Planetary sciences
Class #1

Selected Topics Class #12

star formation: Jeans criterion.

$$2K + U = 0$$

$$U \sim -\frac{3}{5}\frac{GM_c^2}{R_c} \qquad K = \frac{3}{2} NkT \qquad N = \frac{M_c}{\mu m_H}$$

$$\frac{3M_c kT}{\mu m_H} < \frac{3}{5}\frac{GM_c^2}{R}$$

$$R_c = \left(\frac{3M_c}{4\pi\rho_0}\right)^{1/3}$$

$$\frac{\cancel{3}\cancel{M_c} KT}{\mu m_H} < \frac{\cancel{3}}{5}\frac{GM_c^{\cancel{2}}}{R_c}$$

$$M_c^{2/3} > \frac{5KT}{G\mu m_H} \cdot R_c = \frac{5KT}{G\mu m_H} \cdot \left(\frac{3}{4\pi\rho_0}\right)^{1/3}$$

$$M_c^2 = \left(\frac{5KT}{G\mu m_H}\right)^3 \cdot \frac{3}{4\pi\rho_0}$$

$$\boxed{M_J \simeq \left(\frac{5KT}{G\mu m_H}\right)^{3/2} \left(\frac{3}{4\pi\rho_0}\right)^{1/2}}$$

Also in terms of radius

$$R_C > R_J$$

$$R_J \simeq \left(\frac{15 kT}{4\pi G \mu m_H \rho_0}\right)^{1/2}$$

Jean's length.

$$\boxed{M_J \simeq \left(\frac{5kT}{G\mu m_H}\right)^{3/2} \left(\frac{3}{4\pi \rho_0}\right)^{1/2}}$$

$$\left(\frac{5 c_s^2}{G}\right)^{3/2} \cdot \left(\frac{3}{4\pi \rho_0}\right)^{1/2}$$

$$M_J \simeq \left(\frac{5^{\cancel{3}\,2} c_s^6}{G^3} \cdot \frac{\cancel{3}}{\cancel{4\pi}\rho_0}\right)^{1/2} \sim \left(\frac{25\, c_s^6}{G^3\, \rho_0}\right)^{1/2}$$

$$M_J \simeq \frac{5\, c_s^3}{G^{3/2} \rho_0^{1/2}} \qquad P_0 = \rho_0 c_{s0}^2$$

$$\boxed{M_J \simeq \frac{5\, c_s^4}{G^{3/2} P_0^{1/2}}}$$

Example: Hydrogen cloud, $T = 50K$, $n = 5 \times 10^2\ cm^{-3}$

if HI; $\rho_0 = m_H n_H = 8.4 \times 10^{-19} \frac{kg}{m^3} = 8.4 \times 10^{-16} \frac{g}{cm^3}$

$M \simeq 1500\ M_\odot$. Masses are usually $1-100\ M_\odot$

Dense core: $T = 10K$, $n_{H_2} = 10^4\ cm^{-3}$. Molecular hydrogen, $\rho = 2 m_H n_{H_2} = 3 \cdot 10^{-14}\ g/cm^3$; $\mu = 2$

$M_J = 8\ M_\odot$

If including external pressure:

$$M_J \simeq \frac{5\, c_s^4}{G^{3/2}(P_0 + P_{ext})^{1/2}}$$

$$M_J \simeq \frac{5 c_s^4}{G^{3/2} P_0^{1/2} \left(1 + \frac{P_{ext}}{P_0}\right)^{1/2}}$$

$$\frac{P_{ext}}{P_0} \simeq \frac{3 \cdot 10^{-17} \cdot 1\cancel{0}}{8 \cdot 10^{-19} \cdot 5\cancel{0}} \simeq 0.5 \times 10^2 \simeq 50$$

$$M_{BE} \simeq \frac{c_s^4}{G^{3/2} P_0^{1/2}} \quad \text{(Bonnor-Ebert)}$$

Then $M_{BE} \sim 1-2$ solar mass.