Class 5

# Barnard 68

Jeans criterion from Virial theorem: $2K+U=0$

$$U \sim -\frac{3}{5}\frac{GM^2}{R}$$

$K = 3/2\ NkT$ ; $N = \frac{M}{\mu m_H}$

post-class: student asked why 3/2, since that's for monoatomic species but the gas is molecular.

Collapse: $\frac{3M_c kT}{\mu m_H} < \frac{3}{5}\frac{GM_c^2}{R}$

Substitute $R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$ (constant density)

$$\boxed{M > \left(\frac{5kT}{G\mu m_H}\right)^{3/2} \cdot \left(\frac{3}{4\pi\rho}\right)^{1/2} \quad \text{Jeans mass}}$$

For the ISM, cloud of $T = 50K$; $n = 5\times10^2\ cm^{-3}$ ; $\rho = m_H \cdot n \sim 10^{-15}\ \frac{g}{cm^3}$

Substitute: $M_J \sim 1500\ M_\odot$ too high!

The inner core of cloud is dense and shielded: $T = 10K$, $n = 10^4\ cm^{-3}$ ; $\rho \sim 10^{-14}\ g/cm^3$

$M_J \sim 10 M_\odot$ (still too high) Ask students what could help bring $M_J$ down and hint at external pressure.

$$M > \left(\frac{5kT}{G\mu m_H}\right)^{3/2} \cdot \left(\frac{3}{4\pi\rho}\right)^{1/2}$$

$\rho = P/c_s^2$ Rewrite $M_J$ with pressure instead of density

$$M > \left(\frac{5kT}{G\mu m_H}\right)^{3/2} \left(\frac{3}{4\pi P}c_s^2\right)^{1/2} \qquad T = \frac{c_s^2 \cdot m}{\gamma k}$$

$$> \left(\frac{5\cancel{k}\cdot c_s^2 \cdot \cancel{m}}{G\mu \cancel{m_H} k\gamma}\right)^{3/2}\left(\frac{3}{4\pi P}\cdot c_s^2\right)^{1/2} = \frac{5^{3/2}\cdot c_s^3 \cdot c_s \cdot \cancel{3^{1/2}}}{2\cdot \cancel{\pi^{1/2}}\cdot P^{1/2}\cdot G^{3/2}\,\cancel{\gamma^{3/2}}}$$

$$M > \left(\frac{5KT}{G\mu m_H}\right)^{3/2} \left(\frac{3}{4\pi P} c_s^2\right)^{1/2} \qquad T = c_s^2 \cdot \frac{m}{\gamma k}$$

$$> \left(\frac{5 \cancel{k} \cdot c_s^2 \cdot \cancel{m}}{G \mu \cancel{m_H} k \gamma}\right)^{3/2} \left(\frac{3}{4\pi p} \cdot c_s^2\right)^{1/2} = \frac{5^{3/2} \cdot c_s^3 \cdot c_s \cancel{3^{1/2}}}{2 \cdot \cancel{\pi^{1/2}} \cdot P^{1/2} \cdot G^{3/2} \cancel{\gamma^{3/2}}}$$

$$5^{3/2}/2 \sim 5$$

$$\boxed{M_J \sim \frac{5 c_s^4}{G^{3/2} P^{1/2}}}$$

Jeans mass in terms of pressure

Add external pressure:

$$\boxed{M_{BE} = \frac{5 c_s^4}{G^{3/2} \cdot (P + P_{ext})^{1/2}}}$$

Bonnor-Ebert mass

Highlight that's just the Jeans mass with external pressure

If $P_{ext} \sim 50P$, then $M_{BE} = \frac{1}{\sqrt{50}} \cdot M_J \sim \frac{M_J}{7}$

$$\boxed{M_{BE} \sim 1-2\ M_\odot}.$$

Show João's plot of the extinction map.

The $A_V$ profile indicates exponential density $\rho = \rho_0 e^{-r}$, as in the barometric formula. The mass is highly concentrated.

Exponential globule?

$$\frac{dP}{dr} = -\frac{GM}{r^2}\rho \quad ; \quad \frac{dM}{dr} = 4\pi r^2 \rho$$

Derivative

$$\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -G\frac{dM}{dr} = -G 4\pi r^2 \rho$$

$$\therefore \frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho$$

$$P = \rho c_s^2$$

$$\frac{c_s^2}{r^2}\frac{d}{dr}\left(r^2\frac{1}{\rho}\frac{d\rho}{dr}\right) = -4\pi G\rho$$

Exponential profile:

$$\frac{dP}{dr} = -g\rho$$

$$\frac{d\ln\rho}{dr} = -\frac{g}{c_s^2}$$

$$\rho = \rho_0 e^{-g/c_s^2 \cdot r}$$

$$\rho = \rho_0 e^{-\psi}$$

$$\frac{c_s^2}{r^2}\frac{d}{dr}\left(r^2\frac{1}{\rho}\frac{d\rho}{dr}\right) = -4\pi G\rho_0 e^{-\psi}$$

$$\frac{c_s^2}{r^2}\frac{d}{dr}\left(r^2\frac{d\ln\rho}{dr}\right) = -4\pi G\rho_0 e^{-\psi}$$

$$\frac{c_s^2}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) = 4\pi G\rho_0 e^{-\psi}$$

$$\frac{1}{4\pi G \rho_0} \frac{c_s^2}{r^2} \frac{d}{dr}\left(r^2 \frac{d\psi}{dr}\right) = e^{-\psi}$$

Normalize $\xi \doteq \frac{r}{c_s} \cdot \sqrt{4\pi G \rho_0}$

$$\boxed{\frac{1}{\xi^2} \frac{d}{d\xi}\left(\xi^2 \frac{d\psi}{d\xi}\right) = e^{-\psi}}$$

Lane-Emden equation

Numerical solution . $\xi = 6.9 \pm 0.2$ show again the plot

$\Rightarrow$ Radius 12500 AU ($\sim 0.05$ pc)

$M = 2 M_\odot$

$\text{Pressure}_{out} \sim 2.5 \times 10^{-12}$ Pa

Temperature $\sim 16$ K $P_{out} \sim 10\, P_{INT}$

Consistent with theory - Isothermal, pressure-confined, self-gravitating cloud, on the verge of collapse.

Explain the $\xi \sim 6.9$

$\nabla^2\phi = 4\pi G\rho$ $\quad \frac{\text{gravity}}{\text{pressure}} = \frac{\int 4\pi G\rho dr}{\frac{1}{\rho}\nabla \rho c_s^2} \rightarrow \xi^2 = \frac{4\pi G\rho R^2}{c_s^2}$

$\nabla \cdot g = 4\pi G\rho$

$\xi = \frac{R}{c_s}\sqrt{4\pi G\rho}$ Measurement of Jeans length?

Jeans length? $\rightarrow \xi = \frac{R}{L_c} \quad \therefore \quad L_c = \frac{c_s}{\sqrt{4\pi G\rho}} = \frac{1}{\sqrt{4\pi}} \cdot \frac{c_s}{\sqrt{G\rho}}$ (I)

$\lambda_J = \left(\frac{3 M_J}{4\pi\rho}\right)^{1/3}$ (radius of sphere of constant density and mass equal to Jeans mass)

$M_J = \frac{5 c_s^4}{G^{3/2}\rho^{1/2}} \quad \therefore \quad \lambda_J = \left(\frac{3}{4\pi\rho} \cdot \frac{5 c_s^4}{G^{3/2}\rho^{1/2} \cdot c_s}\right)^{1/3}$

$\lambda_J = \left[\frac{15\, c_s^3}{4\pi G^{3/2} \cdot \rho^{3/2}}\right]^{1/3} = \left(\frac{15}{4\pi}\right)^{1/3} \cdot \frac{c_s}{\sqrt{G\rho}}$ (II)

(II) in (I) $\rightarrow$

$$\boxed{\lambda_J = \left(\frac{15}{4\pi}\right)^{1/3} \cdot \sqrt{4\pi} \cdot L_c \approx 7 L_c}$$

difference between Jeans length with constant and exponentially varying density