Class 10

Evolution of high-mass stars

Evolution of high-mass stars: Supernovae

Core-collapse supernovae .

Typically 10^{53} erg $\lesssim$ 0.01% photons (1% K)
→ majority neutrinos

Carbon burning. Onion-layer
Short burning time because less energy comes from each fuel, we're closer to the peak in binding energy!

Alpha ladder

$^{12}C + \alpha \to {}^{16}O + \gamma$ 7.16 MeV
$^{16}O + \alpha \to {}^{20}Ne + \gamma$ 4.73 MeV
$^{20}Ne + \alpha \to {}^{24}Mg + \gamma$ 9.31 MeV
$^{24}Mg + \alpha \to {}^{28}Si + \gamma$ 9.98 MeV
$^{28}Si + \alpha \to {}^{32}S + \gamma$ 6.95 MeV

$^{32}S \to {}^{36}Ar \to {}^{40}Ca \to {}^{44}Ti \to {}^{48}Cr \to {}^{52}Fe \to {}^{56}Ni$

Odd-Z elements

At temperatures of $\sim 10^9$ K, the photon energy is roughly 1 MeV, and there are many photons in excess of that in the high energy tail.

At very high temperatures, the radiation field has photons of MeV, which is comparable to nucleon binding energy. The nuclei can then be photodisintegrated, which is the analog to photoionization in atoms. Consider

$$^{56}Fe + \gamma \rightarrow 13\,^{4}He + 4n$$

$$^{4}He + \gamma \rightarrow 2p^{+} + 2n$$

Photodisintegration effectively turned an iron nucleus into protons and neutrons. The process is fission, trying to undo what the star built through nucleosynthesys. Consider also silicon burning:

Above 3×10^{9} K,

$$^{28}Si + \gamma \rightarrow ^{24}Mg + \alpha$$

$$^{24}Mg + \alpha \rightarrow ^{27}Al + p$$

$$^{27}Al + \alpha \rightarrow ^{30}Si + p$$

The end product are 2 neutrons added to Si, but some odd-Z elements are created and consumed. In quasi-static equilibrium between photodisintegration and particle capture, there is a significant population of odd-Z elements at a given time.

Neutronization (Neutron drip)

(I) $n \rightarrow p + e + \bar{\nu}$ (13 minutes)

(II) $p + e \rightarrow n + \nu$ (needs energy since $m_n > m_p + m_e$)

For the second reaction to occur at high densities, the energy of the electron must exceed the excess rest energy of the neutron. We can write the Fermi energy as

$$E_F = \left(p_F c^2 + m_e^2 c^4 \right)^{1/2}.$$

And the energy needed is that minus the electron rest mass, $E = \left(E_F - m_e c^2 \right)$

if $E \geqslant \left[m_n - (m_p + m_e) \right] c^2$ neutron excess mass

Electrons can combine with protons to form neutrons. Substituting E:

$$\left(p_F^2 c^2 + m_e^2 c^4 \right)^{1/2} - m_e c^2 = \left[m_n - (m_p + m_e) \right] c^2$$

Substituting also $Q = m_n - m_p$, and solving for p_F:

$$p_F = m_e c \left[\left(\frac{Q}{m_e} \right)^2 - 1 \right]^{1/2}$$

Equate it with the definition of the Fermi momentum

$$p_F = \left(\frac{3h^3}{8\pi}\frac{\rho}{m_H}\right)^{1/3}$$

And solving for the density ρ:

$$\rho_c \approx 1.2 \times 10^7 \ \frac{g}{cm^3}$$

When this density is exceeded, neutronization occurs. Notice also that if the gas is fully degenerate, the $n \to p + e^- + \nu$ reaction does not happen, since there are no free states for the electron to occupy below the very high energy Fermi level.

At $T \sim 8 \times 10^9$ K and $\rho \sim 10^{10}$ g cm^{-3} ($\sim 15 M_\odot$), the electrons that gave degenerate pressure are captured by heavy nuclei and protons produced by photodisintegration. The support of degenerate pressure is gone, and the core begins to collapse in free fall!

The collapse will continue until nuclear densities are achieved and neutron degeneracy pressure kicks in, halting the collapse. In the end, a volume the size of Earth is compressed to 50 km.

We can calculate the associated release of gravitational energy:

Release of gravitational energy: $-\frac{\Delta W}{2}$ released

$$W = -\frac{3}{5}\frac{GM^2}{R} \qquad E = -\frac{\Delta W}{2} = -\frac{3}{10}\frac{GM^2}{R^2}\Delta R$$

To produce 10^{53} erg, going from an Earth radius ($R_\oplus$) to 50 km, one needs $\sim 2\,M_\odot$ of material.

The collapse continues to nuclear densities. At about 3x nuclear density, neutron degeneracy kicks in and the core rebounds. The bounce sends pressure waves that quickly become shocks. The shock meets the infalling outer iron core, and dissipates (the energy is used in raising the temperature and photodissociation). The shock stalls.

Neutrino emission mechanisms

Usually matter is extraordinarily transparent to neutrinos. So, if a process generates them, they are an energy loss mechanism.

However, a medium can become optically thick to neutrinos if conditions are extreme enough.

Consider the cross-section of neutrinos:

$$\sigma_\nu \sim 10^{-44}\, \varepsilon_\nu^2 \ \text{cm}^2$$

$\varepsilon_\nu \equiv$ Neutrino energy in MeV

Mean free path: $\lambda = \frac{1}{n\sigma_\nu}$

n: number density of targets. We can write
$\rho = n \cdot m = n \mu m_H$

$$\lambda = \frac{\mu m_H}{\rho \sigma_\nu} \sim \frac{10^{20}}{\rho\, \varepsilon_\nu^2} \ \text{cm}$$

For density of lead ($\sim 10 \ \text{g cm}^{-3}$) and a typical solar neutrino (~ 1 MeV),

$$\lambda \sim \frac{10^{20}}{1 \cdot 10} \sim 10^{19} \ \text{cm}$$

Given $1\,\text{pc} \sim 3 \cdot 10^{18}$ cm, solar neutrinos can traverse light-years of lead without being intercepted.

To get short λ, we need either very high densities, or very high neutrino energies, or both.

Densities in SN approach and then exceed those of nuclear matter. (until degenerate nuclear pressure kicks in)

$$\rho \sim 10^{14}\ g\ cm^{-3}$$

If we take Fermi energies to be representative,

$$p_F = \left(\frac{3h^3 \rho}{8\pi \mu_e m_H}\right) \quad ; \quad E_F = \frac{p_F^2}{2m}$$

For $\rho \sim 10^{14}\ g\ cm^{-3}$, Fermi energies are of $\sim 20\ MeV$

Then, $\boxed{\lambda_\nu \approx 25\ m}$

Neutrinos are thus effectively trapped.

At high energies, $p + e^- \rightarrow n + \nu$

Also possible $e^- + e^+ \rightarrow \nu + \bar{\nu}$

Need a high number of positrons. Possible if $kT \sim 2 m_e c^2$, so photons can $2\gamma \rightarrow e^- + e^+$

$$m_e \sim 0.511\ \frac{MeV}{c^2} \quad ; \quad k = 8.617 \cdot 10^{-5}\ \frac{eV}{K}$$

$T \sim 10^{10}$ K

High-energy tail of Planckian function at $T \sim 3 \cdot 10^{9}$ K can yield an appreciable number of these photons.

Iron core: $T_c \sim 3.7 \times 10^{9}$ K
$\rho_c \sim 4.9 \times 10^{7}$ g/cm^3

$\varepsilon_\nu \sim 10^{13}$ erg g^{-1} s^{-1}

$$\frac{dL}{dr} \propto -\varepsilon_\nu$$

If whole core released it, $L_\nu \simeq 5 \times 10^{12} L_\odot$

Amount of energy escaping in the form of neutrinos

photons $\sim 4 \times 10^{37}$ erg/s
neutrino $\sim 3 \times 10^{45}$ erg/s

Most of the energy is in the form of neutrinos

Under the shock:

Neutrino sphere develops from photodissociation and electron capture.

Opaque to neutrinos

5% of neutrino energy is deposited to shock, so it begins again.

$K \sim 10^{51}$ ergs ; $L_\nu \sim 10^{53}$ erg

$L_\gamma \sim 10^{49}$ ergs

$L \sim 10^{36}$ W $\rightarrow 10^{43}$ erg/s $\rightarrow 10^{10} L_\odot$ (outshine a galaxy).