Planetary Sciences Class #2

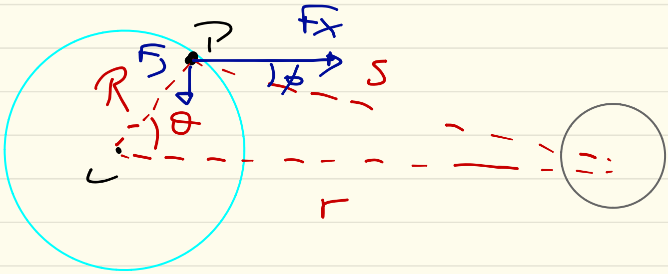
STA → #13

Tides

$$F_m = \frac{GMm}{r^2}$$

$$dF = F_1 - F_2 = \frac{dF}{dr} \cdot dr = -\frac{2GMm}{r^3}\, dr$$

Center: $F_c = \frac{GMm}{r^2}\hat{x} + 0\hat{y}$



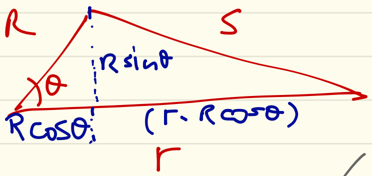
$$F_{px} = \frac{GMm}{s^2}\cos\phi$$

$$F_{py} = -\frac{GMm}{s^2}\sin\phi$$

Differential force between center and surface is

$$\Delta F = F_p - F_c$$

$$= GMm\left(\frac{\cos\phi}{s^2} - \frac{1}{r^2}\right)\hat{x} - \frac{GMm}{s^2}\sin\phi\,\hat{y}$$



$$s^2 = (r - R\cos\theta)^2 + R^2\sin^2\theta$$
$$= r^2 + R^2 - 2rR\cos\theta$$

$$s^2 = r^2\left[1 + \cancel{\frac{R^2}{r^2}} - 2\frac{R}{r}\cos\theta\right]$$

$$s^2 = r^2\left(1 - \frac{2R}{r}\cos\theta\right)$$

$$\Delta F = \frac{GMm}{r^2}\left(\cos\phi\left(1 + \frac{2R}{r}\cos\theta\right) - 1\right)\hat{x}$$
$$- \frac{GMm}{r^2}\left[1 + \frac{2R}{r}\cos\theta\right]\sin\phi\,\hat{y}$$

For $\cos\phi \simeq 1$, $\sin\phi \approx \frac{R\sin\theta}{r}$

$$\Delta F = \frac{GMmR}{r^3}\left(2\cos\theta\,\hat{x} - \sin\theta\,\hat{y}\right)$$

Europa; evidence for non-synchronous rotation

Roche limit

Tidal force supplants internal forces holding body together

$$\underset{\text{gravity on surface}}{\frac{GM_m}{R_m^2}} < \underset{\text{tidal force from planet}}{\frac{2GM_pR_m}{r^3}}$$

$$r^3 < 2\frac{M_p}{M_m}R_m^3$$

$$r^3 < 2\frac{\bar{\rho}_p R_p^3}{\bar{\rho}_m \cancel{R_m^3}}\cancel{R_m^3}$$

$$r < 2^{1/3}\left(\frac{\bar{\rho}_p}{\bar{\rho}_m}\right)^{1/3} R_p$$

Useful because we can measure $\bar{\rho}_p$ and $\bar{R}_p$ (and estimate $\bar{\rho}_m$ better than we can M_m and R_m)

True value not $2^{1/3} = 1.3$ but ~ 2.456.

For saturn; $r \sim 1.23 \times 10^8$ m

Planetesimal formation

Self-gravity of dust to overcome the solar tide

$$\frac{G M_p}{R_p^2} > \frac{2 G M_* R_p}{r^3}$$

$$\frac{\frac{4\pi}{3}\rho_p R^{\cancel{3}}}{\cancel{R^2}} > \frac{2 M_* \cancel{R}}{r^3}$$

$$\rho_p > 2\,\frac{M_*}{r^3}\cdot\frac{3}{4\pi} \simeq 0.5\,\frac{M_*}{r^3}$$

$$\rho_p \simeq 0.5\left(\frac{M_*}{M_\odot}\right) M_\odot \left(\frac{AU}{r}\right)^3 \cdot \frac{1}{AU^3}$$

$$\rho_p \simeq 0.5\left(\frac{M_*}{M_\odot}\right)\left(\frac{r}{AU}\right)^{-3} \frac{M_\odot}{AU^3}$$

$$\boxed{\rho_p \simeq 10^{-7}\,\frac{g}{cm^3}\left(\frac{M_*}{M_\odot}\right)\left(\frac{r}{AU}\right)^{-3}}$$

$$M \approx \rho \cdot V = \rho_R \cdot \frac{4\pi}{3} R^3$$

$$= \rho_\bullet \frac{4\pi}{3} R_p^3$$

$$R = R_H = a \left(\frac{M_p}{M_\otimes}\right)^{1/3}$$

$$M_p = \frac{4\pi}{3} R_p^3 \rho_p$$

$$\rho_R\, a \left(\frac{M_p}{M_\otimes}\right)^{1/3} = \rho_\bullet R_p^3$$

$$\rho_R\, a \left(\frac{4\pi}{3} R_p^3 \rho_\bullet\right)^{1/3} \frac{1}{M_\otimes^{1/3}} = \rho_\bullet R_p^3$$

$$\rho_R\, a \left(\frac{4\pi}{3}\right)^{1/3} \cancel{R_p}\ \cancel{\rho_\bullet^{1/3}}\ \frac{1}{M_\otimes^{1/3}} = \rho_\bullet^{2/3} R_p^{\cancel{3}\,2}$$

$$R_p^2 = \rho_R \cdot \frac{a}{M_\otimes^{1/3}} \left(\frac{4\pi}{3}\right)^{1/3} \cdot \frac{1}{\rho_\bullet^{2/3}}$$

$$R_p = \frac{\rho_R^{1/2}}{\rho_\bullet^{1/3}} \left(\frac{a}{AU}\right)^{1/2} AU^{1/2} \left(\frac{M_\otimes}{M_\odot}\right)^{-2/3} M_\odot^{-2/3} \left(\frac{4\pi}{3}\right)^{2/3}$$

$$\boxed{\rho_p \approx 10^{-7}\ \frac{g}{cm^3} \left(\frac{M_\otimes}{M_\odot}\right) \left(\frac{r}{AU}\right)^{-3}}$$

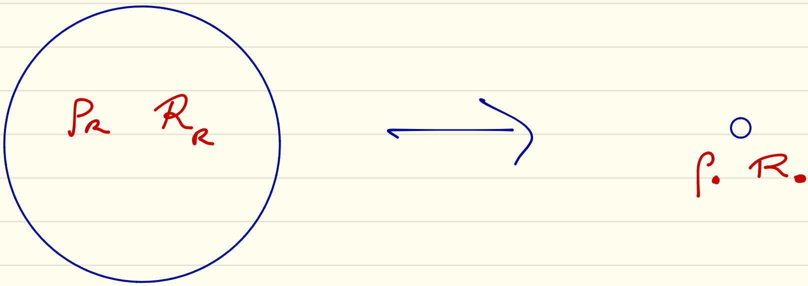
Suppose that the collapse happens vertically, so that ρ changes but Σ stays the same

$$M \sim \frac{\Sigma_d^3}{\rho_R^2} \sim \frac{(10)^3}{(10^{-7})^2}$$

$$M \sim 10^{17}\ g$$

Ceres Mass $\sim 10^{24}$ g

Very low density. Asteroids have densities of the order of 10^{10} g/cm^3.



The mass that collapses $\sim M_c \sim 10^{17}$ g, compared to Ceres Mass $\sim M_{ceres} \sim 10^{24}$ g

$$\frac{M_c}{M_{ceres}} \propto \left(\frac{R_c}{R_{ceres}}\right)^3 \sim 10^{-7}$$

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So $R_c \sim \left(10^{-7}\right)^{1/3} R_{CERES}$ should be the typical radius of a dust ball produced by gravitational instability. Ceres radius being roughly 500 Km, we have

$$R_c \sim 5\times10^{-3} \cdot R_c \sim 2.5 \text{ Km}$$

so, planetesimals should be ~ 5 Km in size.