Planetary Sciences #3

Selected Topics in Astrophysics #14

Planetesimal hypothesys and Core Accretion

$$\frac{M_c}{M_{ceres}} \propto \left(\frac{R_c}{R_{ceres}}\right)^3 \sim 10^{-7}$$

So $R_c \sim (10^{-7})^{1/3} R_{CERES}$ should be the typical radius of a dust ball produced by gravitational instability. Ceres radius being roughly 500 Km, we have

$$R_c \sim 5 \times 10^{-3} \cdot R_c \sim 2.5 \text{ Km}$$

so, planetesimals should be ~ 5 Km in size.

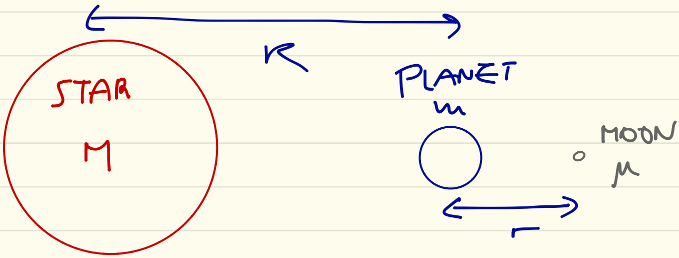
If all the dust mass got converted to planetesimals, their total number should be:

$$Z \cdot \frac{M_{disk}}{M_\odot} \cdot M_\odot \sim 10^{-4} M_\odot \sim 10^{29} g$$

$$\frac{10^{29}}{10^{17}} \sim 10^{12} \sim \text{a trillion planetesimals!}$$

Hill radius

What is the radius of a sphere over which the planet's gravitational field dominates over the star's?



Balance centrifugal force and gravity

Planet: $\cancel{m}\Omega^2 R = \dfrac{G\cancel{m}M}{R^2}$ $\qquad \Omega^2 = \dfrac{GM}{R^3}$

Moon:

$$\mu\Omega^2(R+r) = \frac{G\mu M}{(R+r)^2} + \frac{G\mu m}{r^2}$$

$$\Omega^2 = \frac{GM}{R^3}$$

$$\cancel{\mu}\left(\frac{\cancel{G}M}{R^3}\right)(R+r) = \frac{\cancel{G}\cancel{\mu}M}{(R+r)^2} + \frac{\cancel{G}\cancel{\mu}m}{r^2}$$

$$\frac{M}{R^3}(R+r) = \frac{M}{(R+r)^2} + \frac{m}{r^2}$$

$$\frac{M}{R^3}(R+r) = \frac{M}{(R+r)^2} + \frac{m}{r^2}$$

$$M\frac{(R+r)^3}{R^3} = M + \frac{m}{r^2}(R+r)^2 \qquad \times (R+r)^2$$

$$M(R+r)^3 r^2 = MR^3 r^2 + m(R+r)^2 R^3 \qquad \times r^2 R^3$$

$$m(R+r)^2 R^3 = Mr^2\left((R+r)^3 - \cancel{R^3}\right)$$

$$\cancel{R^3} + 3R^2 r + 3Rr^2 + r^3$$

$$m(R+r)^2 R^3 = Mr^3\left(3R^2 + \cancel{3Rr} + \cancel{r^2}\right)$$

$$mR^{\cancel{5}\,3} = Mr^3 3\cancel{R^2} \qquad r \to r_{HILL} \quad R \to a \text{ (semi-major axis)}$$

$$r^3 = R^3 \frac{m}{3M} \longrightarrow \boxed{r_{HILL} = a\left(\frac{m}{3M}\right)^{1/3}}$$

Let Σ_p be the surface density of planetesimals.
mass in annulus

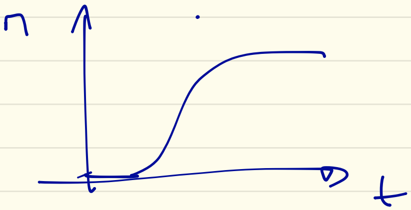
$$M \sim \pi R^2 \cdot \Sigma_p \cdot f_g \qquad 1 < f_g < \left(\frac{R_{HILL}}{R}\right)^{1/2}$$

$$\frac{dM}{dt} = \pi R^2 \cdot \Sigma_g \cdot f_g\, \Omega \qquad \to \Omega \text{ orbital period}$$

Growth until an "isolation mass" is reached

As M increases, Σ decreases.

$\dot{M} \propto M^{2/3}$



Estimate this "isolation mass"

Feeding zone: $f_G^{1/2} a_H = C a_H$ in radius

Mass within feeding zone $\quad \Delta R \sim 2 R_H$

$$M_{zone} = 2\pi R \, \Delta R \, \Sigma_p = 4\pi a C R_H \Sigma_p$$

$R_H \propto M_p^{1/3}$;

So $M_{ZONE} \propto M_p^{1/3}$. When all this mass has been accreted, the isolation mass is achieved. So, set

$$M_{ZONE} = M_{PLANET} = M_{iso} \quad (\text{when } \Sigma = 0 \text{ and } \dot{M} = 0)$$

$$M_{iso} = 4\pi a C R_H \Sigma_p = 4\pi a^2 \left(\frac{M_{iso}}{3M_\odot}\right)^{1/3} \Sigma_p \, C$$

$$M_{iso}^{2/3} = \frac{4\pi a^2 \Sigma_p C}{(3M_\odot)^{1/3}}$$

$$M_{iso}^{2/3} = \frac{4\pi a^2 \Sigma_p C}{(3M_\star)^{1/3}}$$

$$M_{iso} = \frac{(4\pi C\Sigma_p)^{3/2}}{(3M_\star)^{1/2}} \cdot a^3 = \frac{8\pi^{3/2}}{\sqrt{3}} M_\star^{-1/2} \Sigma_p^{3/2} a^3 f_g^{1/3}$$

$$\boxed{M_{iso} = \frac{8\pi^{3/2}}{\sqrt{3}} M_\star^{-1/2} f_g^{1/3} \Sigma_p^{3/2} a^3}$$

ISOLATION MASS

Protoplanets grow to this size.

We now need to estimate f_g, the gravitational focusing. Consider two masses, at relative speed σ at infinity.

$$\frac{1}{4} m \sigma^2 = m v_{MAX}^2 - \frac{Gm^2}{R_c}$$

R_c closest

v_m approach

Angular mom conservation

$$v_{max} = \frac{b\,\sigma}{2R_c}$$

$$l = r \times v = R_c\, v_{max} \qquad \rightarrow \quad v_{max} = \frac{b\,\sigma}{2R}$$
$$= b\,\sigma/2$$

$R_s \rightarrow$ sum of radii

$R_c < R_s \Rightarrow$ Collision

$R_c = R_s = R_1 + R_2$ (for collision)

$$\frac{1}{4} m \sigma^2 = m\left(\frac{1}{4}\frac{b^2\sigma^2}{R_c^2}\right) - \frac{4G m^2}{R_c\,\sigma^2}$$

$$b^2 = R_s^2 + \frac{4GmR_s}{\sigma^2}$$

$$v_{esc}^2 = \frac{4Gm}{R_s} \quad \therefore \quad b^2 = R_s^2\left(1 + \frac{v_{esc}^2}{\sigma^2}\right)$$

use $\sigma \sim v_{rms}$

$$b^2 = R_s^2\left(1 + \frac{v_{esc}}{v_{rms}}\right)$$

Gravitational focusing.

$$f_g = \left(1 + \frac{v_{esc}}{v_{rms}}\right)^{1/2} \Leftarrow b = R_s \cdot f_g$$

For terrestrial planet region:

$a = 1\,AU$; $\Sigma_p = 10\ g/cm^2$ $M_* = M_\odot$

$C = 2\sqrt{3}$

$M_{iso} \simeq 0.1\ M_\oplus$

For Jupiter region:

$M_{iso} \simeq 10 M_\oplus$ if ices are considered.

Gas accretion

$$M_t = M_{core} + M_{env}$$

$$L = \frac{G M_c}{R_c} \dot{M}_c$$

$$\frac{dP}{dr} = -\frac{G M(r)}{r^2}\rho$$

$$\frac{L}{4\pi r^2} = -\frac{16}{3}\frac{\sigma T^3}{\kappa\rho}\cdot\frac{dT}{dr}$$

$$\frac{dT}{dP} = \frac{3\kappa_r L}{64\pi\sigma G M T^3}$$

$$\int_{T_{disk}}^{T} T^3\, dT = \frac{3\kappa_R L}{64\pi\sigma G M_t}\int_{P_{disk}}^{P} dP$$

$T^4 \gg T_{disk}$; $P \gg P_{disk}$

$$T^4 \simeq \frac{3}{16\pi}\frac{\kappa_R L}{\sigma G M_t} P$$

$$P = \frac{k_B}{\mu m_p} \rho T \quad \therefore T \simeq \left(\frac{\mu m_p}{k_B}\right) \frac{GM_t}{4r}$$

$$R_{out} = R_{TH} < R_{HILL} \qquad \rho \simeq \frac{64\pi\sigma}{3\kappa_R L} \left(\frac{\mu m_p GM_t}{4k_B}\right)^4 \frac{1}{r^3}$$

$$r_{TH} \sim \frac{GM_t}{c^2}$$

$$M_{env} = \int_{R_{core}}^{R_{out}} 4\pi r^2 \rho(r)\, dr$$

$$= \frac{256\pi^2\sigma}{3\kappa_R L} \left(\frac{\mu m_p GM_t}{4k_B}\right)^4 \ln\left(\frac{R_{out}}{R_{core}}\right)$$

Strong dependence on M_t, weak dependence on M_{core} (via $L = \frac{GM_c \dot{M}_c}{R_c} \propto M_c^{2/3} \dot{M}_c$

Note $M_c = M_t - M_{env}$

$$M_{core} = M_t - \left(\frac{C}{\kappa_R \dot{M}_c}\right) \cdot \frac{M_t^4}{M_c^{2/3}}$$

For given $\dot{M}$ there exists $M_{core\ crit}$ beyond no solution is possible. Show solution.

Interpretation not clear. Above M_{crit}, no solution is possible because there is no hydro eq. in the envelope.

Envelope will contract and further gas will [illegible] fast as energy (grav) can be radiated.

$$\frac{M_{crit}}{M_\oplus} = 12\left(\frac{\dot{M}_{core}}{10^{-6}\,M_\oplus/yr}\right)^{1/4}\left(\frac{K_R}{1\,cm^2 g^{-1}}\right)^{1/4}$$

about $10\,M_\oplus$ for critical core mass

Show Pollack 96 model of M_{core} and M_{env} vs time