Class 6: 2/11/20

Cloud core scale 0.1 pc ; density $n \sim 10^5 \text{ cm}^{-3}$

$$\beta = \frac{\text{Rotational Kinetic energy}}{\text{gravitational energy}}$$

$$= \frac{1/2 \, I \omega^2}{q GM^2/R}$$

Moment of inertia $I = pMR^2$

Uniform density $p = 2/5$; $q = 3/5$ ∴ $p/q = 2/3$

$$\beta = \frac{1}{2} \frac{p}{q} \frac{\omega^2 R^3}{GM}$$ for constant density $\rho = \frac{3M}{4\pi R^3}$

$$\beta = \frac{1}{3} \frac{\omega^2 R^3}{GM} \cdot \frac{4\pi}{4\pi} = \frac{\omega^2}{4\pi G \rho_0}$$ Both ω and ρ are measureable

$$\beta_0 = \frac{E_{rot}}{|E_{grav}|} \sim 0.02$$ (show figure) ∴ rotational support is irrelevant

Clouds are not rotationally supported, but their angular momentum is still huge!

J of core $J = m\, r \times v = M \omega R^2$

$$\beta = \frac{\omega^2}{3GM} R^3 \longleftrightarrow \omega = \left(\frac{3GM}{R^3}\beta\right)^{1/2}$$

$$J = MR^2 \cdot \left(\frac{3GM}{R^3}\right)^{1/2} \cdot \beta^{1/2}$$

algebra steps (omit)

$$J = G^{1/2} \cdot M^{3/2} \cdot \beta^{1/2} \cdot R^{1/2} \cdot 3^{1/2}$$

$$J = (3G)^{1/2} \cdot \left(\frac{M}{M_\odot}\right)^{3/2} \cdot M_\odot^{3/2} \cdot \left(\frac{\beta}{0.02}\right)^{1/2} (0.02)^{1/2} \left(\frac{R}{0.05\,pc}\right)^{1/2} (0.05\,pc)^{1/2}$$

$$J = \left(\frac{M}{M_\odot}\right)^{3/2} \left(\frac{\beta}{0.02}\right)^{1/2} \left(\frac{R}{0.05\,pc}\right)^{1/2} \left[(3G)^{1/2} \cdot M_\odot^{3/2} (0.02)^{1/2} \cdot (0.05\,pc)^{1/2}\right]$$

$$J = 2 \times 10^{54}\ g\ cm^2\ s^{-1} \left(\frac{M}{M_\odot}\right)^{3/2} \left(\frac{\beta}{0.02}\right)^{1/2} \left(\frac{R}{0.05\,pc}\right)^{1/2}$$

Show again Solar System figure. This angular momentum is 10^4x larger than the Sun's.

Solar System → 3-4 orders of magnitude less

Jupiter angular momentum:

$$J = M_J \times \omega \times R^2$$

$$= 2 \times 10^{30} g \times \frac{2\pi}{T_{JUP}} \times (5.2\ AU)^2$$

$$\boxed{J_{SS} = 2 \times 10^{50}\ g\ cm^2 s^{-1}}$$

The angular momentum was lost from the molecular cloud core to the Solar system. How??

Binaries?
Large disks?

If all this angular momentum got stored in a dish, how large would the disk be?

Specific angular momentum: $\ell_{core} = \frac{J_{core}}{M_{core}} \approx \frac{2\times10^{54}\,g\,cm^2 s^{-1}}{4\times10^{33}\,g} \approx 5\times10^{20}\,cm^2 s^{-1}$

For the dish, the specific angular momentum is

$$\ell_{dish} = \Omega r^2 = \sqrt{\frac{GM}{r^3}} \cdot r^2 = \sqrt{GMr}$$

M is central mass, r is arbitrary radius

Equate ℓ_{core} and ℓ_{dish} and solve for r

$$\sqrt{GMr} = 5\times10^{20}\,cm^2 s^{-1} \quad \therefore \quad r = \frac{(5\times10^{20})^2}{GM} \sim 1.8\times10^{15}\,cm$$

$1\,AU \sim 1.5\times10^{13}\,cm$

$r \sim 100\,AU$

Collapsing clouds should produce disks of typical size of the order of ~100 AU.
(show observational evidence for this size)

Introduce T-Tauri stars if there is time

Key points so far:
- Size of disks
- Masses of disks
- Ang Momentum of disks
- Jeans mass