Minimum Mass Solar Nebula

Class 3

The refractories composition of the planets is chondritic (show plot meteorites vs solar)
Add volatiles until solar composition is reached.

MMSN: Jupiter's enrichment → 2-40 times solar for weidenschilling 77
zone's boundaries → arithmetic mean of adjacent orbit

$$\Sigma(r) = 1.7\times10^{3} \left(\frac{r}{AU}\right)^{-3/2} g\,cm^{-2}$$

Integrating this to 30 AU

$$M = 2\pi\int_{r_0}^{r_1} \Sigma\, r\, dr = 2\pi\Sigma_0 \int_{r_0}^{r_1} \left(\frac{r}{AU}\right)^{-3/2} \cdot r\, dr = 2\pi\,\Sigma_0\, AU^{3/2} \int_{r_0}^{r_1} r^{-1/2}\, dr$$

$$= 2\pi\,\Sigma_0\, AU^{3/2}\, 2r^{1/2}\Big|_{r_0}^{r_1}$$

$$M = 4\pi\,\Sigma_0\, AU^{3/2} \left(\sqrt{r_1} - \sqrt{r_0}\right) = 4\pi\,\Sigma_0\, AU^{2}\sqrt{30} \simeq 2.6\times10^{31} g$$

$$M_{MMSN} \simeq 0.01\, M_\odot$$

Minimum mass to form the planets.
Point out it assumes 100% efficiency, and ignores migration.

Estimate pressure:

$\Sigma = 1500\ g/cm^2$

$T = 280\ K$

$P = \rho(\gamma-1)c_v T$

$= \rho(\gamma-1)\ c_v \cdot T$

$\rho = \frac{\Sigma}{H} = \frac{1500}{1\times10^{12}}$

$c_v \sim R$ ($R \sim 8.\times10^7$ erg K mol)

$P = \frac{1500}{10^{12}} \cdot 10^8 \cdot 200 = \frac{10^3 \cdot 10^8 \cdot 10^2}{10^{12}} = 10\ \frac{dyne}{cm^2}$

$\simeq 1\ Pa$

$P = 10^{-5}\ bar$

$c_p - c_v = \frac{R}{M} = \frac{k_B}{m}$

water sublimation temperature/pressure (show phase diagram)

Highlight the water delivery problem

Water ice under these pressures are at $\sim < 200K$. At Earth, liquid water needs $273K < T < 373K$. 100 difference!
Planets in the habitable zone form from water-poor material!
Water must come from somewhere.

$$\Sigma_{rock} \simeq 7.1 \left(\frac{r}{AU}\right)^{-3/2} g\,cm^{-2} \text{ for } r < 2.7\,AU$$

$$\Sigma_{ice/rock} \simeq 30 \left(\frac{r}{1\,AU}\right)^{-3/2} g\,cm^{-2} \text{ for } r > 2.7\,AU$$

Evidence for water snowline at middle of asteroid belt:
Meteorites, Ceres, Jupiter Moons/Mars/Pressure

Deuterium to hydrogen in water from Earth: 156 ppm

Carbonaceous chondrites: 159 ± 10 ppm
Oort Cloud comets: 300 ppm

Water on Earth: $3 \times 10^{-4} M_\oplus$
Mass of asteroid belt: $3 \times 10^{-4} M_\oplus$

10% of carbonaceous chondrites is water, so only

$3 \times 10^{-5} M_\oplus$ of water at present.

Asteroid belt should been AT LEAST 10x more massive before, If all flux arrived at Earth. Another factor 10 at least, so originally belt should hv been a few $\times 10^{-3} M_\oplus$, 100x more massive than today.