Planetary Sciences

Class 20

Hadley circulation

At low inclinations, a planet's equator receives more heat; hot air rises and flows towards regions of low pressure (poles).

This circulation forms a cell, known as Hadley cell. For a slow rotator, or no rotation, there is only one Hadley cell. For Earth, Coriolis forces deflect the flow and breaks it into 3 cells.

Interesting detail: rising hot air expands adiabatically, cools and gives out its humidity. This is why we have rain forests in the equator, and in the Pacific Northwest. The air that is coming back at $\simeq 30^\circ$ latitude is dry, and forms the deserts. Show in map that the deserts are in a line: California, American Southwest, Sahara, Arabia, Iran, Gobi. In the Southern Hemisphere: Atacama, Namibia, Australia.

Storms

Conservation of vorticity $\omega = \nabla \times u$

Take the momentum equation and curl it:

$$\frac{D\omega}{Dt} = -\omega(\nabla \cdot u) + (\omega \cdot \nabla) u + \frac{1}{\rho^2} \nabla \rho \times \nabla p + \nu \nabla^2 \omega$$

For rotational flow

$$u_x = -\Omega y$$
$$u_y = \Omega x$$

$\nabla \cdot u = 0$ by construction, and the first term (compression) cancels.

The second term also cancels because $\omega = \nabla \times u = 2\Omega \hat{z}$ points in the z-direction, so $\omega \cdot \nabla = \omega \, \partial_z$, yet the flow does not depend on z. So, we are left with

$$\frac{D\omega}{Dt} = \frac{1}{\rho^2} \nabla\rho \times \nabla p + \nu \nabla^2 \omega$$

If the flow is polytropic, $p = K\rho^\gamma$, so $\nabla\rho \times \nabla p = 0$.

If the Reynolds number is high we can ignore the viscous term, and then

$$\boxed{\frac{D\omega}{Dt} = 0}$$

i.e. vorticity is conserved. Because of this property, storms in the atmosphere of the giant planets (such as the Great Red Spot) are stable over long periods of time.

Subsurface Oceans in Icy moons

Consider the heat conduction equation

$$q = -k \nabla T$$

where K is the thermal conductivity and q is the heat flux.

$$\frac{\Delta T}{\Delta x} = -\frac{q}{K} \quad \therefore \quad \Delta x = -\Delta T \frac{K}{q}$$

for $\Delta T = T_s - T_{melt}$; $\Delta x = d =$ depth of ocean

$$\boxed{d = (273K - T_s) \cdot \frac{K}{q_s}}$$

T_s: surface temperature

q_s: surface heat flux.

For Europa, $T \approx 100K$; $K \approx 3 \frac{W}{mK}$

$F \sim 10-100 \frac{mW}{m^2}$; $\rightarrow D \rightarrow 5-50$ km

$= 5 \frac{mW}{m^2}$

Radiogenic heating

The Earth is measured to produce

$$\boxed{H_r = (4.5 \pm 0.5) \cdot 10^{-12} \ W/kg}$$

from radioactive decay in its bulk. To convert this to an energy flux we do simple dimension analysis:

$$[q_H] = \frac{W}{m^2} = \frac{W}{kg} \cdot \frac{kg}{m^2} = \left(\frac{W}{kg}\right)(kg) \cdot \left(\frac{1}{m^2}\right) = \frac{[H_r][M]}{[A]}$$

So $$q_H = \frac{H_r \cdot \rho \, 4/3 \pi R^3}{4 \pi R^2}$$

$$\boxed{q_H = \frac{1}{3} \left(H_r \rho R \right)}$$

For Europa, this is 5 mW/m².

Notice that even without tidal heating, this radiogenic heating would still produce an ocean. The difference is that in this case, the ocean would lie at greater depths and we would not see tectonics in Europa. In fact, all icy bodies should have oceans.