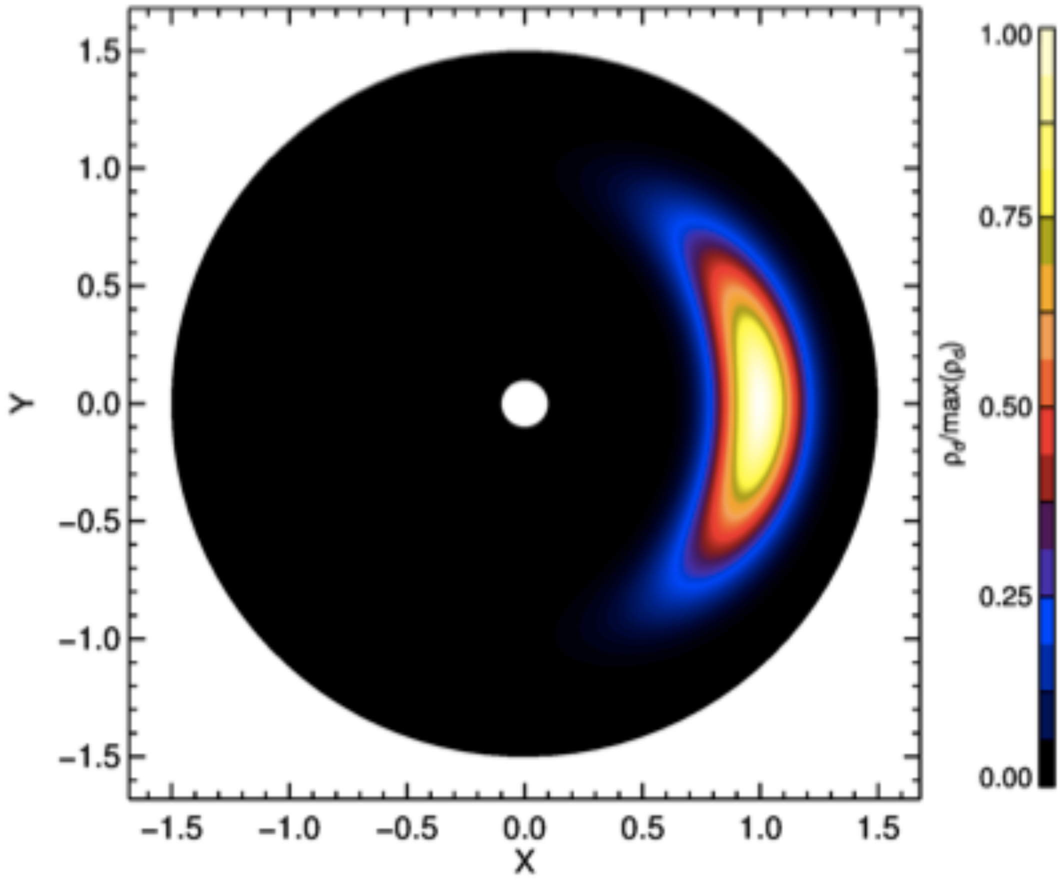


Kida vortex of

aspect ratio $\chi=4$

In a disk of $h=0.1$

and $\delta/\mathrm{St} = 1$

Lyra & Lin (2013, submitted)

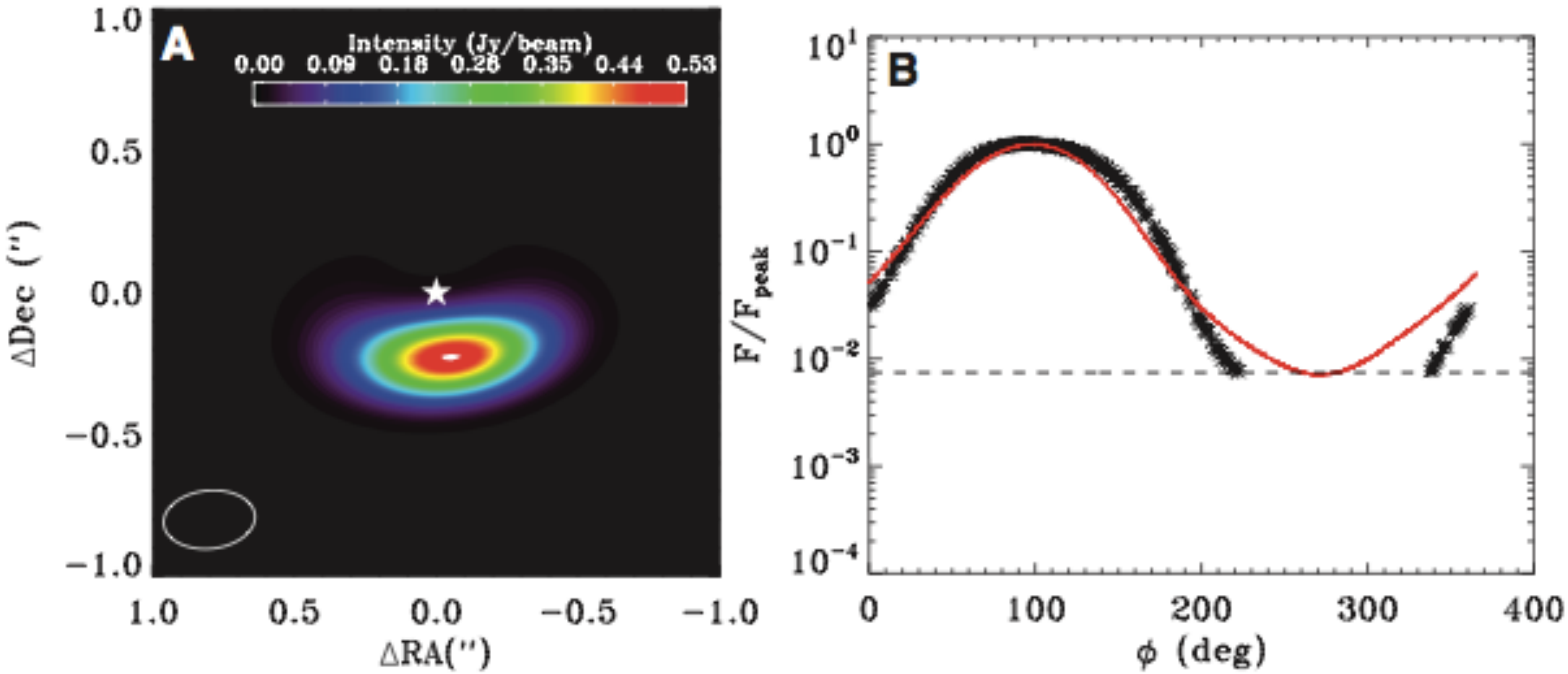
h = $\underline{H/r}$ = disk aspect ratio
χ = vortex aspect ratio
δ = diffusion parameter
St = Stokes number (particle size)
$f(\chi)$ = model-dependent scale function

$$\rho_d(a) = \rho_{d0} \exp\left(-\frac{a^2}{2H_V^2}\right),$$

$$H_V = \frac{H}{f(\chi)}\sqrt{\frac{\delta}{\mathrm{St}}}.$$

Observational predictions

Density contrast



$$\rho_{\max} / \rho_{\min} = \exp\left(\frac{\pi^2}{h^2}\frac{f^2(\chi)}{\chi^2}\frac{St}{\delta}\right)$$

Lyra & Lin (2013, submitted)

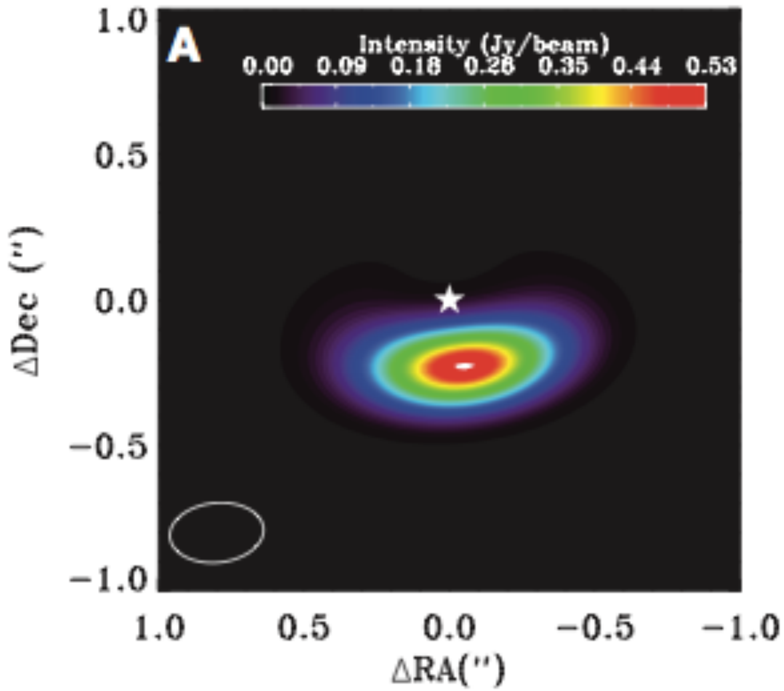
h = $\underline{H/r}$ = disk aspect ratio
χ = vortex aspect ratio
δ = diffusion parameter
St = Stokes number (particle size)
$f(\chi)$ = model-dependent scale function

$$\rho_d(a) = \rho_{d0} \exp\left(-\frac{a^2}{2H_V^2}\right),$$

$$H_V = \frac{H}{f(\chi)}\sqrt{\frac{\delta}{\mathrm{St}}}.$$

Observational predictions

Total trapped mass



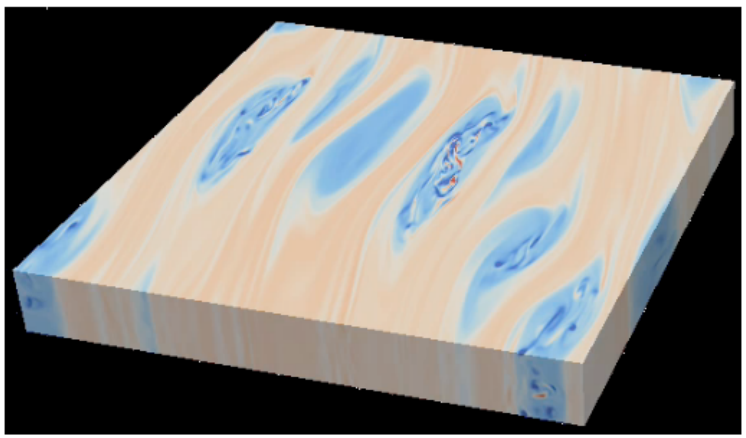
$$M(St) = \int_V \rho_d \, dV$$

$$= \sqrt{\frac{\pi^3}{2}} \chi H_V^2 H_z \rho_{\max}$$

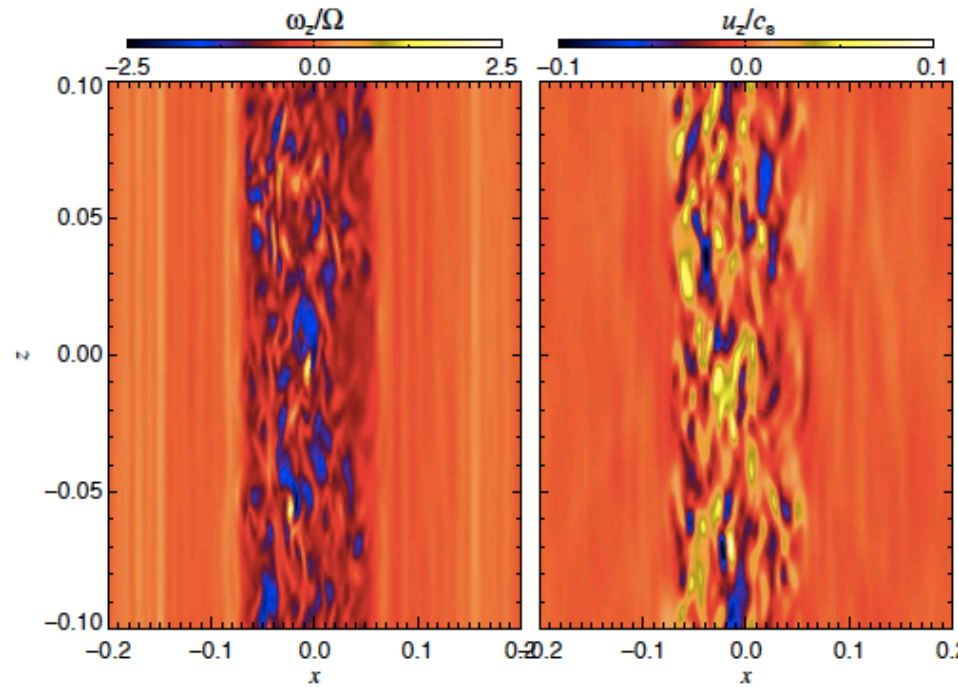
Lyra & Lin (2013, submitted)

Observational predictions

Measuring the diffusion parameter δ
Turbulence in the Core – Doppler broadening



Lesur & Papaloizou (2010)



Lyra & Klahr (2011)

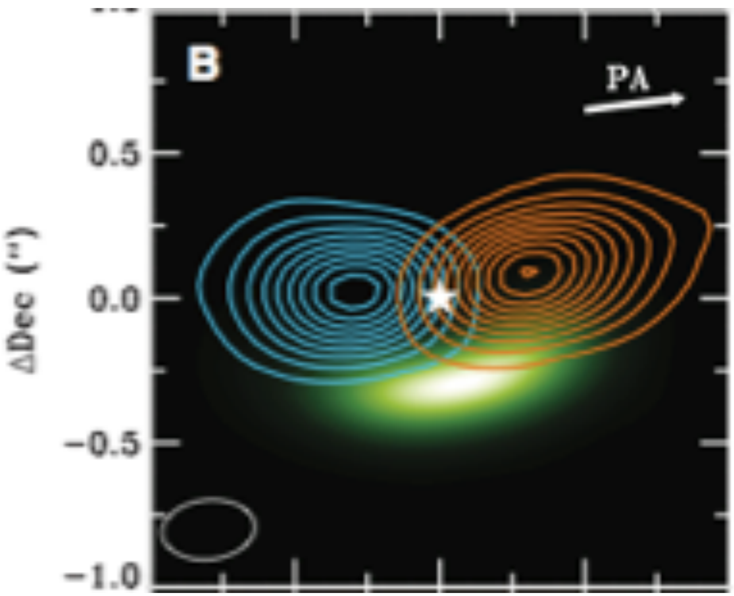
Elliptic instability in vortex core:
turbulence at 10% of sound speed

For a temperature of 60K,
should lead to **0.3 km/s Doppler signature**.

At the "hairy edge" of what ALMA can do (0.2 sensitivity).

Observational predictions

Measuring the vortex model:
Rotational Broadening

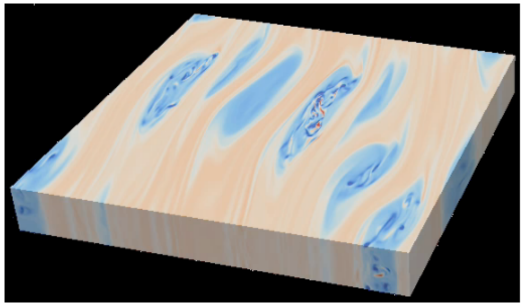


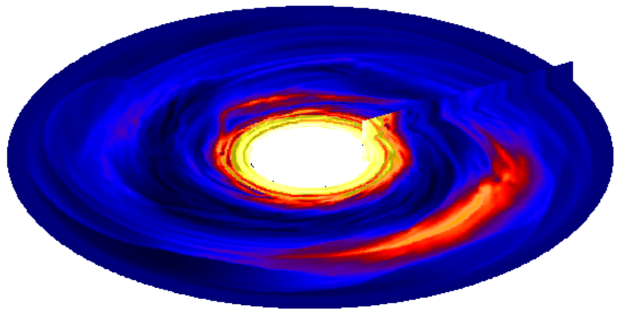
Vortex motion for size H:
$v = \Omega_V H \sim \Omega H \sim c_s$

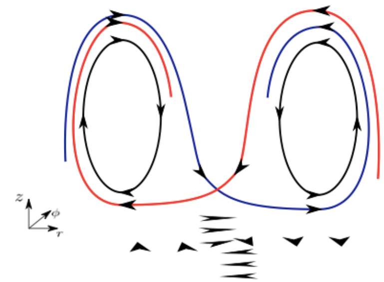
For a temperature of 60K,
should lead to **~1 km/s rotational broadening** signature.

Plus, this should be spatially resolvable (!)

Conclusions







We have come a long way since 2D models of vortex trapping

ALMA is returning images that may
finally give observational constrains to our models