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On the angle of sunset

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Abstract

We calculate the angle ψ between the sun path at sunset and the horizon to be $\psi = \arctan\left[\sqrt{1 - \sin^2\delta/\cos^2\phi}\,\tan(90^\circ - \phi)\right]$, where δ is the declination of the sun, and ϕ is the latitude of the place. We dub this angle the *ghorub* angle, after غروب , the Arabic word for sunset.

INTRODUCTION

The sunset has marveled humans since time immemorial. The inhabitants of sub-polar latitudes experience long sunsets and the phenomenon of white nights, where twilight persists throughout the night near the summer solstice; the duration of which is associated with the angle ψ between the path of the sun (the day arc) at sunset and the horizon. We set here to calculate this angle, for which, to the best of our knowledge, lacks a standard name. We could use a Latin appellation, the angle of *solis occasum*; or Greek, the angle of *heliovasilema*. Instead, we will keep with astronomical tradition and use an Arabic name. “Sunset” in Arabic is غروب (Latin alphabet transliteration *ghrwb*, and approximate pronunciation *GHO-roob*, with “gho“ as in “ghost”), thus we call it the *ghorub* angle.

The geometry of the ghorub angle is visualized in Fig. 1a. The Sun is at altitude h, and azimuth A. Also shown is A_0, the azimuth of the Sun at sunset. NCP refers to North Celestial Pole, and N is the north cardinal point. In what follows the situation refers to the northern hemisphere, but can be trivially generalized to the south hemisphere as well. We consider as reference point the altitude and azimuth at sunset, and measure $\Delta h = h - h_0$ and $\Delta A = A - A_0$. By definition, $h_0 = 0$. Let ψ denote the ghorub angle. When the sun is close to zero altitude, Δh and ΔA become infinitesimal, and

$$\tan\psi = \lim_{\Delta\to 0}\frac{\Delta h}{\Delta A} \equiv \left.\frac{dh}{dA}\right|_{h=0} \tag{1}$$

The question then is to calculate dh/dA at zero altitude. For that we consider the relations between alt-azimuthal and equatorial coordinates (c.f., Smart & Green 1977)

$$\sin h = \sin\phi\sin\delta + \cos\phi\cos\delta\cos H \tag{2}$$
$$\cos h\sin A = \cos\delta\sin H \tag{3}$$
$$\cos h\cos A = \sin\phi\cos\delta\cos H - \cos\phi\sin\delta \tag{4}$$

We isolate $\cos H$ from Eq. (4) and substitute in Eq. (2)

$$\sin h = \sin\phi\sin\delta + \frac{\cos^2\phi\sin\delta}{\sin\phi} + \frac{\cos\phi\cos h\cos A}{\sin\phi} \tag{5}$$

On the timescales of the sunset (< 1 hour), the Sun can be approximated as at constant declination. Therefore, the first two terms in the RHS are constant. Taking the time derivative

$$\dot{h}\left(\tan\phi\cos h + \sin h\cos A\right) = -\dot{A}\cos h\sin A \tag{6}$$

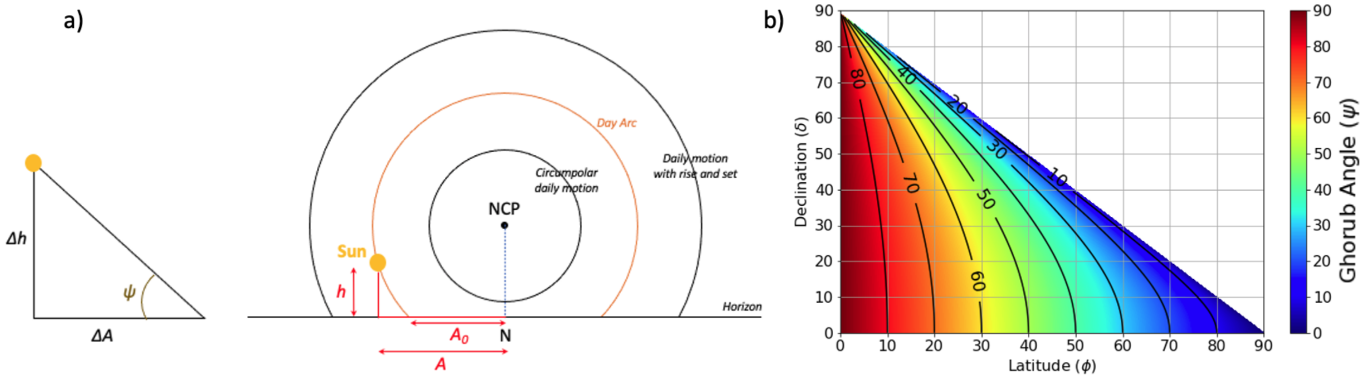

Figure 1. **a.** Geometry of the ghorub angle ψ. A_0 is the azimuth of the sun at sunset (ghorub azimuth). **b.** The ghorub angle as a function of latitude and declination. The solar declination varies only from $|\delta| \leq \varepsilon$, where ε is the obliquity of the ecliptic, but the angle is defined for any celestial object. Vertical lines in this plot correspond to colatitude; oblique contours to the azimuth correction.

given $\dot{h}/\dot{A} = dh/dA$

$$\frac{dh}{dA} = -\frac{\cos h \sin A}{(\tan\phi\cos h + \sin h \cos A)} \tag{7}$$

since we are interested in the moment of setting, when $h = 0$

$$\begin{aligned}\tan\psi &\equiv \left.\frac{dh}{dA}\right|_{h=0} \\ &= -\sin A_0 \tan\left(90^\circ - \phi\right)\end{aligned} \tag{8}$$

When a star is barely circumpolar, i.e., setting tangent to the horizon, $A_0 = 0$, and then $\psi = 0$.

Eq. (8) is the ghorub angle, which means that in principle we are done. Yet, we can generalize the equation by calculating the sunset azimuth A_0 in a general way.

The ghorub azimuth can be calculated as follows. From Eq. (2) we determine the hour angle H_0 at setting ($h = 0$)

$$\cos H_0 = -\tan\phi\tan\delta \tag{9}$$

and substitute this into Eq. (4) to find $\cos A_0$

$$\cos A_0 = -\frac{\sin\delta}{\cos\phi} \tag{10}$$

finally leading to (assuming A_0 lies in the first quadrant)

$$\sin A_0 = \sqrt{1 - \frac{\sin^2\delta}{\cos^2\phi}} \tag{11}$$

Substituting into Eq. (8)

$$\boxed{\tan\psi = \sqrt{1 - \frac{\sin^2\delta}{\cos^2\phi}}\tan\left(90^\circ - \phi\right)} \tag{12}$$

Fig. 1b plots the angle as a function of latitude and declination. The empty areas of the plot correspond to circumpolar situations. Vertical lines in this plot correspond to colatitude; oblique contours to the azimuth correction. Notice that only in specific circumstances the ghorub angle of the Sun is equal to the co-latitude. That is for an observer at the Equator ($\phi = 0$), which will observe all celestial bodies set vertically ($\psi = 90^\circ$) regardless of their declination, or during an equinox ($\delta = 0$) for any observer location.

REFERENCES

Smart, W. M., & Green, E. b. R. M. 1977, Textbook on Spherical Astronomy