Physics of Accretion Disks

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Keplerian disks can't develop hydrodynamical turbulence

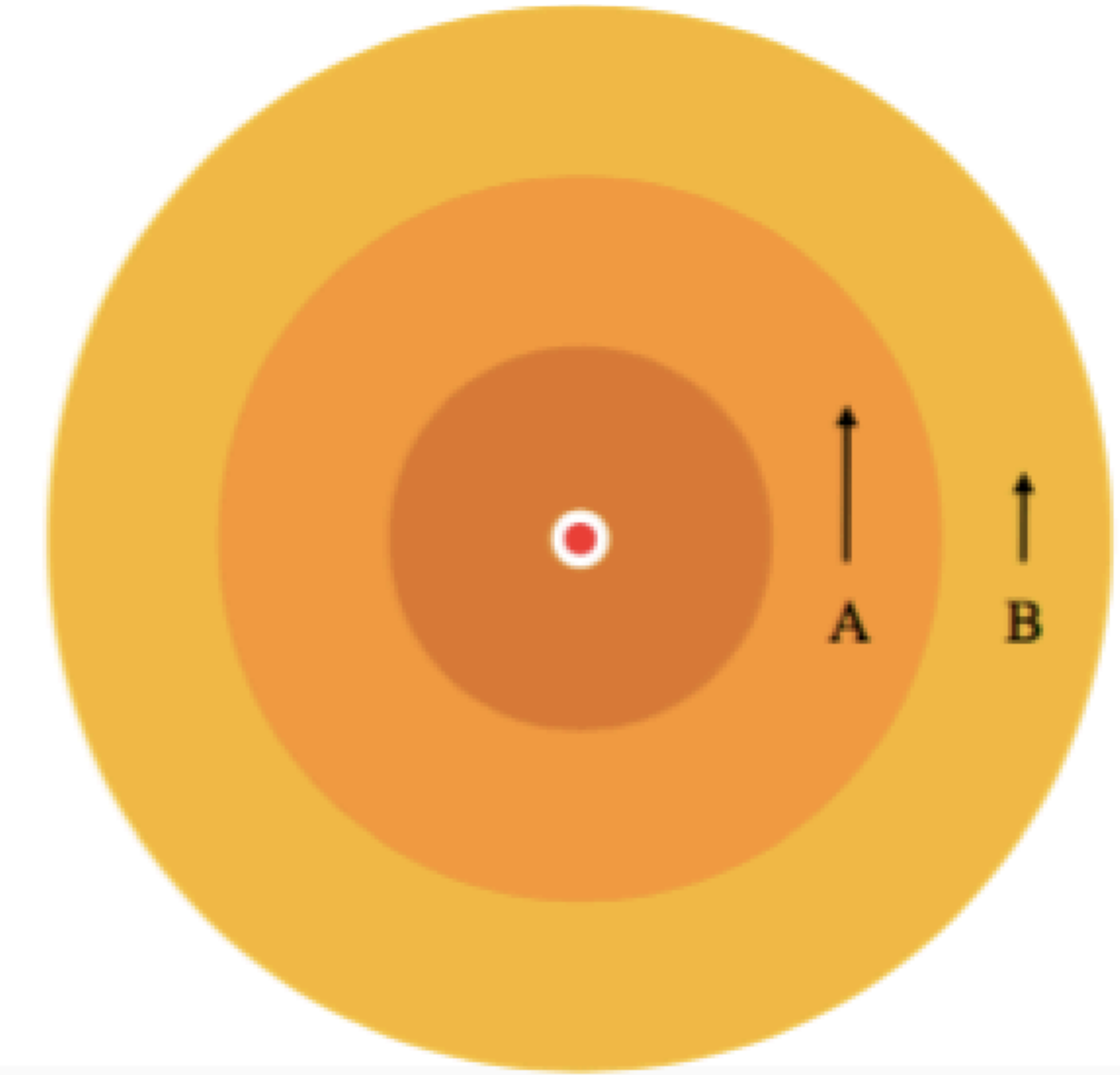
Most astrophysical discs are close to Keplerian

- nearly circular
- angular velocity profile $\Omega \propto r^{-3/2}$
    - angular momentum $\left(r^2 \Omega\right)$ increases outwards

Stable to axis-symmetric disturbances

(Rayleigh criterion)

Rayleigh instability



Normally, angular velocity decreases outwards but angular momentum increases outwards.

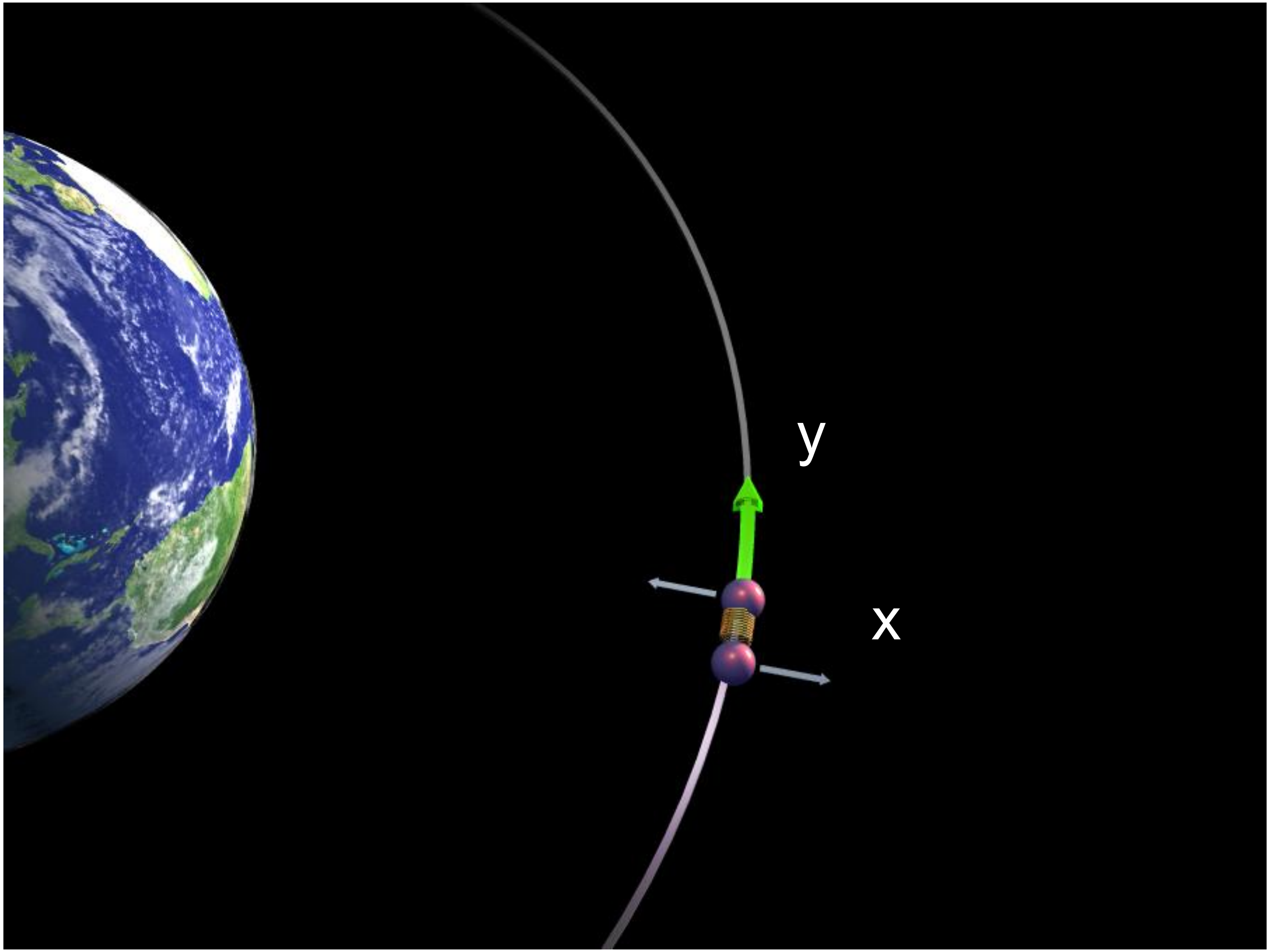
Ring A losing angular momentum needs to jump to an orbit of lower angular momentum, which is inwards.

But IF *both* angular velocity and momentum decrease outwards, then when ring A loses angular, the orbit of lower angular momentum it must jump to is past B.

Likewise, B gaining angular momentum must jump to a higher angular momentum orbit, which is past A.

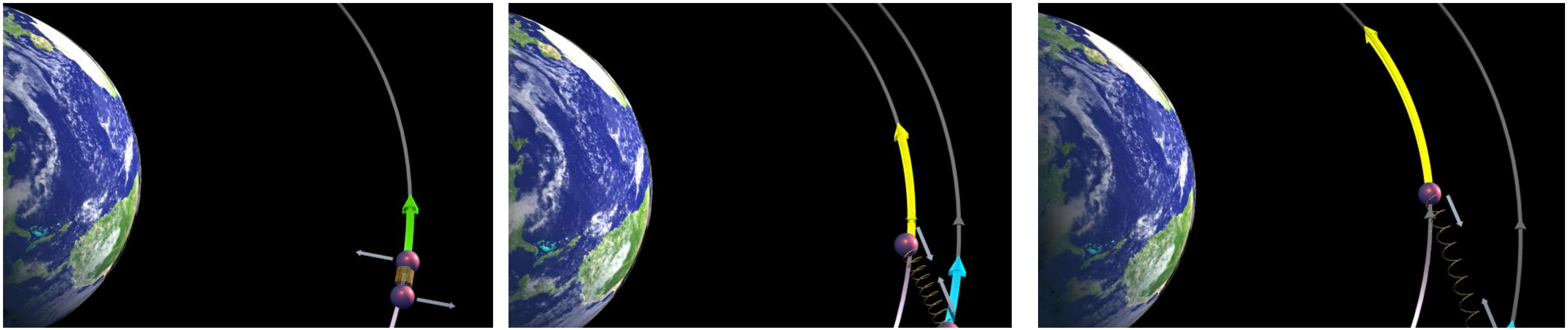
A and B must swap. The situation is UNSTABLE.

Gas parcels joined by a spring



The restoring force destabilizes the flow

Otherwise Rayleigh-stable, Keplerian motion is destabilized
if a restoring force connects two gas parcels.



The restoring force resists the shear, trying to enforce rigid rotation.

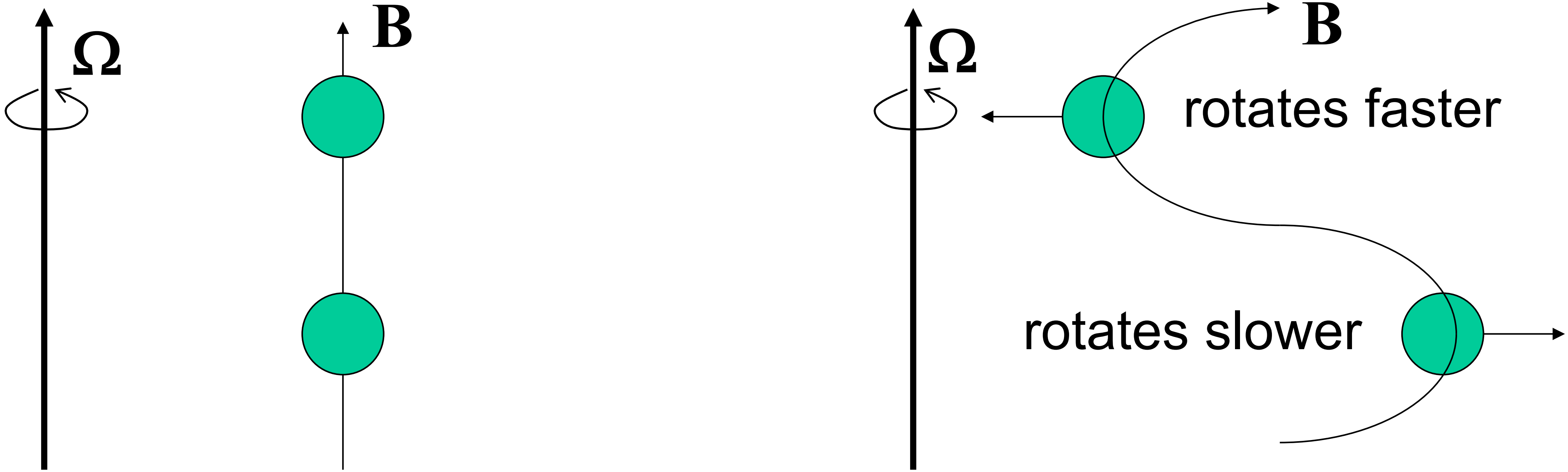
The parcel inwards is tugged back, which enforces it to rotate at the angular velocity it had at the equilibrium position.

The angular momentum it loses is given to the outer gas parcel. Losing angular momentum, it jumps to an inner orbit.

The outer parcel gains angular momentum and jumps to an even more outward orbit.

The situation is unstable.

Magnetorotational Instability (MRI)

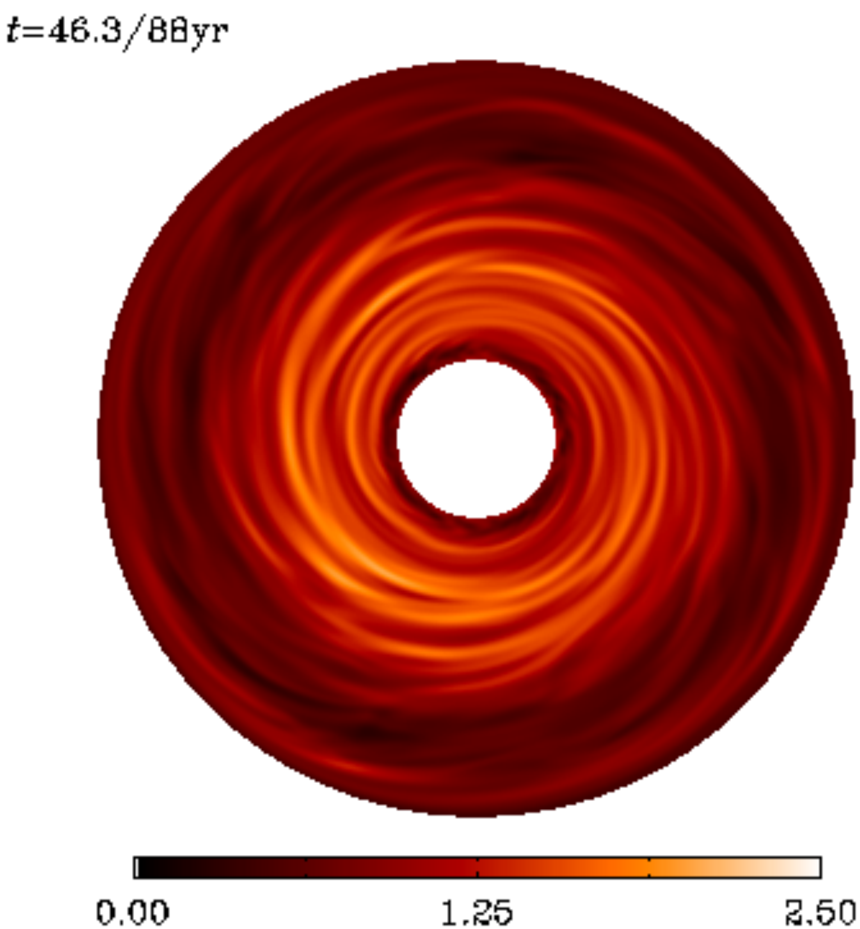


Magnetic fields in a conducting, rotating plasma behave EXACTLY like springs!

Turbulence and Accretion in 3D Global MHD Simulations of Stratified Protoplanetary Disk

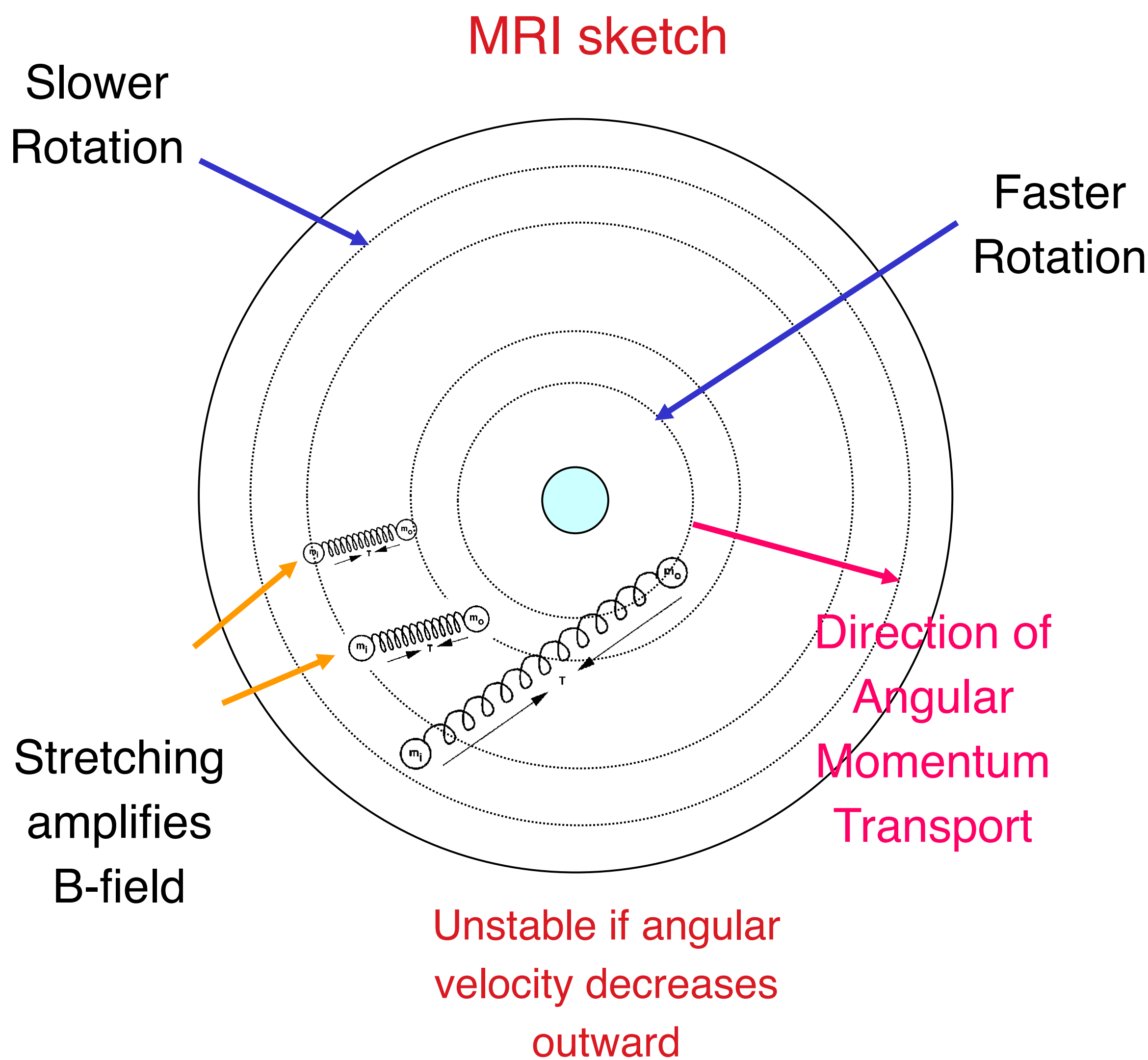
Magneto-Rotational Instability

Turbulence in disks is enabled by the Magneto-Rotational Instability (Balbus & Hawley, 1991)

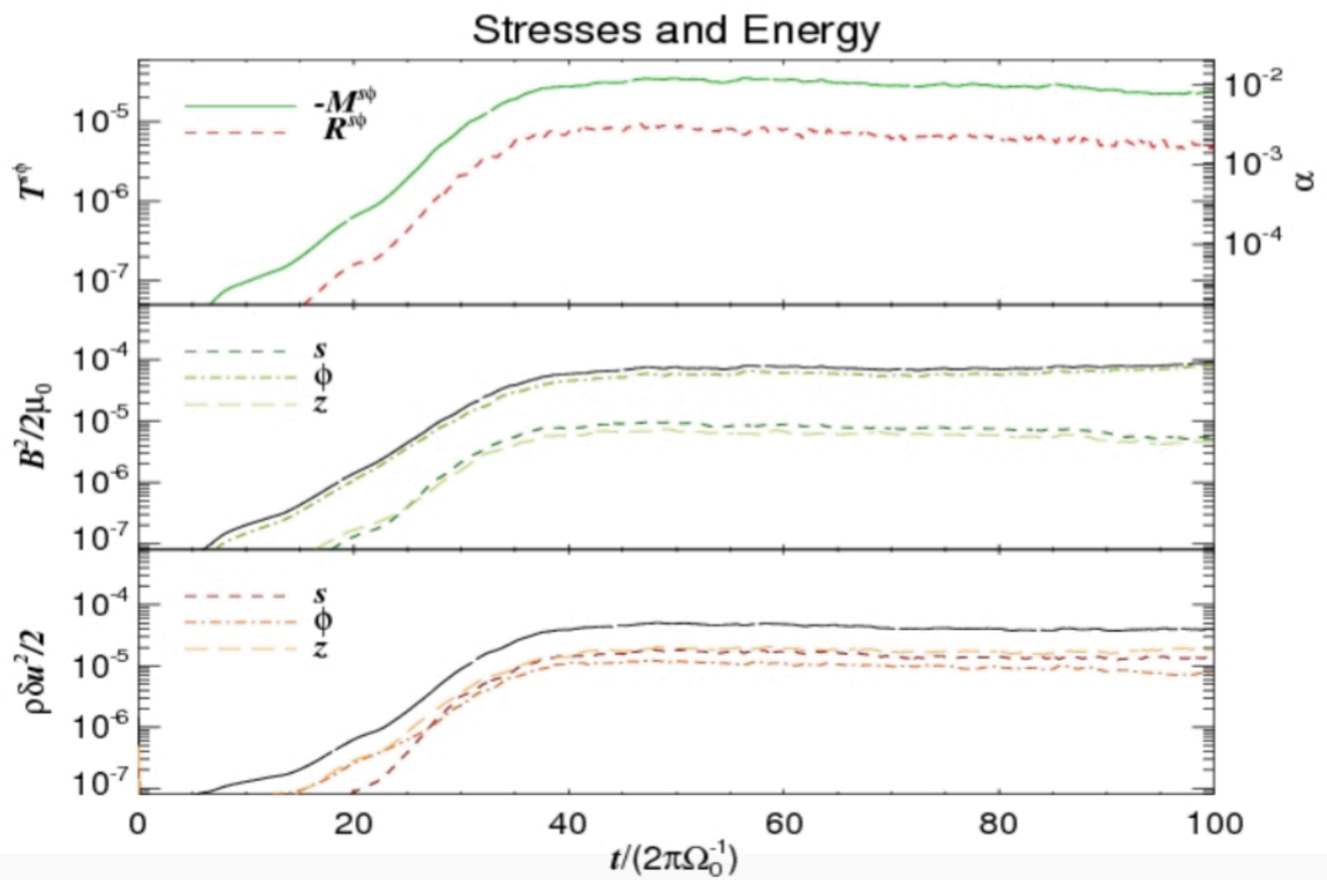


Magnetized disk

Lyra et al. (2008a)



Magneto-Rotational Instability



The MRI

Leads to alpha values of the order of 10^{-2}, as needed to explain mass accretion rates.

Generates large scale azimuthal (toroidal) magnetic fields in the disk.

Saturated State of MRI

Energy budget

FIG. 6.—Sketch of the energy budget. Energy is tapped from the Keplerian motion and goes into magnetic and kinetic energy, and is finally converted into heat. The numbers give the approximate energy fluxes in units of $\langle \frac{1}{2}B^2\Omega \rangle$.

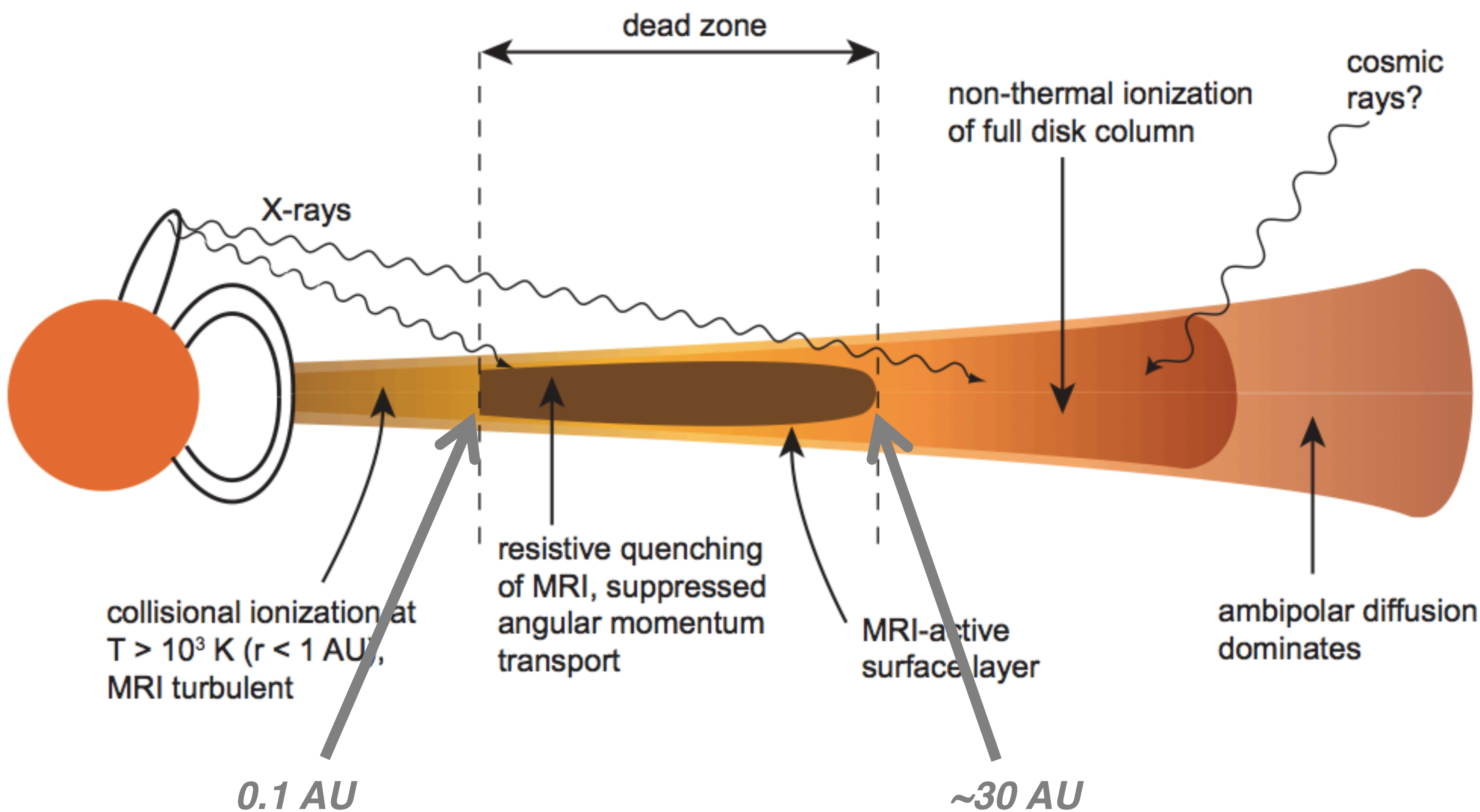
$$\frac{d}{dt}\left(\frac{1}{2\mu_0}B^2\right) = -\frac{3}{2}\Omega_0 \frac{1}{\mu_0}B_x B_y - \boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B}) - \eta\mu_0 J^2$$

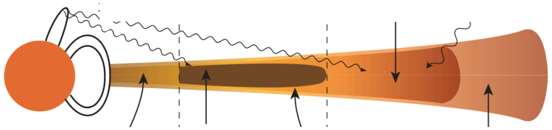
$$\frac{d}{dt}\left(\frac{1}{2}\rho u^2\right) = \frac{3}{2}\Omega_0 \rho u_x u_y + \rho\boldsymbol{u}\cdot\boldsymbol{g} - \boldsymbol{u}\cdot\nabla p + \boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B}) - 2\nu\rho S^2$$

$$\frac{d}{dt}\rho e = -p\nabla\cdot\boldsymbol{u} + 2\nu\rho S^2 + \eta\mu_0 J^2 + \rho Q$$

$$\frac{d}{dt}E_{tot} = \frac{3}{2}\Omega_0\left(\rho u_x u_y - \frac{1}{\mu_0}B_x B_y\right) + \rho Q$$

Dead zones

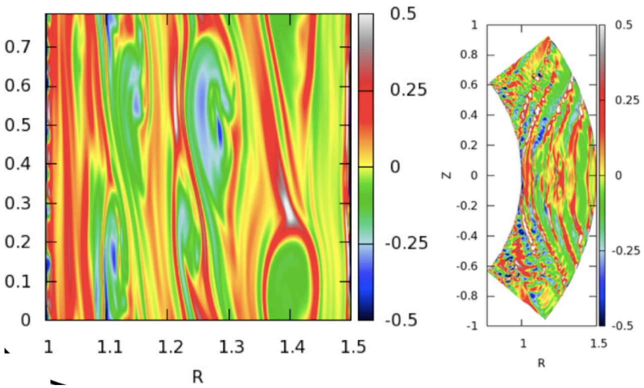

dead zone
non-thermal ionization
of full disk column
cosmic
rays?
X-rays
collisional ionization at
T > 10³ K (r < 1 AU),
MRI turbulent
resistive quenching
of MRI, suppressed
angular momentum
transport
MRI-active
surface layer
ambipolar diffusion
dominates
0.1 AU
~30 AU



Next class

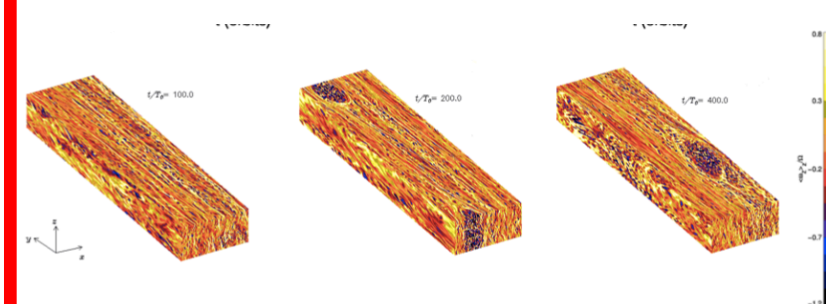
The MRI is dead
Long live the *Thermal Instabilities*

Vertical Shear Instability



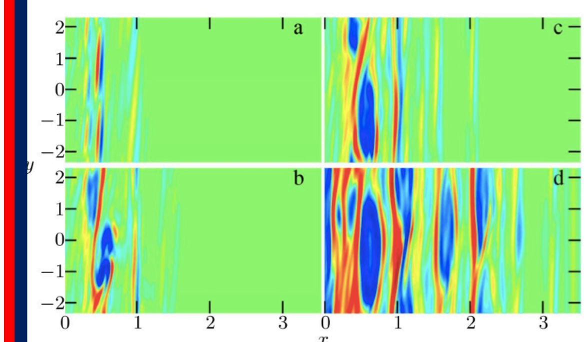
Nelson et al. 2013, Lin & Youdin 2015, Umurhan et al. 2016

Convective Overstability



Klahr 2003, Lesur & Papaloizou 2010, Lyra & Klahr 2011, Lyra 2014

Zombie Vortex Instability



Marcus et al. 2012, 2013, 2015, 2016
Umurhan et al 2016, Lesur & Latter 2016

Vertical shear instability

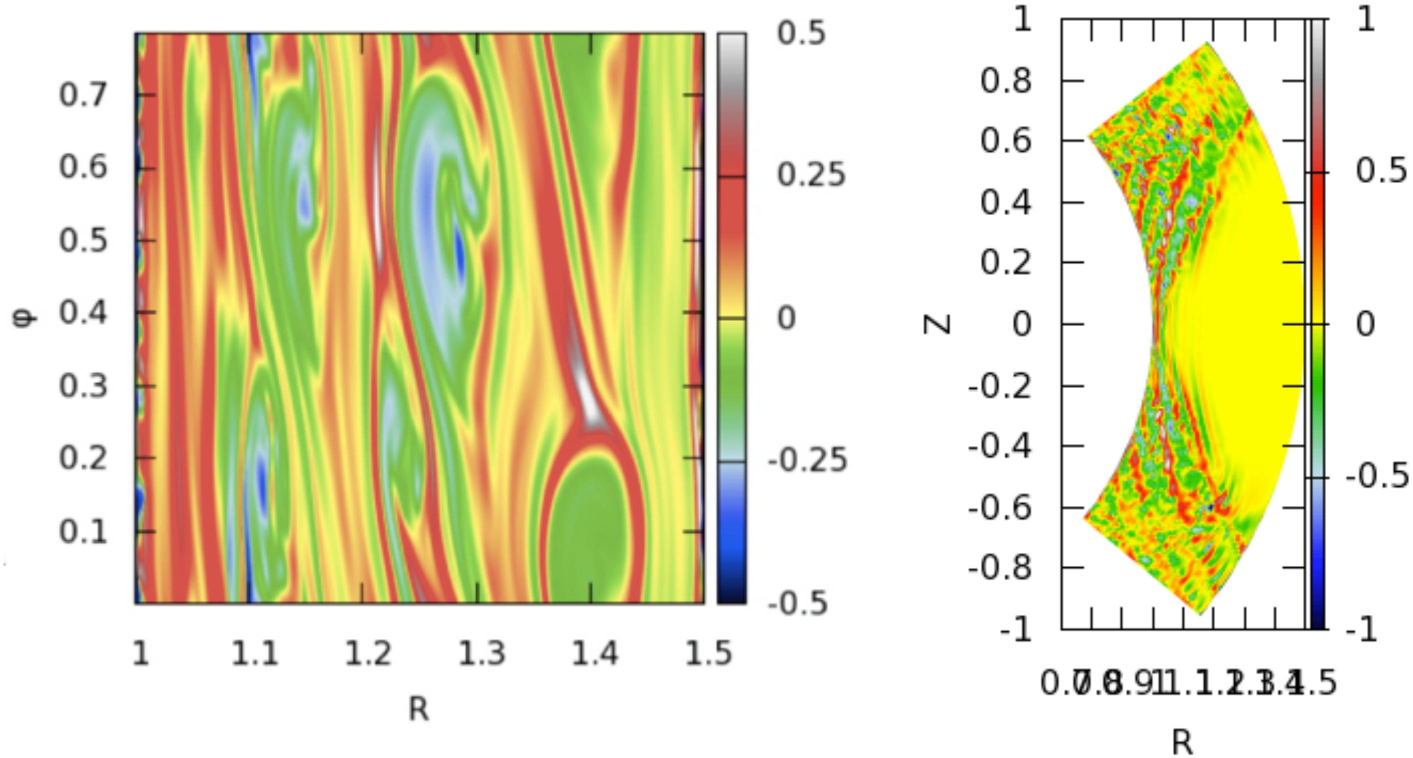
$$\rho_{\mathrm{mid}} = \rho_0 \left(\frac{R}{R_0}\right)^p,$$

$$c_s^2 = c_0^2 \left(\frac{R}{R_0}\right)^q,$$

$$\Omega = \Omega_K \left[1 + \frac{1}{2}\left(\frac{H}{R}\right)^2 \left(p + q + \frac{q}{2}\frac{Z^2}{H^2}\right)\right]$$

$$d\Omega / dz \neq 0 \, ; \, \kappa_z^2 < 0$$

Violates the Rayleigh criterion: **Unstable**



Nelson et al. (2013)