Class 9

Evolution of low-mass stars

Stellar Evolution

As H → He, mean molecular weight of gas increases

$$P = \frac{R}{\mu} \rho T$$

if μ increases, if ρ and T remain the same, P will decrease, and the ~~P~~ cannot support the outer layers. It must compress.

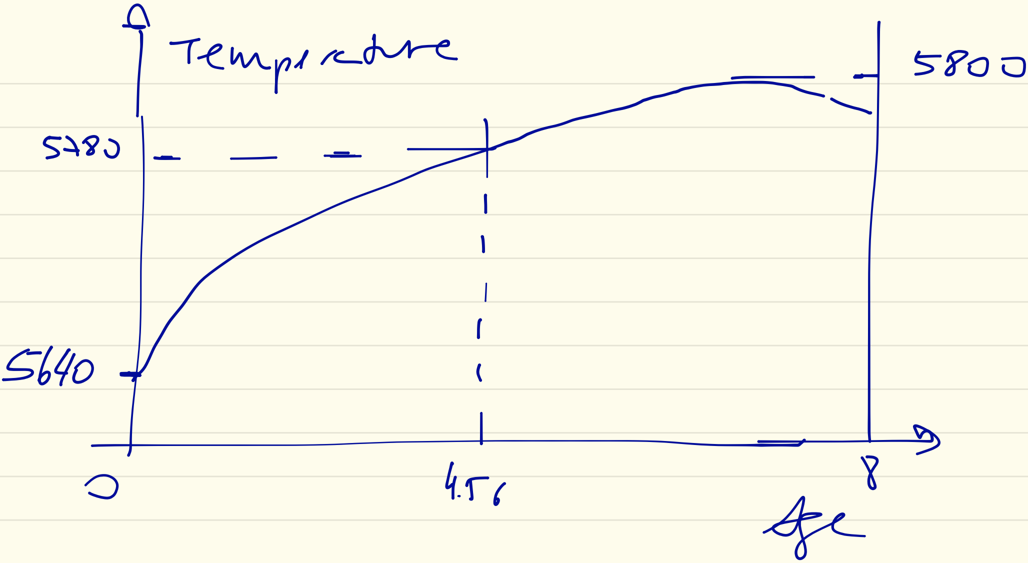
ρ increases, core shrinks, releases luminosity and heats up.

As T increases, the region of the star that can undergo nuclear reactions increases slightly during the main sequence phase of evolution

pp-chain goes as $\rho X^2 T_6^4$, the increase in temperature and density compensate the decrease in hydrogen mass fraction, and the star produces more energy

show sun's evolution figure

Faint sun paradox / Ocean boil-off



Faint sun paradox.

Once H gone, temperature has increased to the point that burning occurs around the inert He shell.

Inert He core isothermal;

$$L = 0 \quad , \quad \text{so } \frac{dT}{dr} \propto L = 0$$

supports gravity due to density gradient alone.

Shell burning ⇒ More energy. Envelope expands T_{eff} decreases ⇒ subgiant branch.

Ash from nuclear burning increases mass of He core. Will continue until He core is too massive to support its weight.

Maximum mass: Schönberg - Chandrasekhar limit

Show track in evolutionary track and go labeling.

Degeneracy: when density is high, electrons are forced to occupy the lowest energy levels. But they are fermions and cannot occupy the ground state. They are stacked into larger energy states. The resulting pressure is due to electron motion and completely independent of temperature.

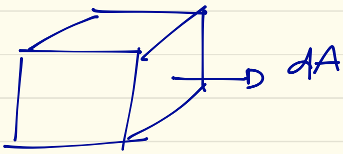
$$P = K\rho^{5/3}$$

uncertainty principle.

In a unit volume h^3 in $x-p$ space there cannot be more than 2 fermions (one spin up one spin down).

$$n(p)dp = 4\pi f(p)\, p^2 dp$$

$$P = \frac{1}{3}\int v p\, f(p)\, 4\pi p^2 dp$$



$$P = \frac{F}{A} = \frac{d\vec{p}}{dt\, dA}$$

$$d\vec{F} = \frac{1}{6}\cdot 2 \cdot 4\pi f(p)\, p^2 dp$$

6 faces

twice momentum elastic

$$dp = n(p) \cdot dV$$
$$= n(p)\cdot dA\, v \cdot dt$$

Substitute $f(p)$ by Maxwell-Boltzmann, the pressure is $n k_B T$.

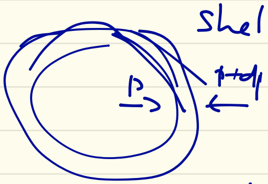
Due to degeneracy, a fermion gas has pressure even at $T=0$.

Real space dV, momenta dp

$$N = 2 \frac{dV}{h^3} d^3p \longrightarrow \text{N of states}$$

spin

shell: $d^3p = 4\pi p^2 dp$



$$n = \frac{N}{dV} = 8\pi p^2 \frac{dp}{h^3} \quad \text{(per unit volume)}$$

$$n = \int_0^{p_F} \frac{8\pi}{h^3} p^2 dp = \frac{8\pi}{3h^3} p_F^3$$

$$f(p) = \begin{cases} 2/h^3 & ; \text{ if } p < p_F \\ 0 & ; \text{ if } p > p_F \end{cases}$$

$$p = m\gamma v \qquad \therefore \; v = \frac{p}{m\gamma} = \frac{pc^2}{E} = \frac{pc^2}{\sqrt{p^2c^2 + m^2c^4}}$$

$$P = \frac{8\pi}{3h^3}\int_0^{p_F} v\, p^3\, dp$$

$$= \frac{8\pi}{3h^3}\int_0^{p_F} \frac{p^4 c^2}{\sqrt{p^2c^2 + m^2c^4}}\, dp$$

$n_H = \frac{\rho X}{m_H}$ (number density of hydrogen)

if fully ionized, then $n_e \approx n_H$

He and others usually contribute $\frac{A}{2}$ electrons, because $A \approx 2Z$. So, they contribute

$(1-X)\rho$ (mass) $\rightarrow \frac{(1-X)\rho}{m_H}$ nucleons,

and $\frac{(1-X)\rho}{2m_H}$ electrons.

$$n_e = \frac{X\rho}{m_H} + \frac{(1-X)\rho}{2m_H} = \frac{\rho}{2m_H}(1+X)$$

$n_e = \frac{\rho}{\mu_e m_H}$ $\qquad \mu_e = \frac{2}{1+X}$ (electron mean weight)

$$p_F = \left(\frac{3h^3\rho}{8\pi\mu_e m_H}\right)$$

for a non-relativistic gas,

$$\left(p^2c^2 + m_e^2c^4\right)^{1/2} \approx m_e c^2$$

So the integral gives

$$P = \frac{8\pi}{15h^3 m_e} p_F^5$$

$$P = K_1 \rho^{5/3}$$

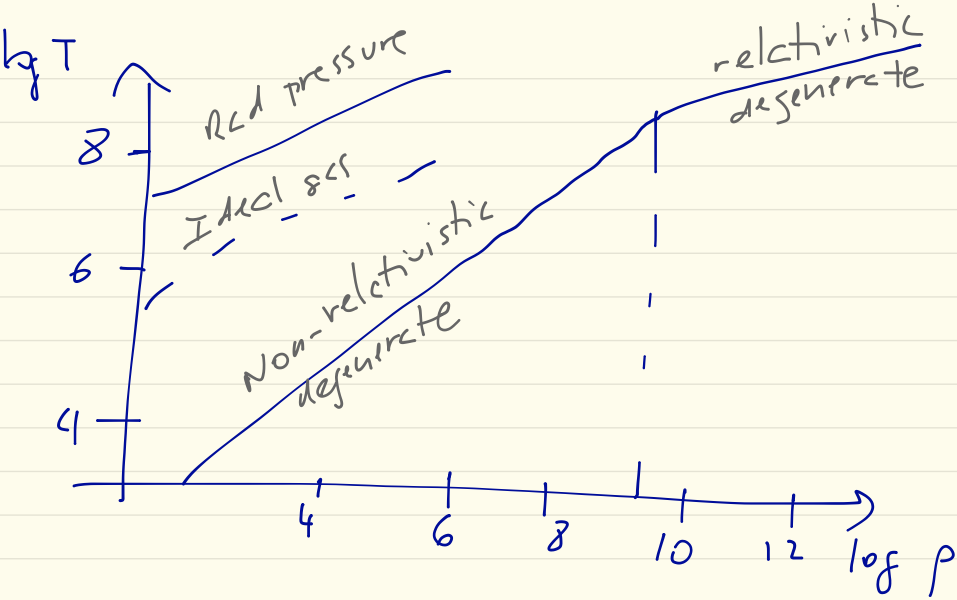
$$K_1 = \frac{3^{2/3}}{20\pi^{2/3}} \frac{h^2}{m_e \, m_H^{5/3} \, \mu_e^{5/3}} = \left(\frac{3}{\pi m_H \mu_e}\right)^{2/3} \cdot \frac{h^2}{20 m_e m_H \mu_e} \propto \mu_e^{-5/3}$$

For fully relativistic, $E = pc$

$$P = \frac{2\pi c}{3h^3} p_F^4$$

$$P = K_2 \rho^{4/3}$$

$$K_2 = \left(\frac{3}{\pi m_H \mu_e}\right)^{1/3} \cdot \frac{hc}{8 m_H \mu_e} \propto \mu_e^{-4/3}$$



Fermi energy: put one electron at a time, when all electrons are put, the kinetic energy of the highest occupied state is the Fermi energy.

This means that even if you cool down to absolute zero, this energy remains. This can be converted to a measurement of velocity and a measurement of temperature. Only when the temperature is higher than the Fermi temperature do the electrons move much faster than at absolute zero.

$$T_F = \frac{E_F}{K_B} \; ; \; p_F = \sqrt{2 m_e E_F} \; ; \; v_F = \frac{p_F}{m_e}$$

Schönberg - Chandrasekhar limit

$$\frac{dP}{dM_r} = -\frac{GM_r}{4\pi r^4}$$

$$4\pi r^3 \frac{dP}{dM_r} = -\frac{GM_r}{r}$$

$$= \frac{d(4\pi r^3 P)}{dM_r} - 12\pi r^2 P \frac{dr}{dM_r}$$

$$= \frac{d(4\pi r^3 P)}{dM_r} - \frac{3P}{\rho} \qquad \frac{P}{\rho} = \frac{kT}{\mu m_H}$$

$$\underbrace{\int_0^{M_c} \frac{d}{dM_r}(4\pi r^3 P)\, dM_r}_{4\pi R_c^3 P_c} - \underbrace{\int_0^{M_c} \frac{3P}{\rho}\, dM_r}_{\frac{3M_c k T_c}{\mu_c m_H} = 3N_c k T_c = 2K_c} = -\int_0^{M_c} \frac{GM_r}{r}\, dM_r$$

$$N_c = \frac{M_c}{\mu_c m_H} \; ; \quad K_c = \frac{3}{2} N_c k T_c \text{ (thermal energy of core)}$$

RHS is grav energy of core, or

$$-\int_0^{M_c} \frac{GM_r}{r}\, dM_r = U_c$$

$$\boxed{4\pi R_c^3 P_c - 2K_c = U_c}$$

c subscript: pressure on surface of core

Virial Theorem. Integrating from center to surface, where $P_c = 0$, we would recover original form.

$$U_c \sim -\frac{3}{5}\frac{GM_c^2}{R_c} \quad ; \quad K_c = \frac{3}{2}\frac{M_c k T_c}{\mu_c m_H}$$

$$P_c = \frac{3}{4\pi R_c^3}\left(\underbrace{\frac{M_c k T_c}{\mu_c m_H}}_{\text{Thermal energy}} - \underbrace{\frac{1}{5}\frac{GM_c^2}{R_c}}_{\text{gravity}}\right)$$

Max pressure: $\frac{dP}{dM_c} = 0 \quad \Big\} \quad R_{ic} = \frac{2}{5}\frac{GM_c \mu_c m_H}{kT_c}$

$$P_c = \frac{375}{64\pi}\frac{1}{G^3 M_c^2}\left(\frac{kT_c}{\mu_c m_H}\right)^4$$

Pressure decreases with mass!

At some point, as one adds mass, the core cannot support itself.

$$P_{env} = \int_0^{P_{env}} dP$$

$$= -\int_{M}^{M_c} \frac{GM_r}{4\pi r^4}\, dM_r \approx \frac{G}{8\pi \bar{r}^4}\left(M_c^2 - M^2\right)$$

Assume $M_c^2 \ll M^2$ $\langle r^4 \rangle \approx R^4/2$

$$P_{env} \approx \frac{GM^2}{4\pi R^4}$$

R^4?

$$\rightarrow T_c = \frac{P_{env}\,\mu_{env}\, m_H}{\rho_{env}\, k} \qquad \rho_{env} \sim \frac{M}{4\pi R^3/3}$$

$$R \sim \frac{1}{3}\,\frac{GM}{T_c}\,\frac{\mu_{env} m_H}{k} \qquad \therefore\ P_{env} \sim \frac{81}{4\pi}\,\frac{1}{G^3 M^2}\left(\frac{kT_c}{\mu_{env} m_H}\right)^4$$

So $$\frac{M_c}{M} \sim 0.54\left(\frac{\mu_{env}}{\mu_c}\right)^2$$

For $X = 0.68$ $Y = 0.3$ and $Z = 0.02$,

$\mu_{env} \simeq 0.63$

For Helium core, $\mu_c = 1.34$

So,
$$\left(\frac{M_c}{M}\right) \simeq 0.08$$

He core will collapse if its mass exceeds 8% of the star's total mass.

Point 2-3: overall contraction (H depleted)

Point 3-4: shell burning in thick shell. More energy than in the main sequence phase. Not all energy reaches the surface. Most is used to expand the envelope. Redward trend!

Point 4: Shell burning increases mass of He core. At #4 S-C limit reached. Core begins to contract rapidly. Grav energy released, star expands, cools. Subgiant branch

Subgiant Branch is shell burning.

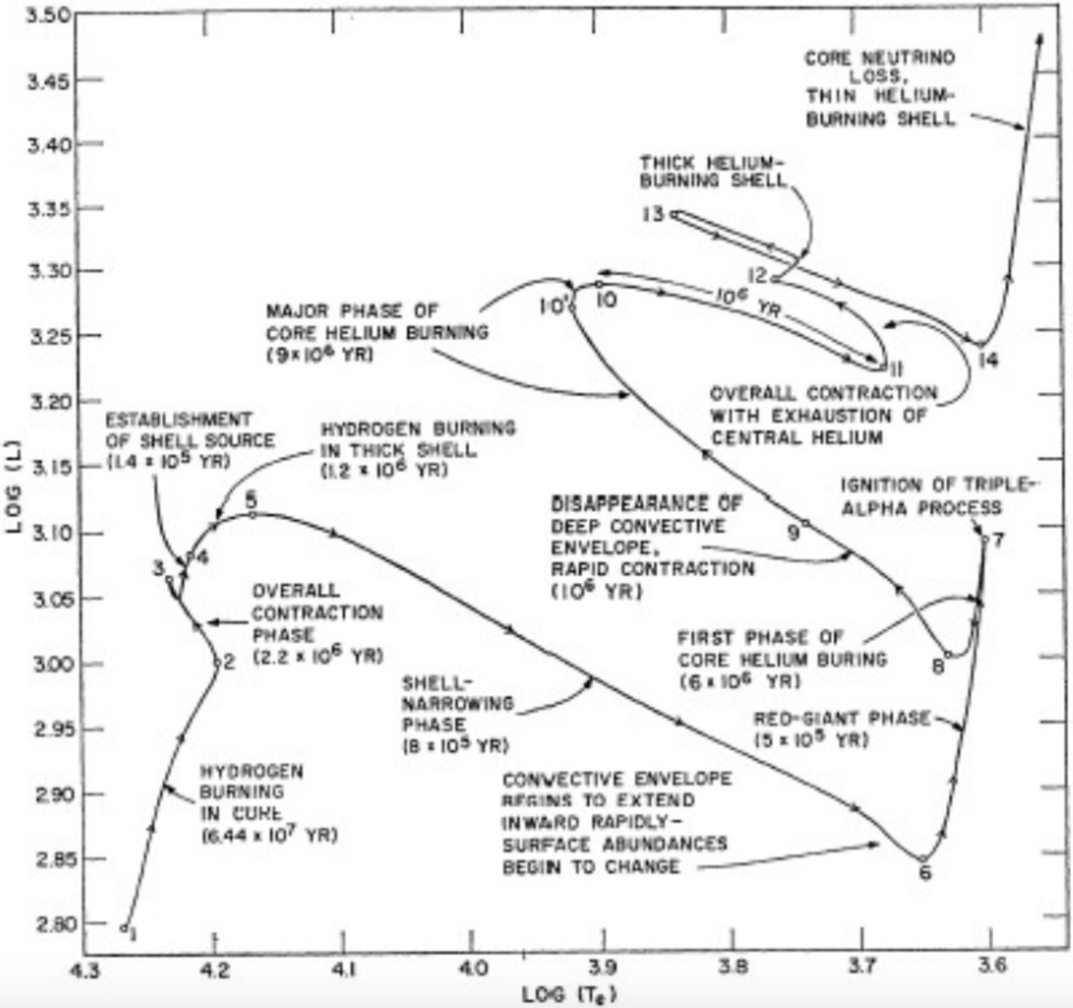
4-5: Contraction of core increases luminosity (KH). Star expands and reddens.
Shell narrows, but increases in density and temperature.

T falls, opacity increases (H^- ion). Convection zone deepens.

Convection reaches very deep and starts carrying photons more efficiently.

6: At the tip of the RGB, temperature and density have become high enough that quantum-tunelling allows Triple alpha to happen.

Helium flash



LOG (L)
LOG (T_e)
3.50
3.45
3.40
3.35
3.30
3.25
3.20
3.15
3.10
3.05
3.00
2.95
2.90
2.85
2.80
4.3
4.2
4.1
4.0
3.9
3.8
3.7
3.6
CORE NEUTRINO LOSS, THIN HELIUM-BURNING SHELL
THICK HELIUM-BURNING SHELL
10^6 YR
MAJOR PHASE OF CORE HELIUM BURNING (9×10^6 YR)
OVERALL CONTRACTION WITH EXHAUSTION OF CENTRAL HELIUM
ESTABLISHMENT OF SHELL SOURCE (1.4×10^5 YR)
HYDROGEN BURNING IN THICK SHELL (1.2×10^6 YR)
IGNITION OF TRIPLE-ALPHA PROCESS
DISAPPEARANCE OF DEEP CONVECTIVE ENVELOPE, RAPID CONTRACTION (10^6 YR)
OVERALL CONTRACTION PHASE (2.2×10^6 YR)
FIRST PHASE OF CORE HELIUM BURING (6×10^6 YR)
SHELL-NARROWING PHASE (8×10^5 YR)
RED-GIANT PHASE (5×10^5 YR)
HYDROGEN BURNING IN CORE (6.44×10^7 YR)
CONVECTIVE ENVELOPE BEGINS TO EXTEND INWARD RAPIDLY- SURFACE ABUNDANCES BEGIN TO CHANGE
1
2
3
4
5
6
7
8
9
10
10'
11
12
13
14

Horizontal Branch

He ignition pushes H-shell burning outward. Star loses energy, contracts, lower R, high T. Self-regulated, L roughly constant

The helium-main sequence.

AGB → He-shell burning analog of the RGB.

Shell burning: zero luminosity core. Isothermal.

Thermal pulse:

H-burning ignites. Dumps He onto helium layer below. Mass of He layer increases, becoming slightly degenerate. When T at base of He increases sufficiently, He flash occurs.

Drives burning H shell outwards. Cools and turns off for a while.

Eventually, He shell burning diminishes, H-shell recovers, and process repeats.

Thermal pulses: series of helium flashes

He shell → pushes H-burning out → L decreases.