Class 7 2/14/20

Classification of T-Tauri stars

$$\boxed{\alpha_{IR}(\lambda) = \frac{d \log \lambda f(\lambda)}{d \ln \lambda}}$$ Spectral index

Ask them again why $\lambda f(\lambda)$ instead of $f(\lambda)$ and press. Highlight that I find is confusing - wait for answers that hint at

1) $\frac{d \ln \lambda f(\lambda)}{d \ln \lambda} = \frac{d \ln \nu f(\nu)}{d \ln \lambda}$ while $\frac{df(\lambda)}{d\lambda} \neq \frac{df(\nu)}{d\nu}$ Invariance of shape

2) surrogate for bolometric flux (approximation of $\int f(\nu) d\nu$)

1) Spectrum shape is the same: $\lambda f(\lambda) = \nu f(\nu)$

Proof: given $f(\nu) = \frac{dF}{d\nu}$ and $f(\lambda) = \frac{dF}{d\lambda}$ then

$$\frac{dF}{d\nu} = \frac{dF}{d\lambda}\left|\frac{d\lambda}{d\nu}\right| \; ; \text{ given } c = \lambda\nu \;\therefore\; \left|\frac{d\lambda}{d\nu}\right| = \frac{c}{\nu^2} = \frac{\lambda}{\nu}$$

$$f(\nu) = f(\lambda)\frac{\lambda}{\nu} \Rightarrow \nu f(\nu) = \lambda f(\lambda) \quad \text{QED}$$

2) Invariance of shape: $\alpha(\lambda) = \alpha(\nu)$. Consider the black body spectrum:

$$f(\nu) = \frac{2h\nu^3}{c^2} \frac{1}{(e^{h\nu/kT} - 1)} \qquad f(\lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{(e^{hc/\lambda kT} - 1)}$$

$$= -4 + \frac{hc}{kT\lambda} \frac{e^{hc/\lambda kT}}{(e^{hc/\lambda kT} - 1)}$$

$$\lambda f(\lambda) = \frac{2hc^2}{\lambda^4} \frac{1}{(e^{hc/\lambda kT} - 1)} \iff \nu f(\nu) = \frac{2h\nu^4}{c^2} \frac{1}{(e^{h\nu/kT} - 1)}$$

Take the log

$$\log \lambda f(\lambda) = \log(2hc^2) - 4\log\lambda - \log\left(e^{hc/\lambda kT} - 1\right)$$

$$\log \nu f(\nu) = \log(2h/c^2) + 4\log\nu - \log\left(e^{h\nu/kT} - 1\right)$$

Take the derivative. For the frequency formulation

$$\frac{d\log \nu f(\nu)}{d\log\nu} = 4 - \nu\frac{d}{d\nu}\left[\log\left(e^{h\nu/kT} - 1\right)\right] = 4 - \frac{h\nu}{kT} \frac{e^{h\nu/kT}}{(e^{h\nu/kT} - 1)}$$

As for the wavelength formulation

$$\frac{d\log \lambda f(\lambda)}{d\log\lambda} = -4 - \lambda\frac{d}{d\lambda}\left[\log\left(e^{hc/\lambda kT} - 1\right)\right] = -4 + \frac{hc}{kT\lambda} \frac{e^{hc/\lambda kT}}{(e^{hc/\lambda kT} - 1)}$$

Given $c = \lambda\nu$, $\frac{d\log\lambda f(\lambda)}{d\log\lambda} = -\frac{d\log\nu f(\nu)}{d\log\nu}$

The minus sign is only because they're varying in opposite directions:

$$\boxed{\left|\frac{d\log\lambda f(\lambda)}{d\log\lambda}\right| = \left|\frac{d\log\nu f(\nu)}{d\log\nu}\right|} \therefore \boxed{|\alpha(\lambda)| = |\alpha(\nu)|}$$

optical will plot λf_λ because they think in units of wavelength.

cultural difference: mm or radio will plot νf_ν because the wavelength is long and they think frequency

$$f(\nu) = A(\nu/\nu_0)^\alpha$$

what are the actual values?

Black body value?

For the star, the infrared is the Rayleigh-Jeans tail $e^{h\nu/kT} \approx 1 + h\nu/kT$

$$\therefore \frac{\nu}{(e^{h\nu/kT}-1)} \cdot \frac{h}{kT} e^{h\nu/kT} \approx -\frac{\nu}{(\cancel{1} + h\nu/kT - \cancel{1})} \cdot \frac{h}{kT} \cdot (1 + \cancel{h\nu/kT}) \approx -1$$

Black body value for star in IR: $\boxed{\dfrac{d \log \lambda f(\lambda)}{d \log \lambda} = -3}$

$\alpha_\lambda = -3$ (or $\alpha_\nu = 3$) is the Rayleigh-Jeans tail. $\leftarrow$

For higher energies or photons closer to thermal $e^{h\nu/kT} - 1 \approx e^{h\nu/kT}$

$$\frac{d\log \lambda f(\lambda)}{d\log \lambda} = -4 + \frac{hc}{kT\lambda} \frac{\cancel{e^{hc/\lambda kT}}}{\cancel{(e^{hc/\lambda kT} - 1)}}$$

at 10 μm:

$$\alpha_\lambda = -4 + 4.79 \left(\frac{\lambda}{10\,\mu m}\right)^{-1} \left(\frac{T}{300K}\right)^{-1}$$

At constant wavelength α_λ is a measurement of temperature!

α correlates with infrared excess! show how α correlates with N band (20 μm) excess

ΔN (Strutskie+90) $\log\left(\frac{F}{F_*}\right)_{N\text{-band}}$ N-band → 10 μm

ΔN is then the difference between the observed spectrum and the reference blackbody spectrum at 10 μm.

α good proxy for ΔN as the photospheres can be heavily extincted.

Young stellar objects (YSOs)
Lada & Wilking (1984) Classification:

Class I	$\alpha_{IR} > 0$
Class II	$-2 < \alpha_{IR} < 0$
Class III	$\alpha_{IR} < -2$

Features of the spectrum

Between near-infrared (K band, 2.2 μm and mid-infrared 10 μm - 24 μm)

Physical characteristics: André & Montmerle

Class II and III → drop at 10 μm. Collapse of IR flux. Devoid of circumstellar material.

Class I and II: class I bigger envelopes, class II smaller (disks)
Difference is only on the spatial distribution of dust.

Class 1 hv higher masses than Class II: class I must be younger.

Circumstellar matter < 0.1 $M_\odot$ in Class 1 & 2; the central object already formed.

Class 0 → 0.5 $M_\odot$, so central mass did not form yet.
Significantly younger. Cold blackbody. No flux < 20 μm.

Interpreted as an evolutionary sequence:

Class 0:	Cloud core, star not formed.	
Class 1:	Accreting envelope, star deeply embedded	$\alpha_\nu > -0.3$
Class 2:	Star + disk (classical T-Tauri)	$-1.6 \leq \alpha_\nu \leq -0.3$
Class 3:	Most circumstellar material gone (debris disk)	$\alpha_\nu \leq -1.6$

Show André & Montmerle plot of mm-flux vs infrared

Sharp boundary -> No Class 3 has high mm flux $\alpha_{IR} = -1.5$

↳ dust mostly gone

CTTS → $W_{H\alpha} > 10\,\text{Å}$

WTTS $W_{H\alpha} < 10\,\text{Å}$

Emission is optically thin at 1.3 mm (mm flux)

mm flux vs infrared flux

ALL DUST HOT DUST