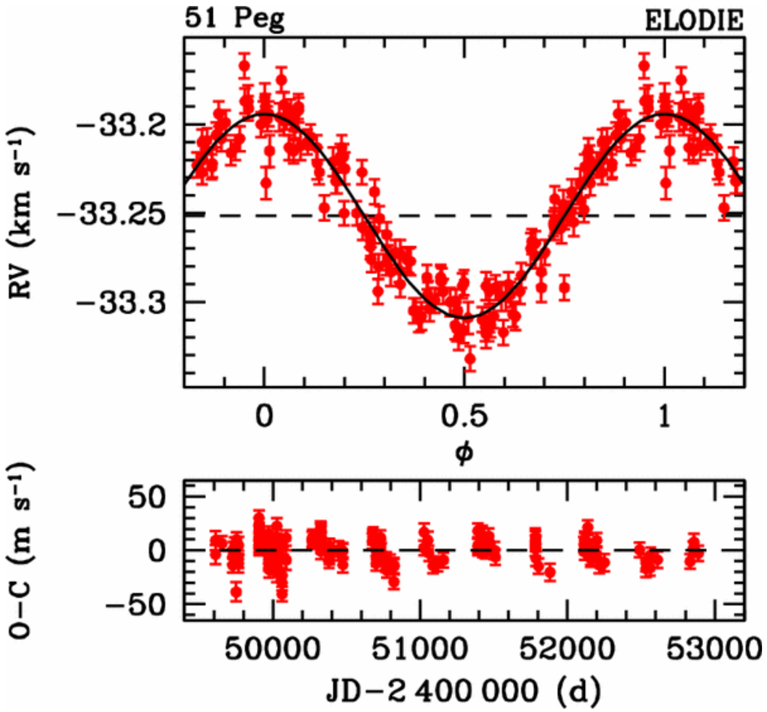
Class 25 – Apr 30th, 2020

- The need for migration and orbital evolution
  - Problems with in situ formation
    - Hot Jupiters
    - Planets in Mean Motion Resonance

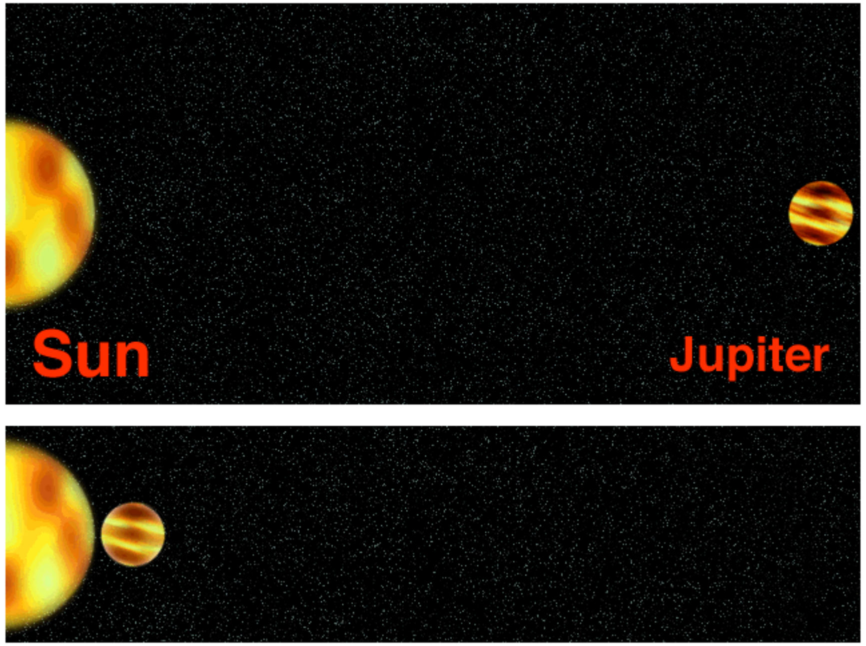
- Gas-Driven migration
  - Disk Torques
    - Impulse Approximation
    - Type I migration (low mass planets)
    - Gap Formation and Type II migration (high-mass planets)

- Planetesimal-Driven migration
  - Nice Model
  - Grand Tack model

# The Surprise




Original detection
(Mayor & Queloz 1995)




A **HOT** Jupiter!

$a$ = 0.052 AU
$P$ = 4.23 days
$M \sin I = 0.468\ M_J$

# Planet Migration was not new…

ApJ, 241, 425 (October 1, 1980)

## DISK-SATELLITE INTERACTIONS

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AND
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*Received 1980 January 7; accepted 1980 April 9*

ABSTRACT

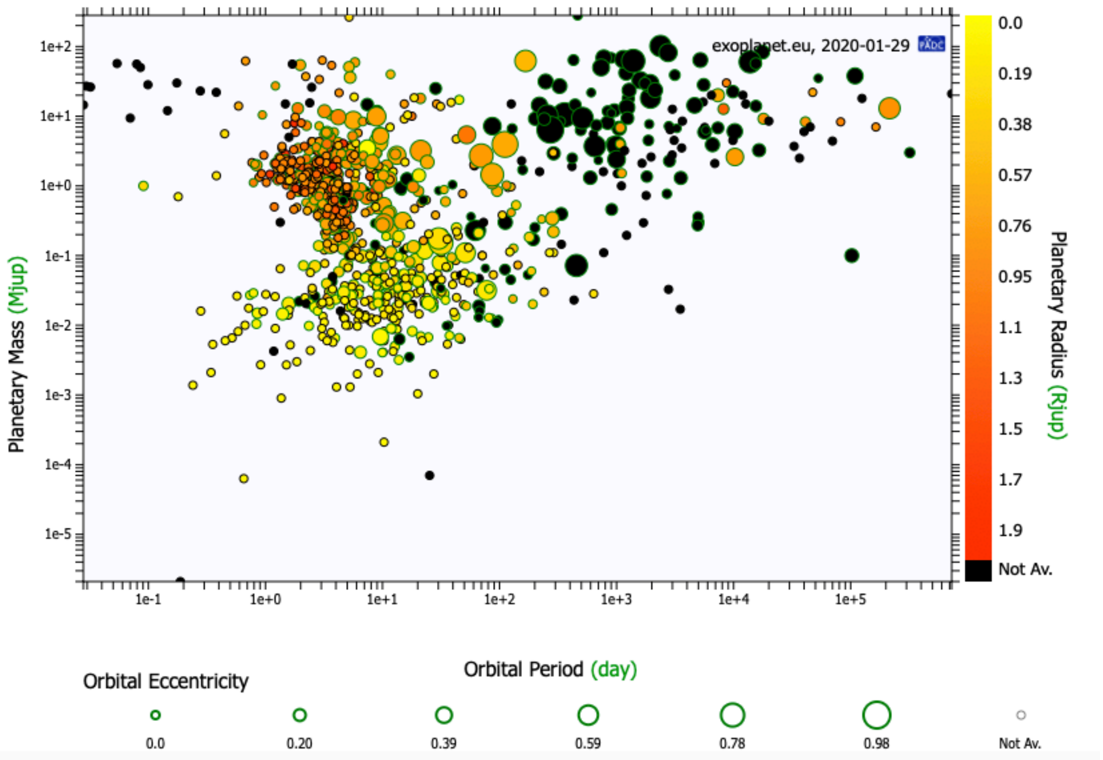
We calculate the rate at which angular momentum and energy are transferred between a disk and a satellite which orbit the same central mass. A satellite which moves on a circular orbit exerts a torque on the disk only in the immediate vicinity of its Lindblad resonances. The direction of angular momentum transport is outward, from disk material inside the satellite's orbit to the satellite and from the satellite to disk material outside its orbit. A satellite with an eccentric orbit exerts a torque on the disk at corotation resonances as well as at Lindblad resonances. The angular momentum and energy transfer at Lindblad resonances tends to increase the satellite's orbit eccentricity whereas the transfer at corotation resonances tends to decrease it. In a Keplerian disk, to lowest order in eccentricity and in the absence of nonlinear effects, the corotation resonances dominate by a slight margin and the eccentricity damps. However, if the strongest corotation resonances saturate due to particle trapping, then the eccentricity grows.

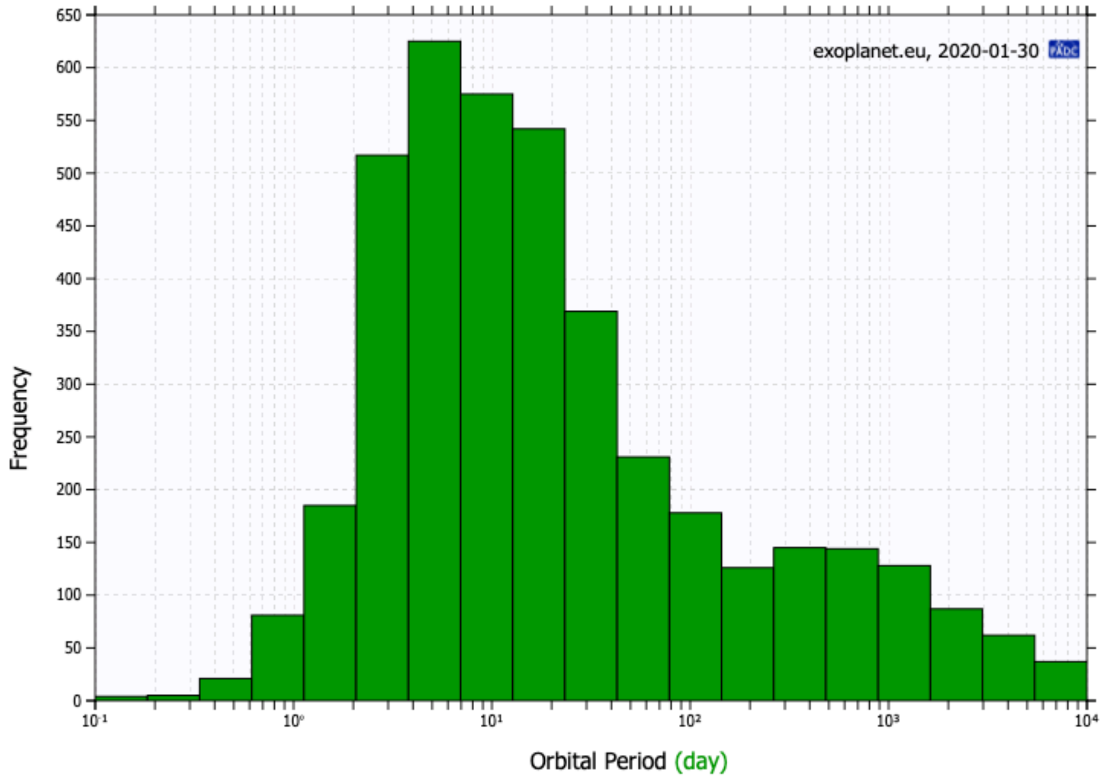
We present an illustrative application of our results to the interaction between Jupiter and the protoplanetary disk. The angular momentum transfer is shown to be so rapid that substantial changes in both the structure of the disk and the orbit of Jupiter must have taken place on a time scale of a few thousand years.

*Subject headings:* hydrodynamics — planets: Jupiter — planets: satellites — solar system: general

*discovered 15 years earlier... by theorists!*

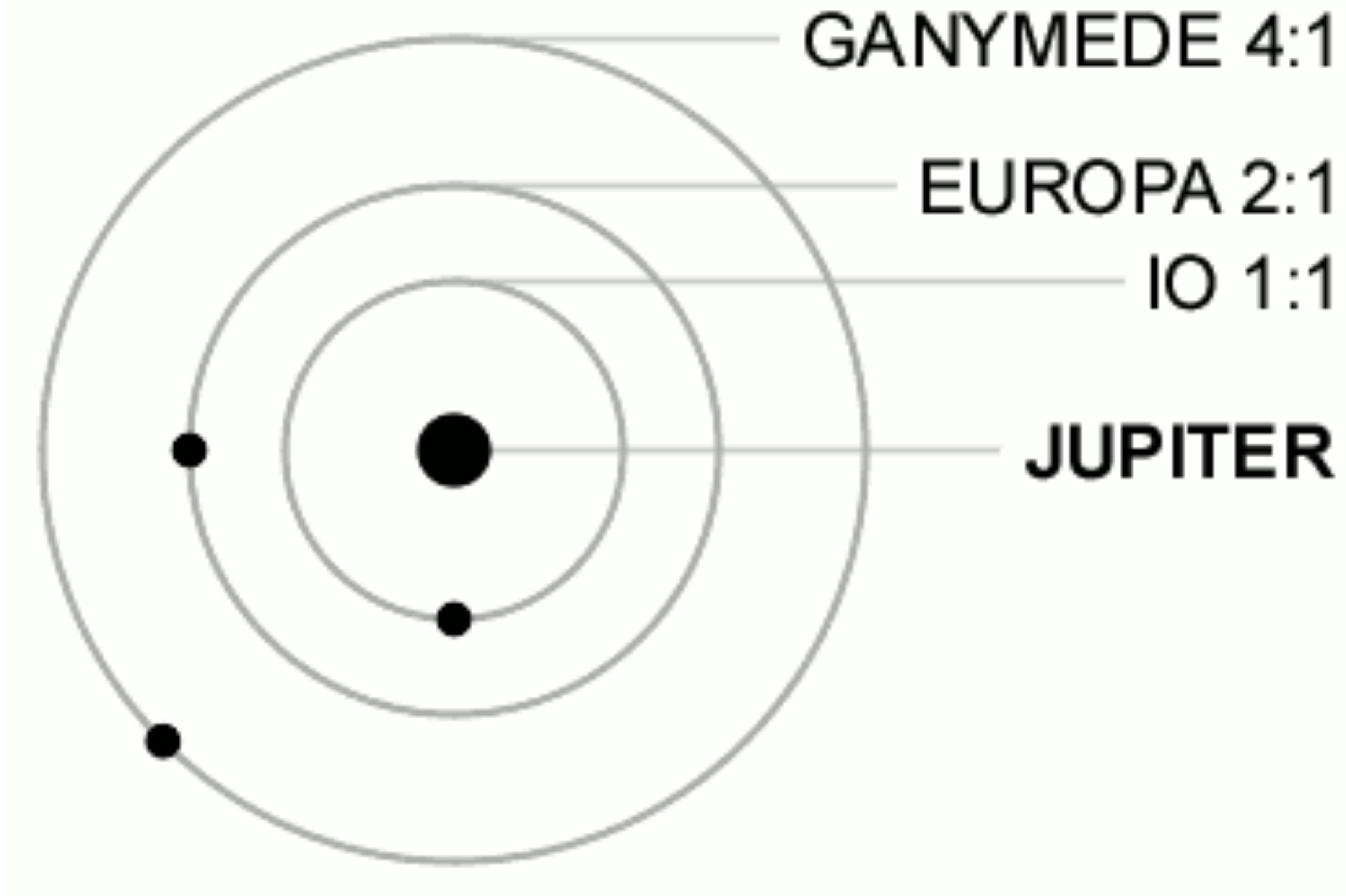
# The Exoplanet Landscape



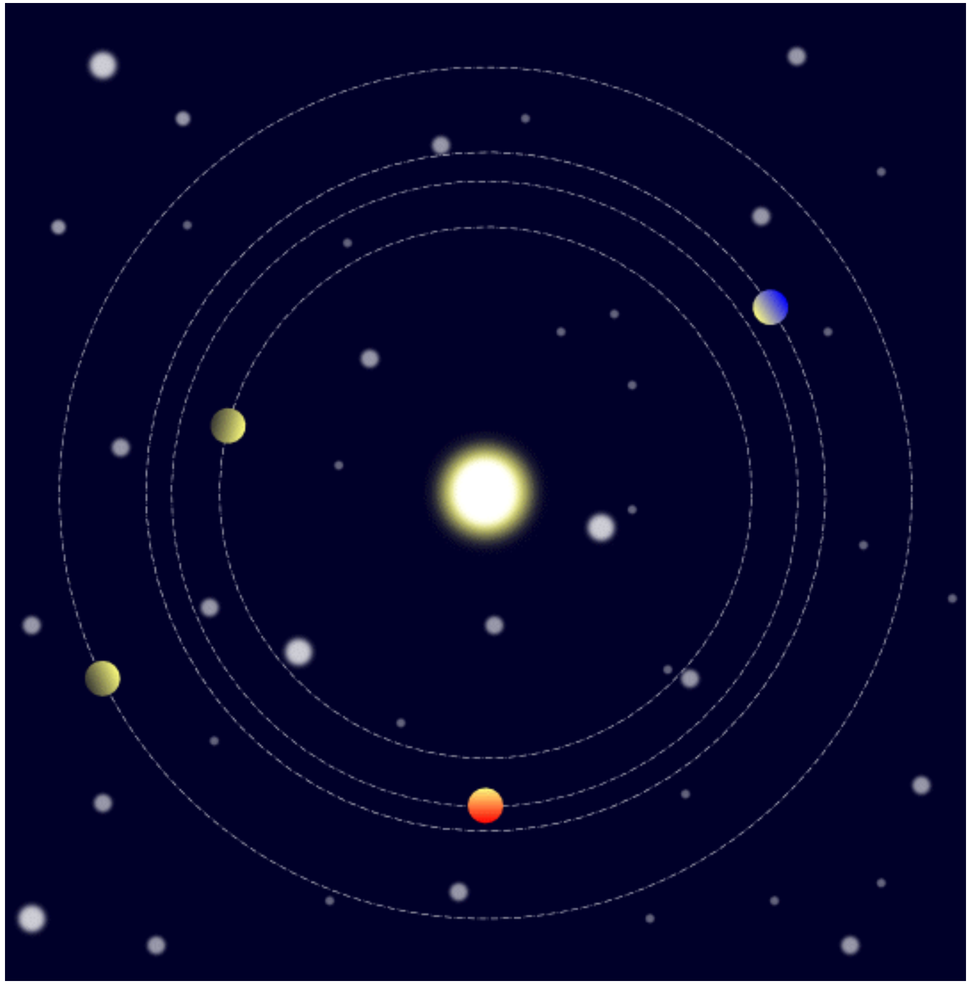

**3-day pile-up**

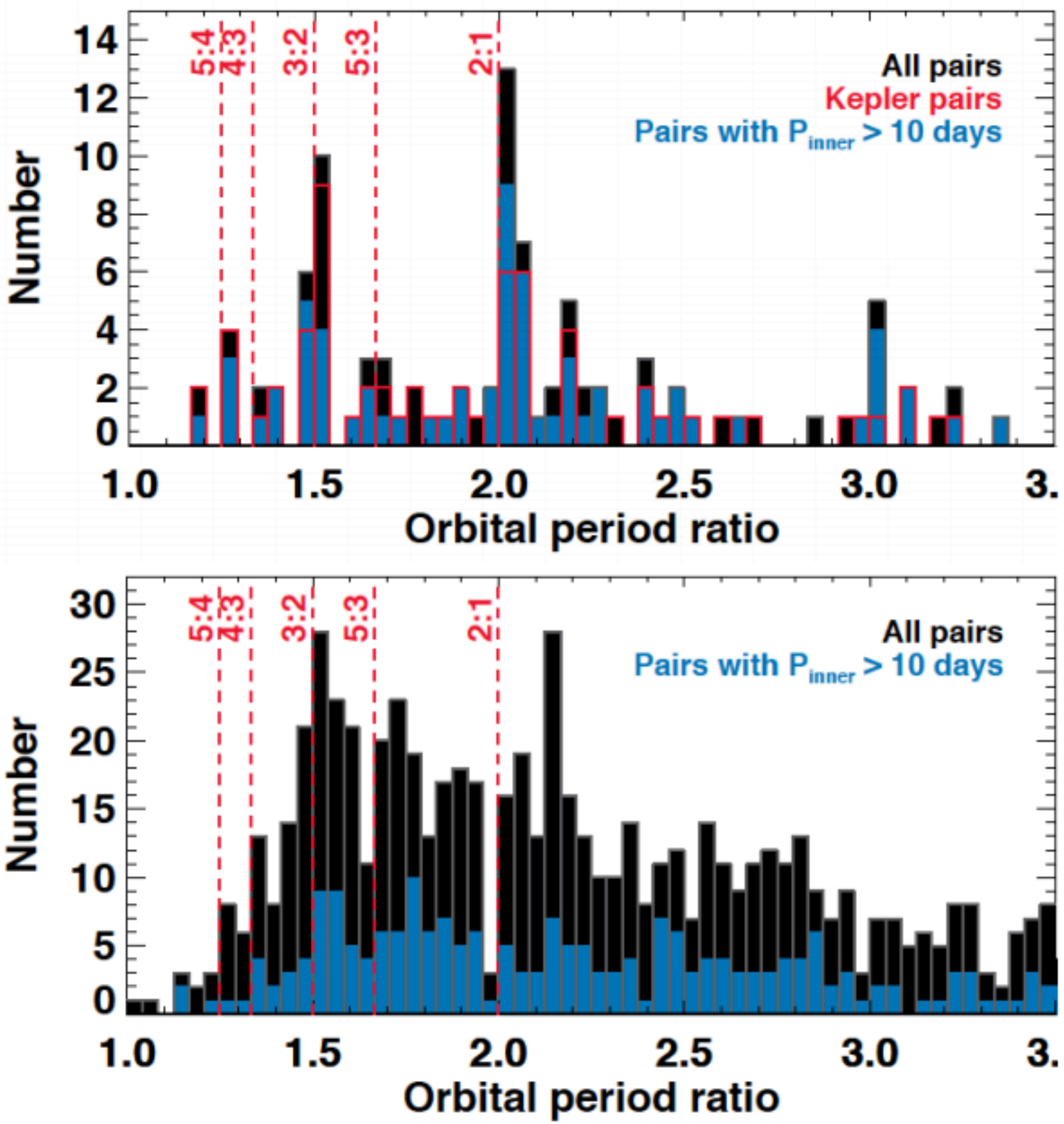
## Laplace Resonance



# Kepler 223

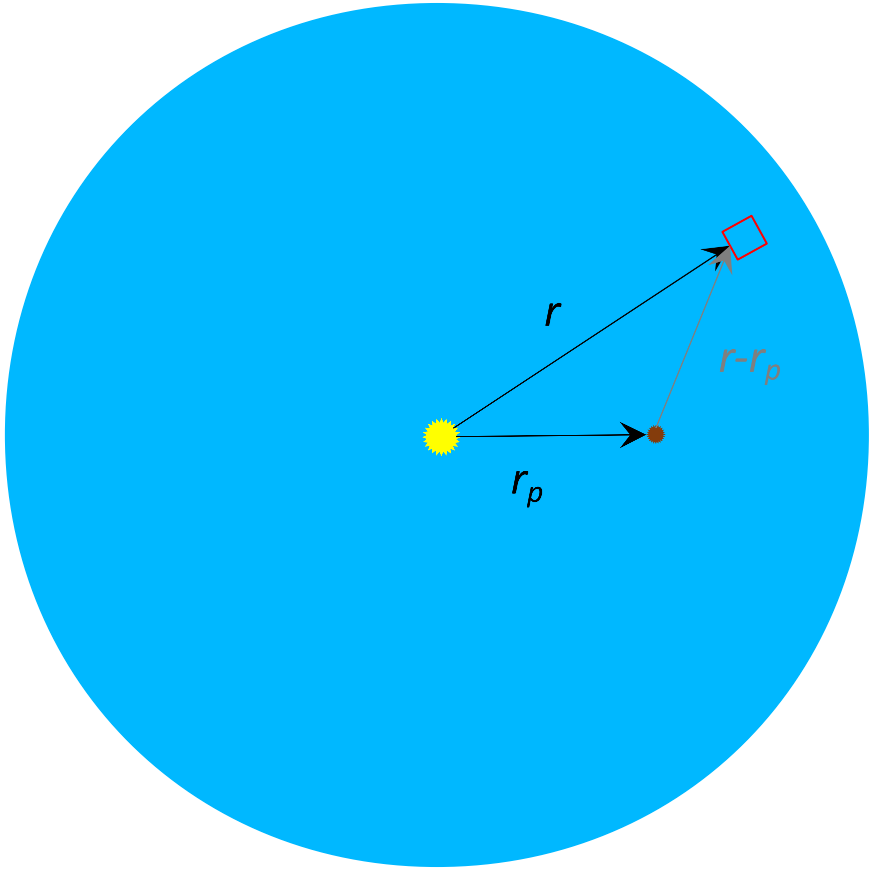
## Four planets in 8:6:4:3 resonance





5:4
4:3
3:2
5:3
2:1
All pairs
Kepler pairs
Pairs with $P_{inner}$ > 10 days
Number
Orbital period ratio
5:4
4:3
3:2
5:3
2:1
All pairs
Pairs with $P_{inner}$ > 10 days
Number
Orbital period ratio

# Gas-Driven Migration




Angular momentum transfer to planet by disk-planet interaction

$$\frac{d\boldsymbol{J}}{dt} = \boldsymbol{\Gamma} = \boldsymbol{r_p} \times \boldsymbol{F}$$

Driven by gravitational torques

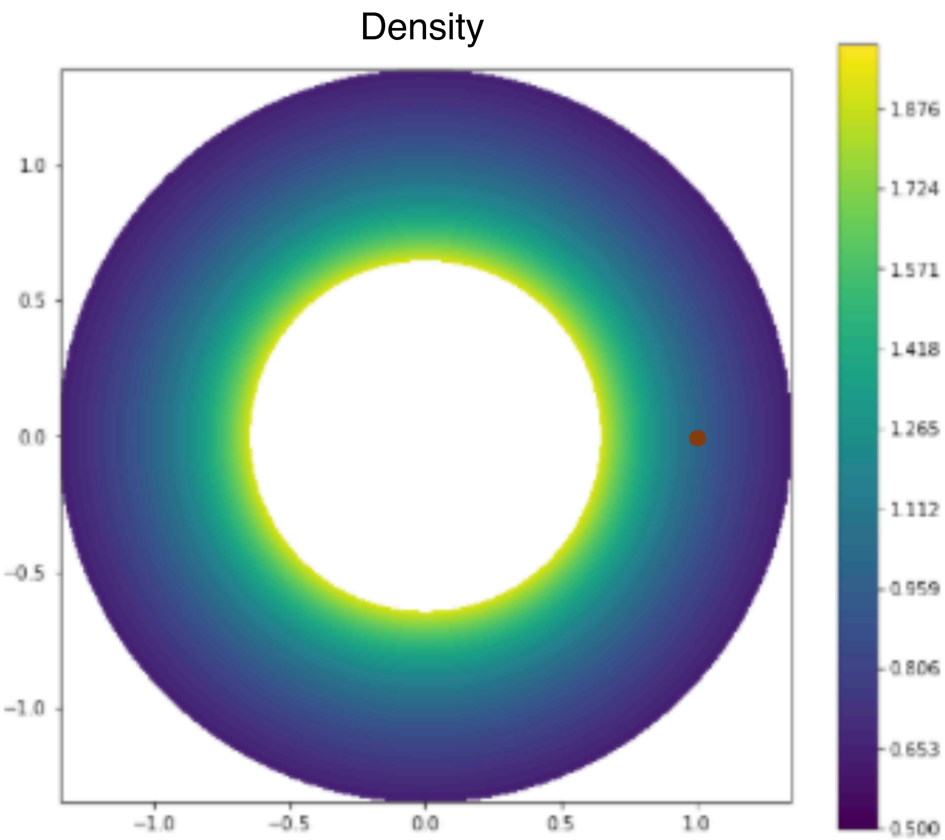
$$\mathrm{d}\boldsymbol{F} = -Gm_p \frac{\rho\, dV}{|\boldsymbol{r}-\boldsymbol{r_p}|^3}(\boldsymbol{r}-\boldsymbol{r}_p)$$

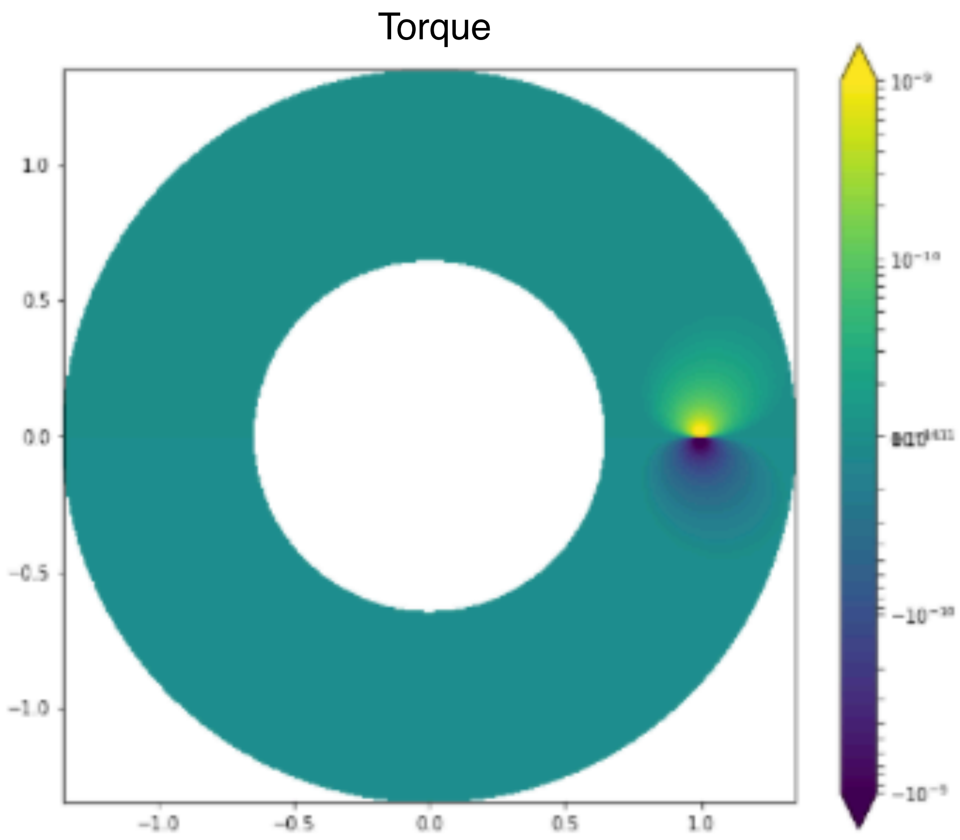
$$\boldsymbol{\Gamma} = \int d\boldsymbol{\Gamma} = \int \boldsymbol{r_p} \times d\boldsymbol{F}$$

$$\boxed{\boldsymbol{\Gamma} = x_p G m_p \iint y \frac{\Sigma\, dxdy}{|\boldsymbol{r}-\boldsymbol{r_p}|^3}}$$

# Migration Torques

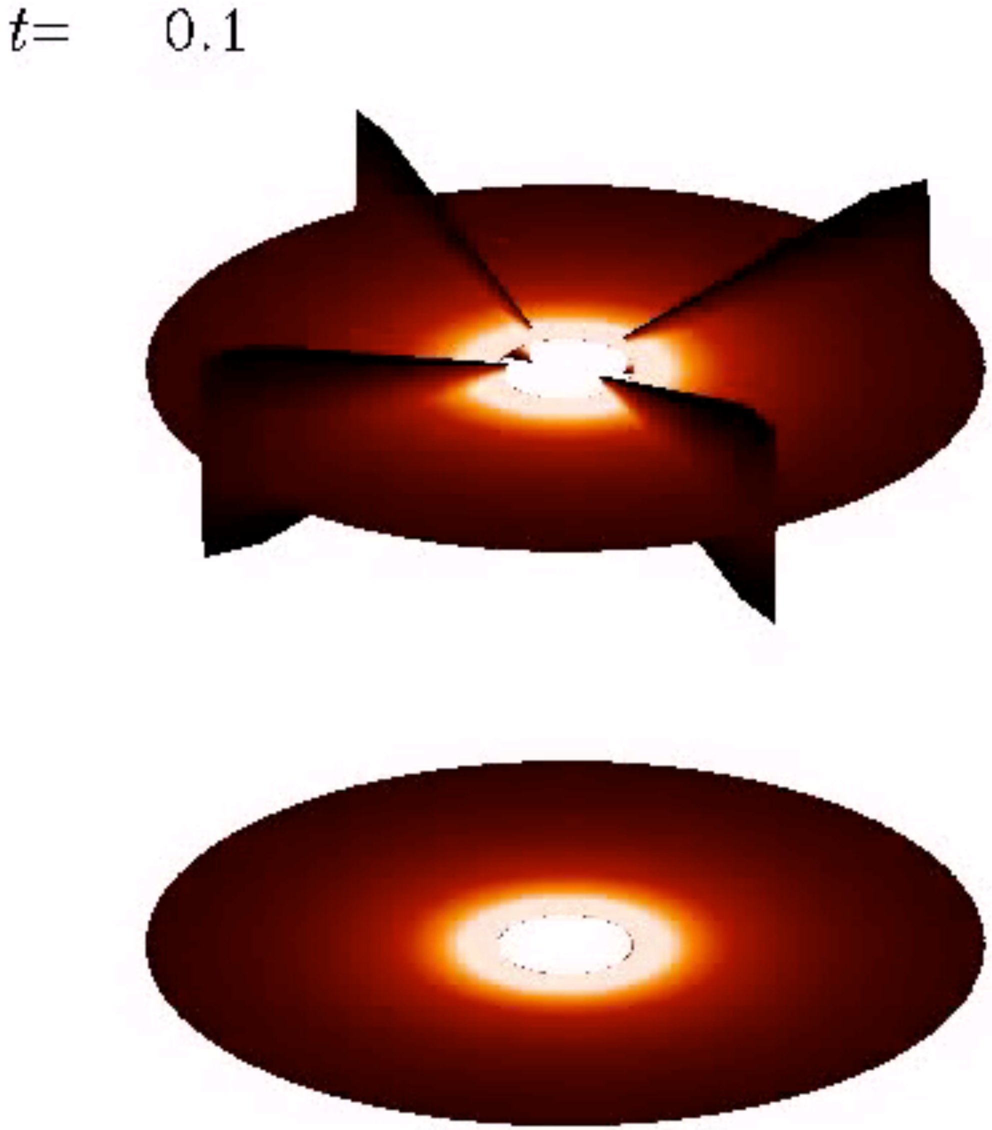
$$\mathbf{\Gamma} = r_p G m_p \iint \Sigma \frac{r^2}{|\boldsymbol{r} - \boldsymbol{r_p}|^3} \sin\phi \, dr d\phi$$



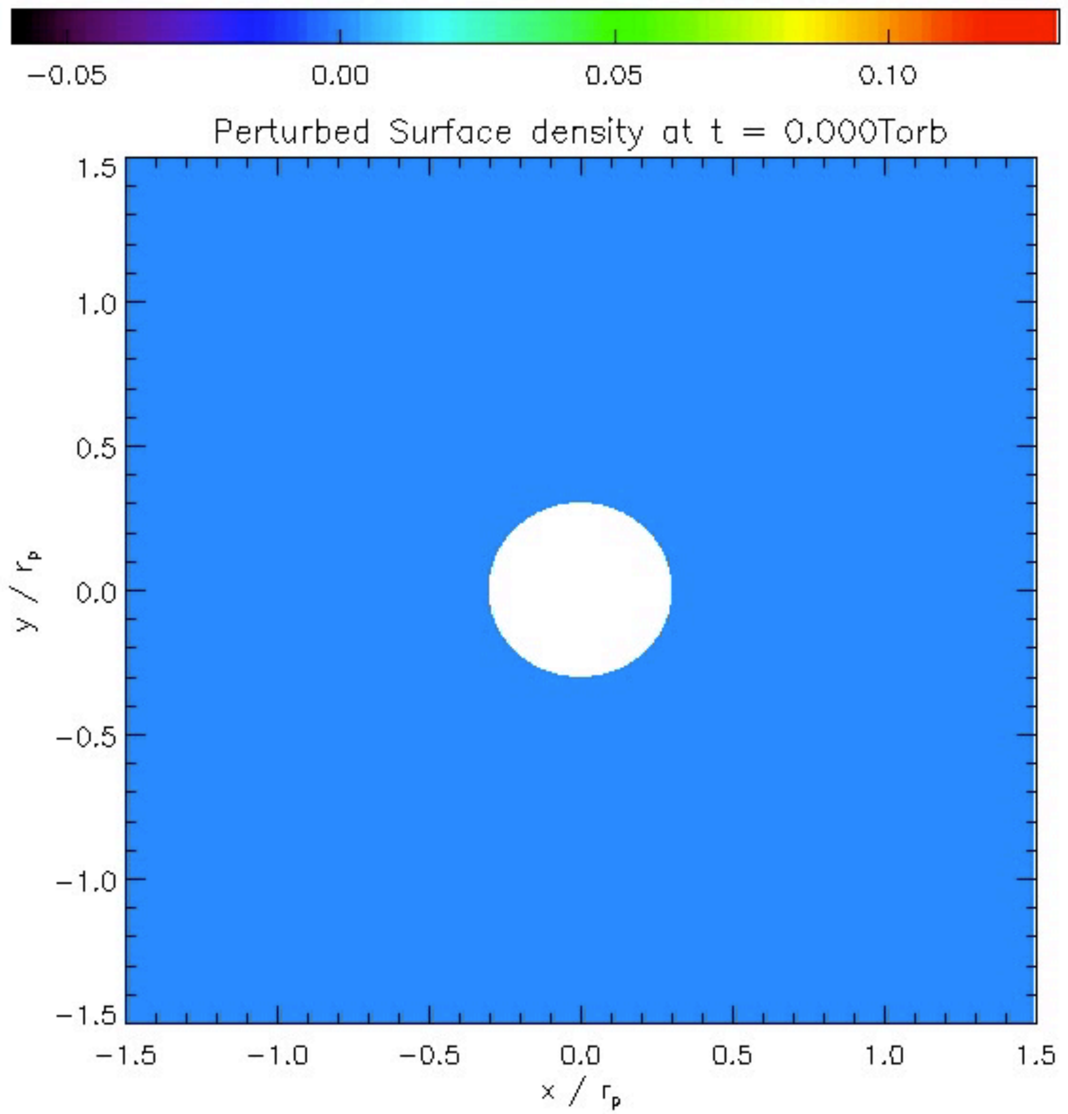




- Most of the torque arises from a region close to the planet
  - For axisymmetric density, the torque cancels

# A planet in a disk



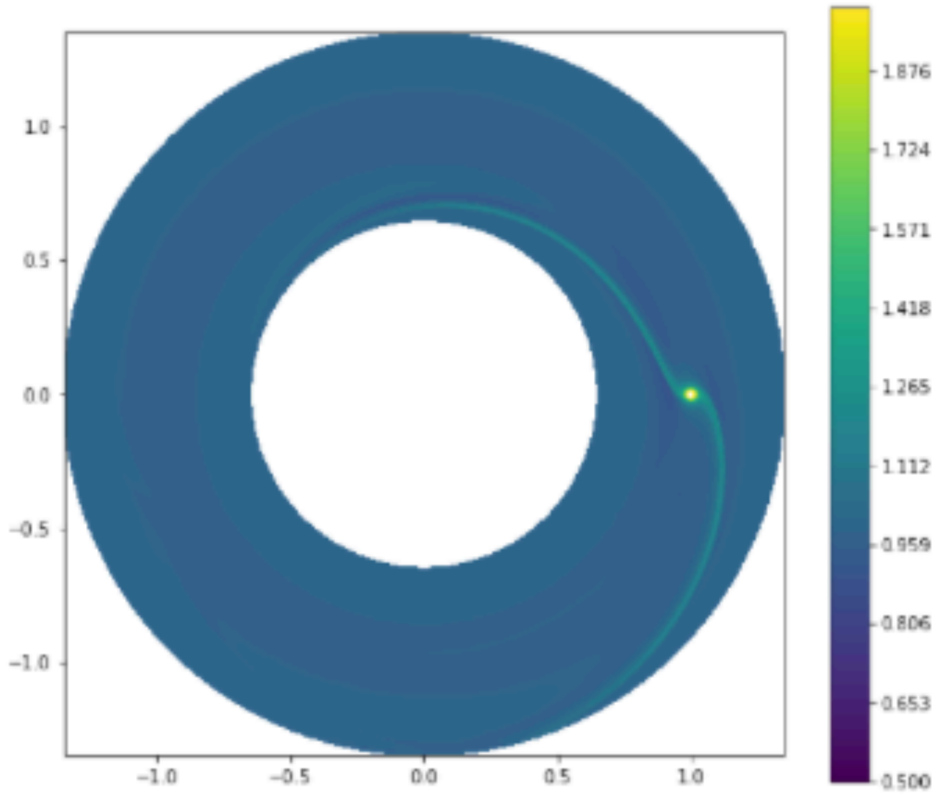
# The planet's wake in the disk: Corotating frame



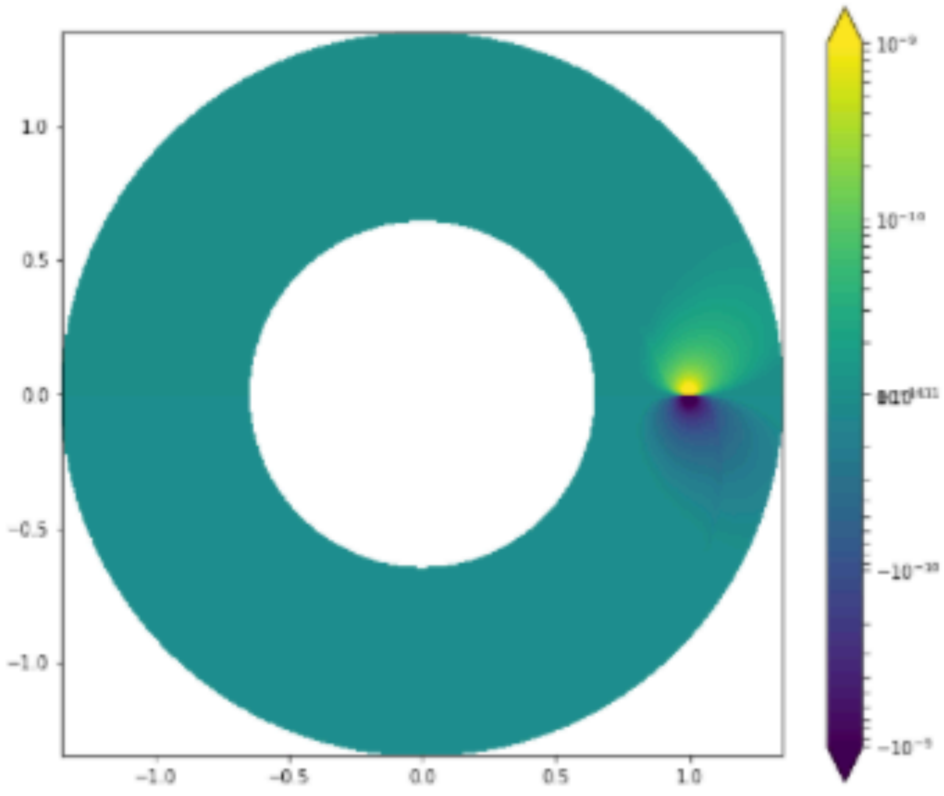
# Migration Torques

$$\boldsymbol{\Gamma} = r_p G m_p \iint \Sigma \frac{r^2}{|\boldsymbol{r} - \boldsymbol{r_p}|^3} \sin\phi \, dr d\phi$$

Density



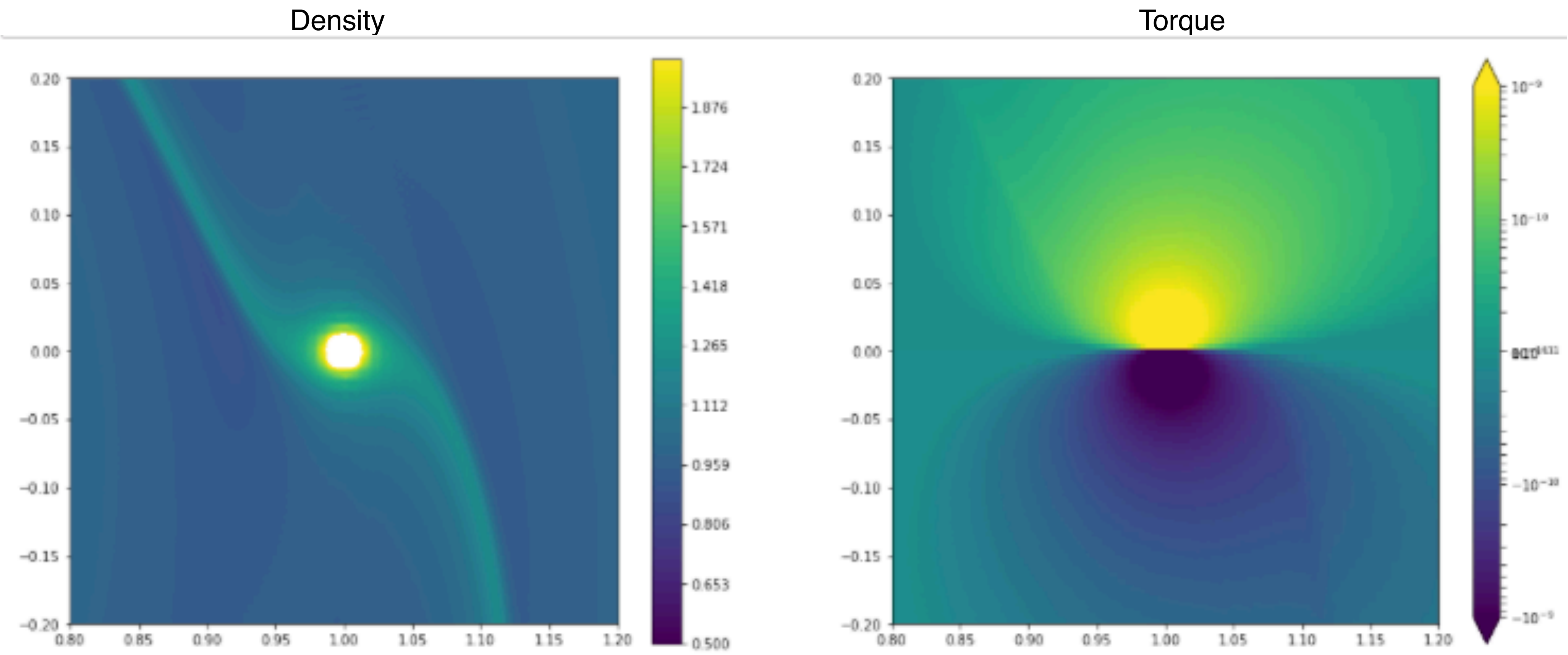

Torque




- The planet generates a non-axisymmetric wake
  - Non-zero torque

# Migration Torques

$$\boldsymbol{\Gamma} = r_p G m_p \iint \Sigma \frac{r^2}{|\boldsymbol{r} - \boldsymbol{r_p}|^3} \sin\phi \, dr d\phi$$

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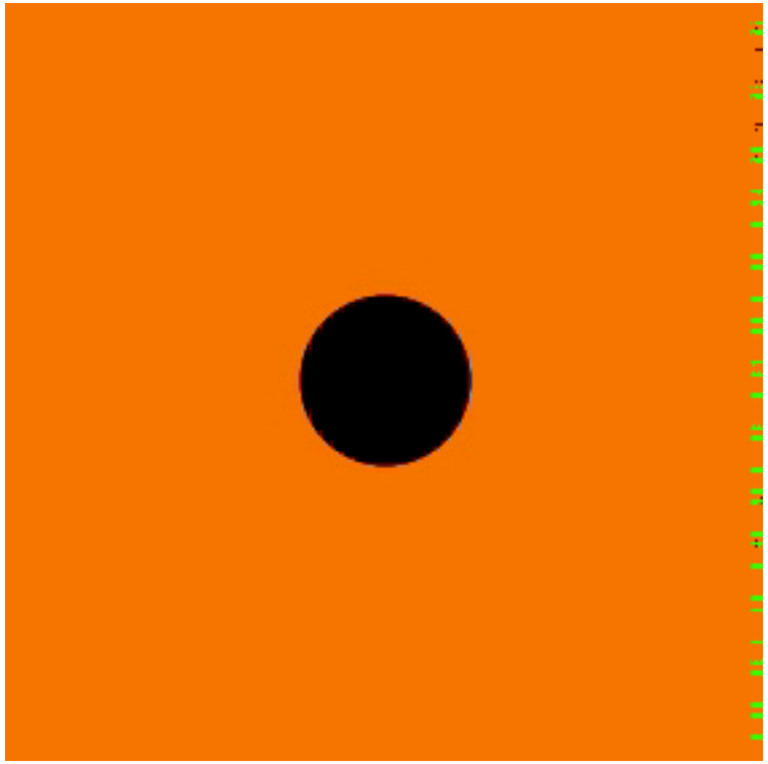
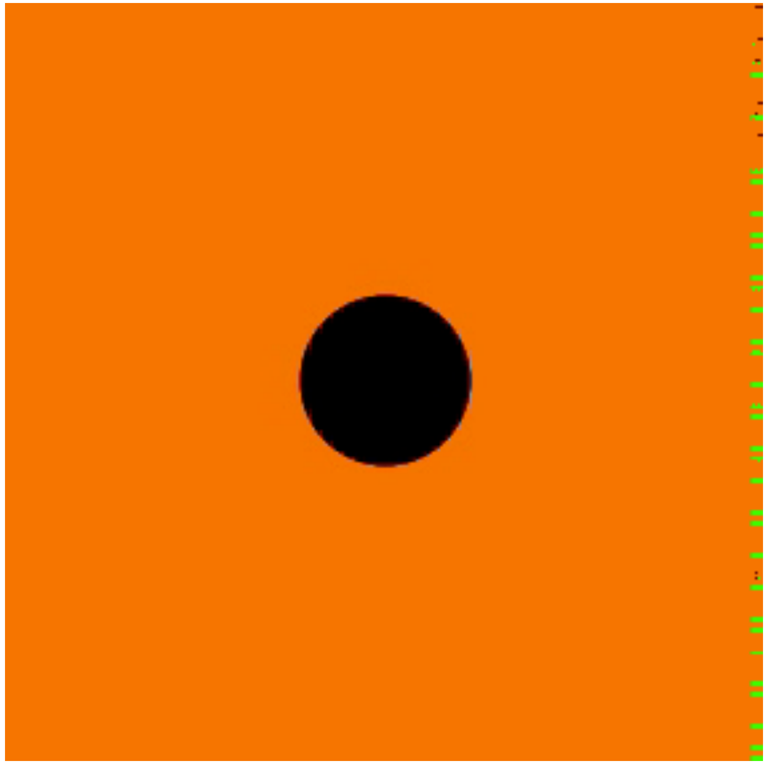
# Migration

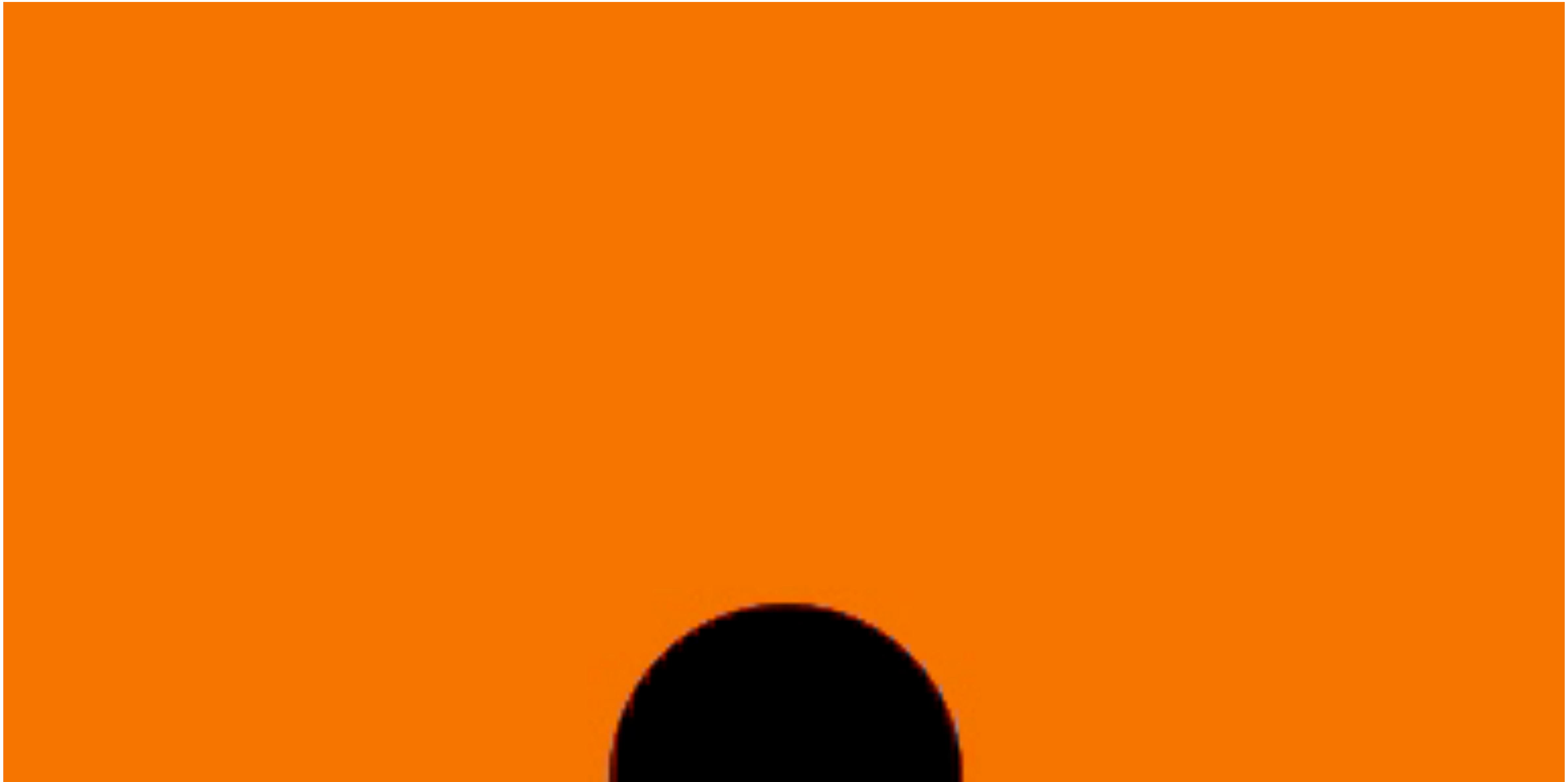
## Inertial frame



## Corotating frame



# Migration



# Two main ways to calculate torque:

1. Follow gas packets *in time*, and see how they exchange angular momentum with the planet.
   – Impulse approximation

2. Analyse how *azimuthal asymmetries* in the steady-state gas distribution in the disk $\Sigma(r,\phi)$ gravitationally pull on the planet.

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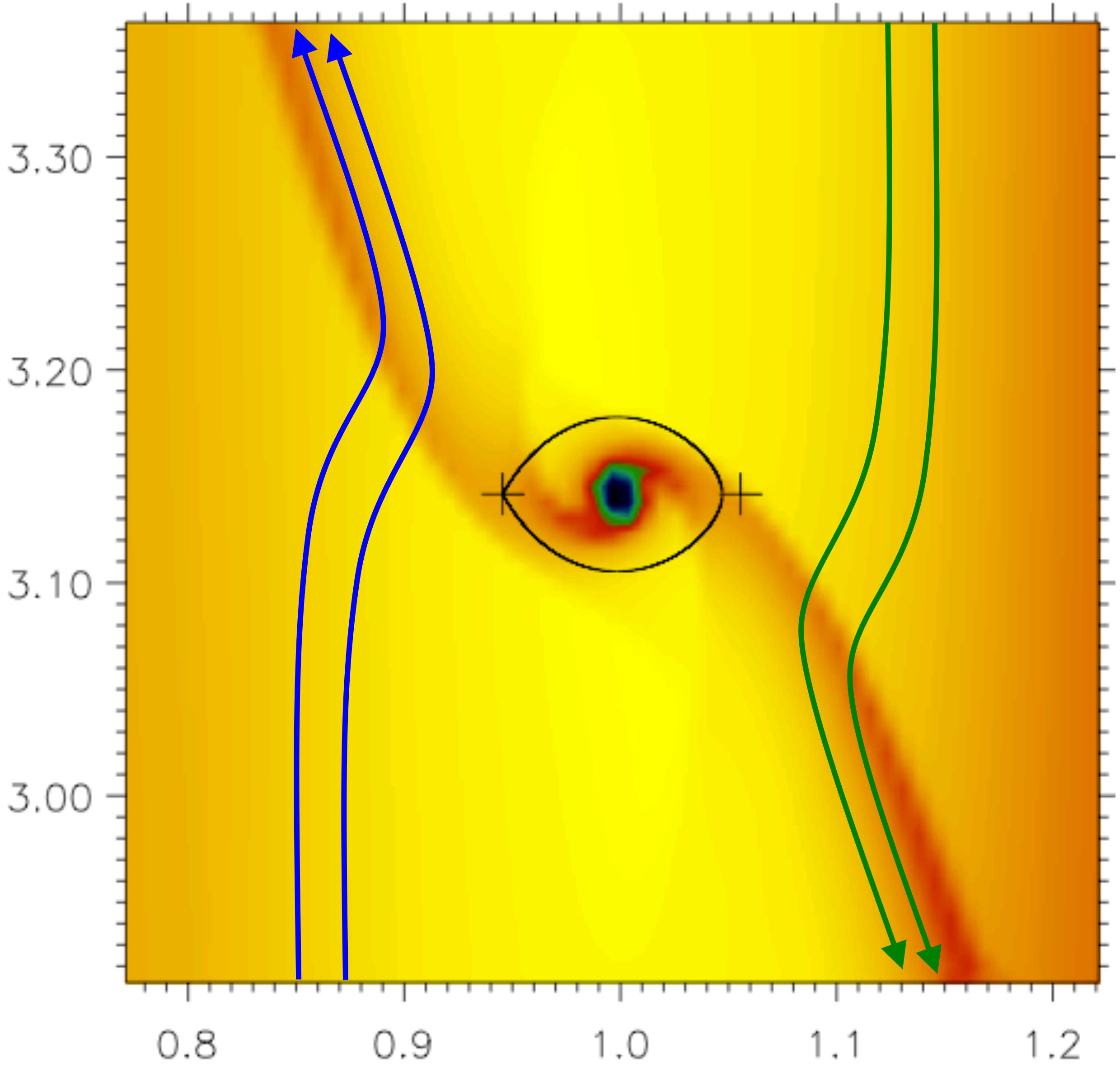
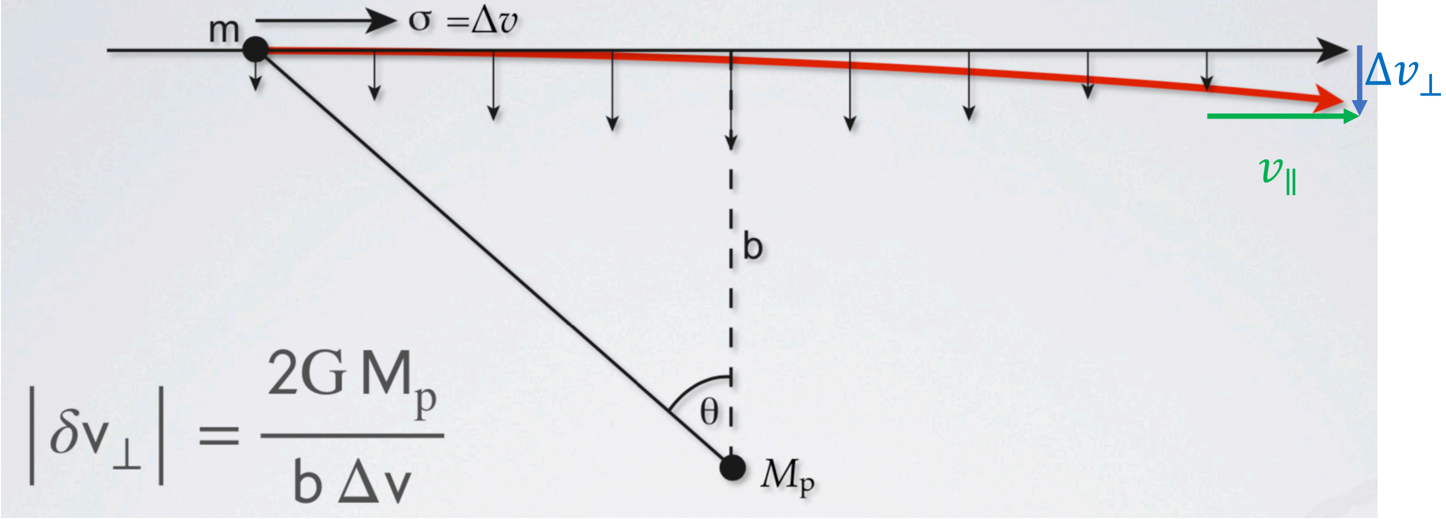


# Migration: Impulse approximation




Energy conservation before and after encounter

$$\Delta v^2 = \Delta v_\perp^2 + (\Delta v - \Delta v_\parallel)^2$$

$\Delta v_\perp$ does not change angular momentum

Solve for $\Delta v_\parallel$

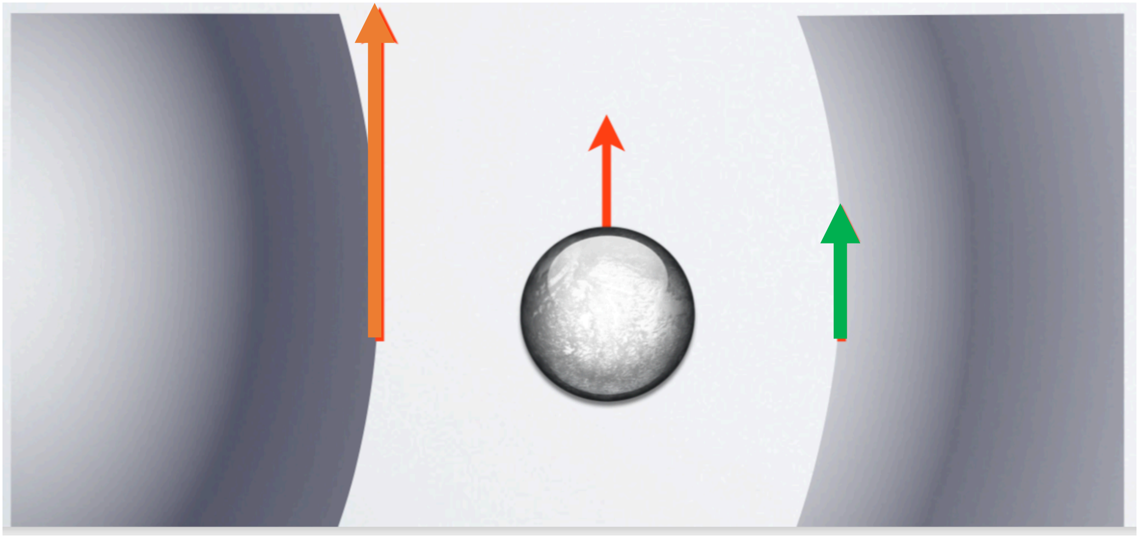
$$\Delta v_\parallel = \frac{1}{2\Delta v}\left(\frac{2GM_p}{b\Delta v}\right)^2$$

**Angular momentum change**

$$\Delta j = a \times \Delta v = a\Delta v_\parallel = \frac{2G^2M_p^2}{b^2\Delta v^3}a$$

# Sign of the torque

$$\Delta j = \frac{2G^2 M_p^2}{b^2 \Delta v^3} a$$



$$v_k = \sqrt{\frac{GM_*}{a}}$$

$$j_k = \sqrt{GM_* a}$$

For gas interior to the planet orbit

- Increase in $\Delta v_\parallel$ means decrease in $j$
- *The gas loses angular momentum*
- *The planet gains angular momentum*
- Planet moves outwards
- Gas moves inwards (gas is repelled from the planet)

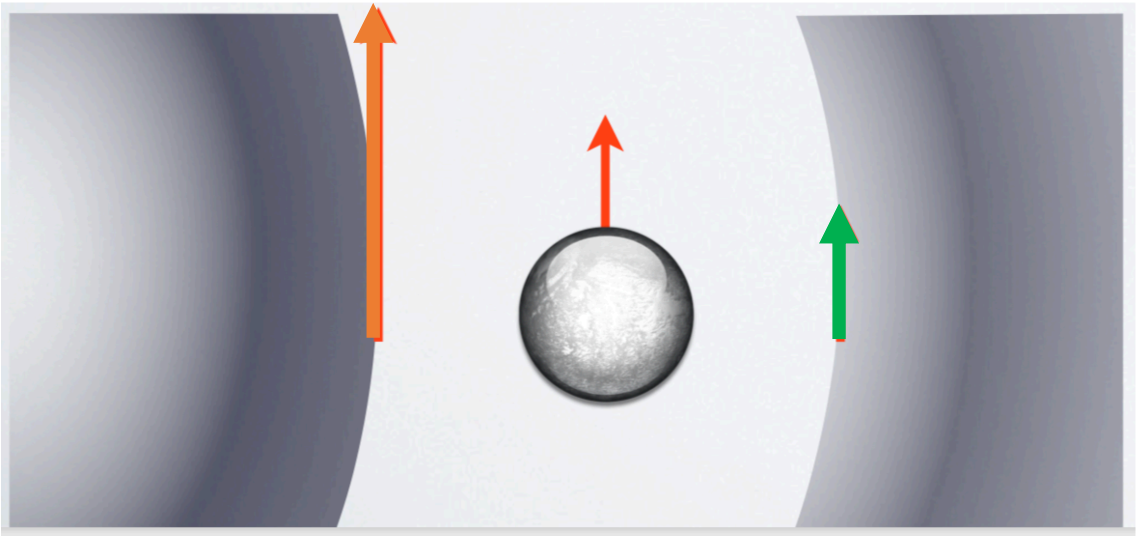
For gas exterior to the planet orbit

- Decrease in $\Delta v_\parallel$ means increase in $j$
- *The gas gains angular momentum*
- *The planet loses angular momentum*
- Planet moves inwards
- Gas moves outwards (gas is repelled from the planet)

# Total Torque

Sign and direction of migration
depends on difference between positive inner torque
and negative outer torque

$$\Delta j = \frac{2G^2M_p^2}{b^2\Delta v^3}a$$

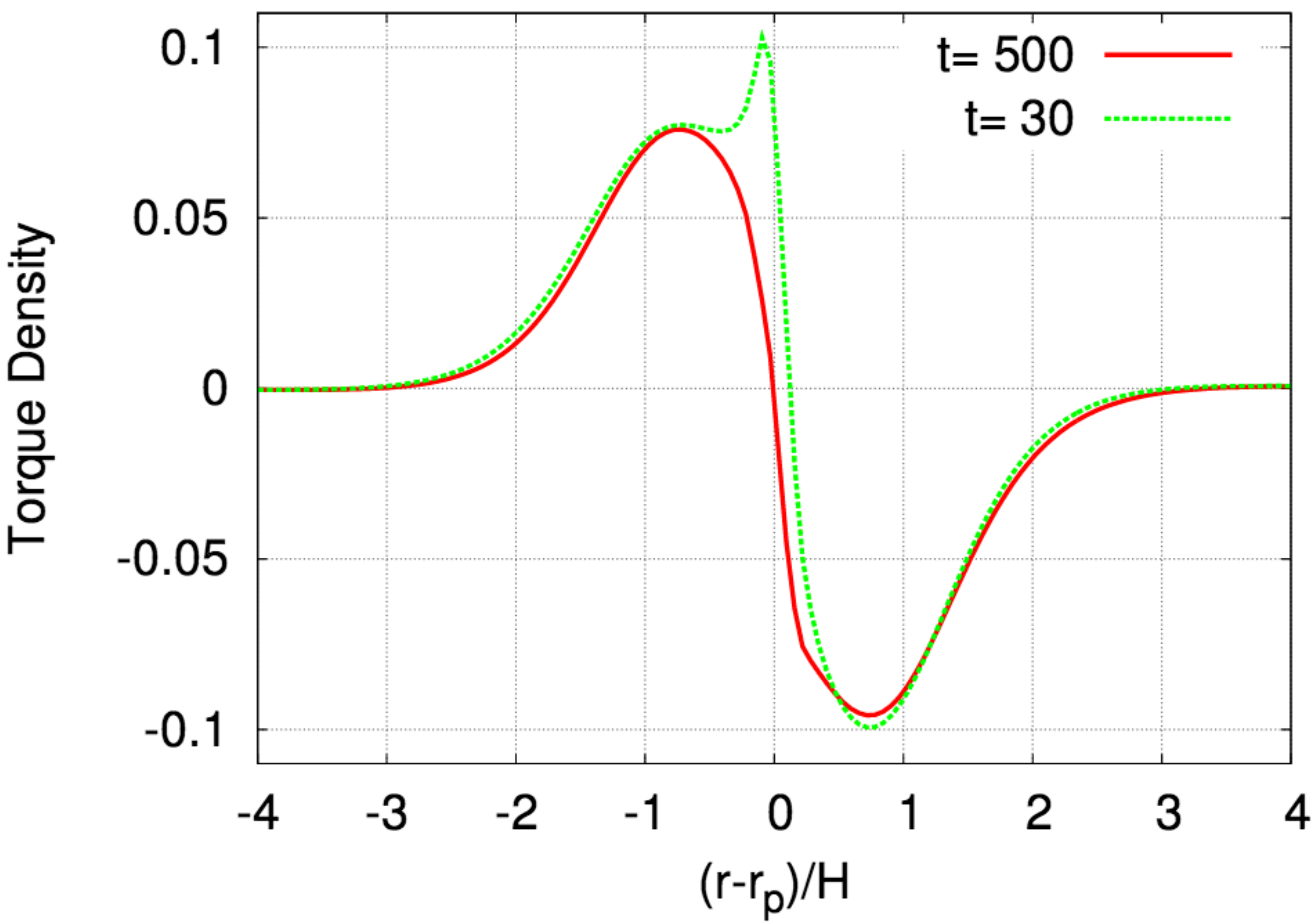


Integrate in impact parameter

$$\Gamma = \frac{dJ}{dt} = -\frac{8}{27}\frac{G^2M_p^2}{\Omega_p^2 b_{min}^3}a\Sigma$$

Proportional to $M_p^2$

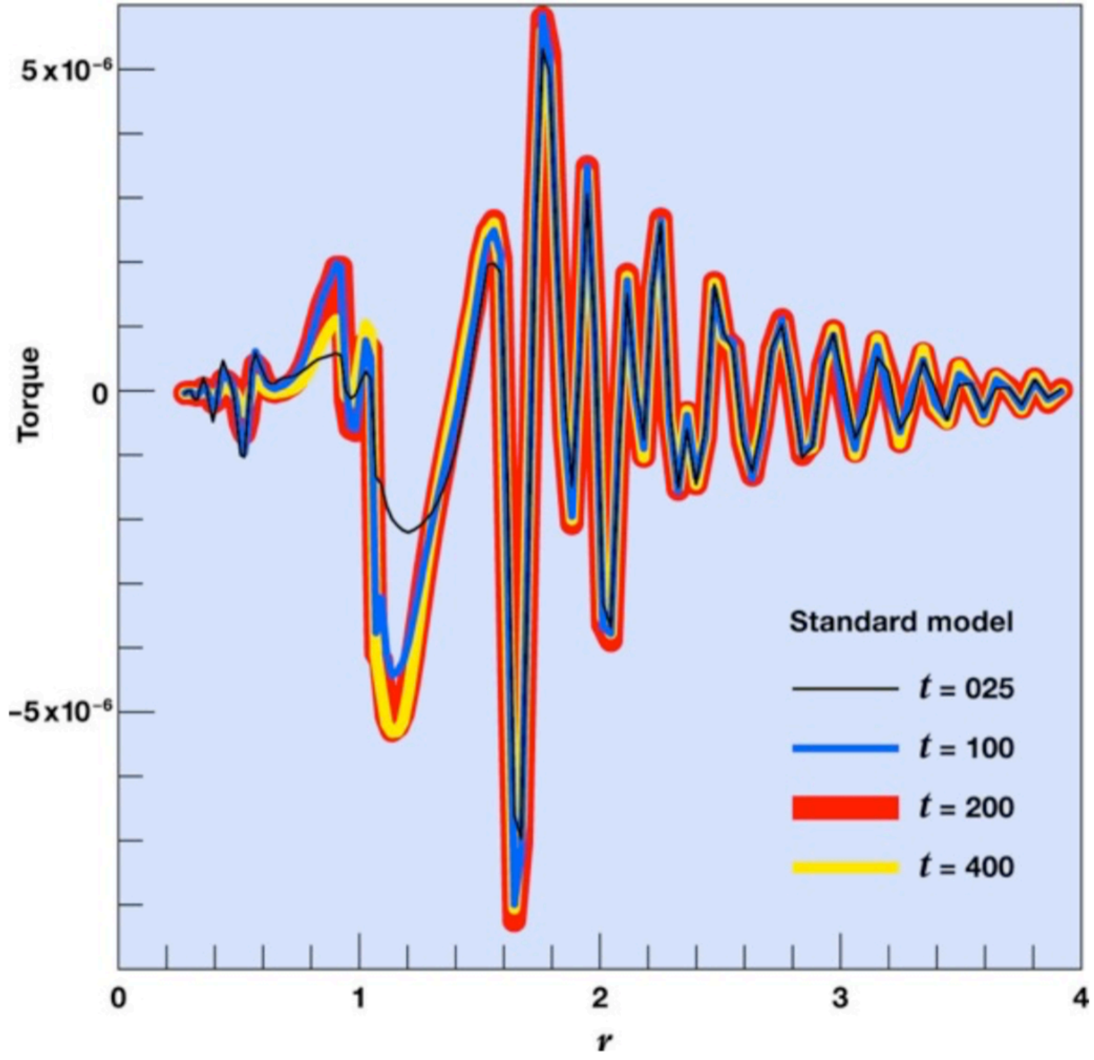
$$\tau_{mig} = \left(\frac{1}{J}\frac{dJ}{dt}\right)^{-1} \sim 1\,\text{Myr}$$



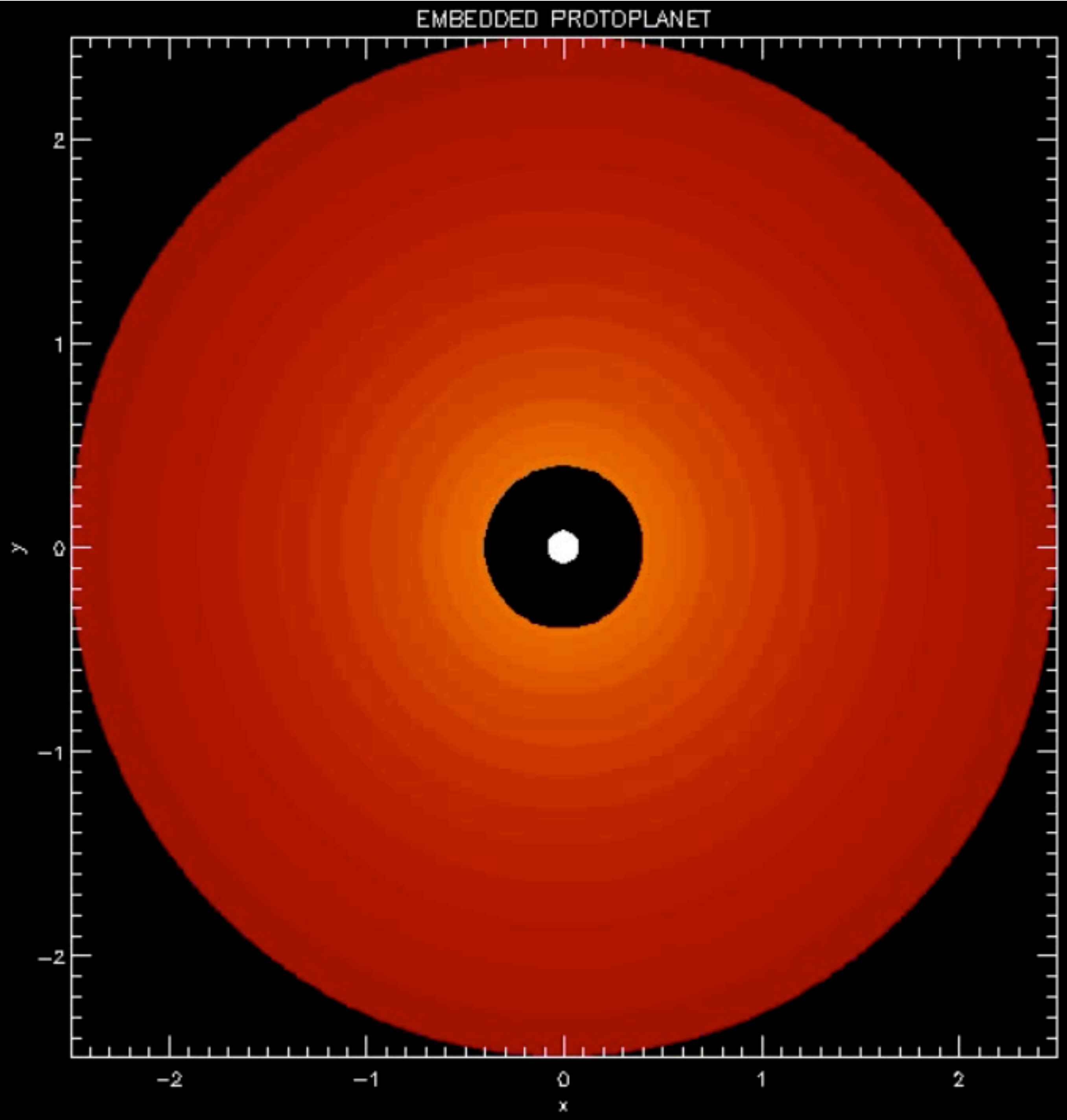
t= 500
t= 30
0.1
0.05
0
-0.05
-0.1
Torque Density
-4
-3
-2
-1
0
1
2
3
4
(r-r_p)/H

# *Net torque*

The net torque is the sum of all the torques. For most of the disk structures, this net torque is such that it induces inward migration.




W. Kley



EMBEDDED PROTOPLANET
2
1
0
−1
−2
y
−2
−1
0
1
2
x

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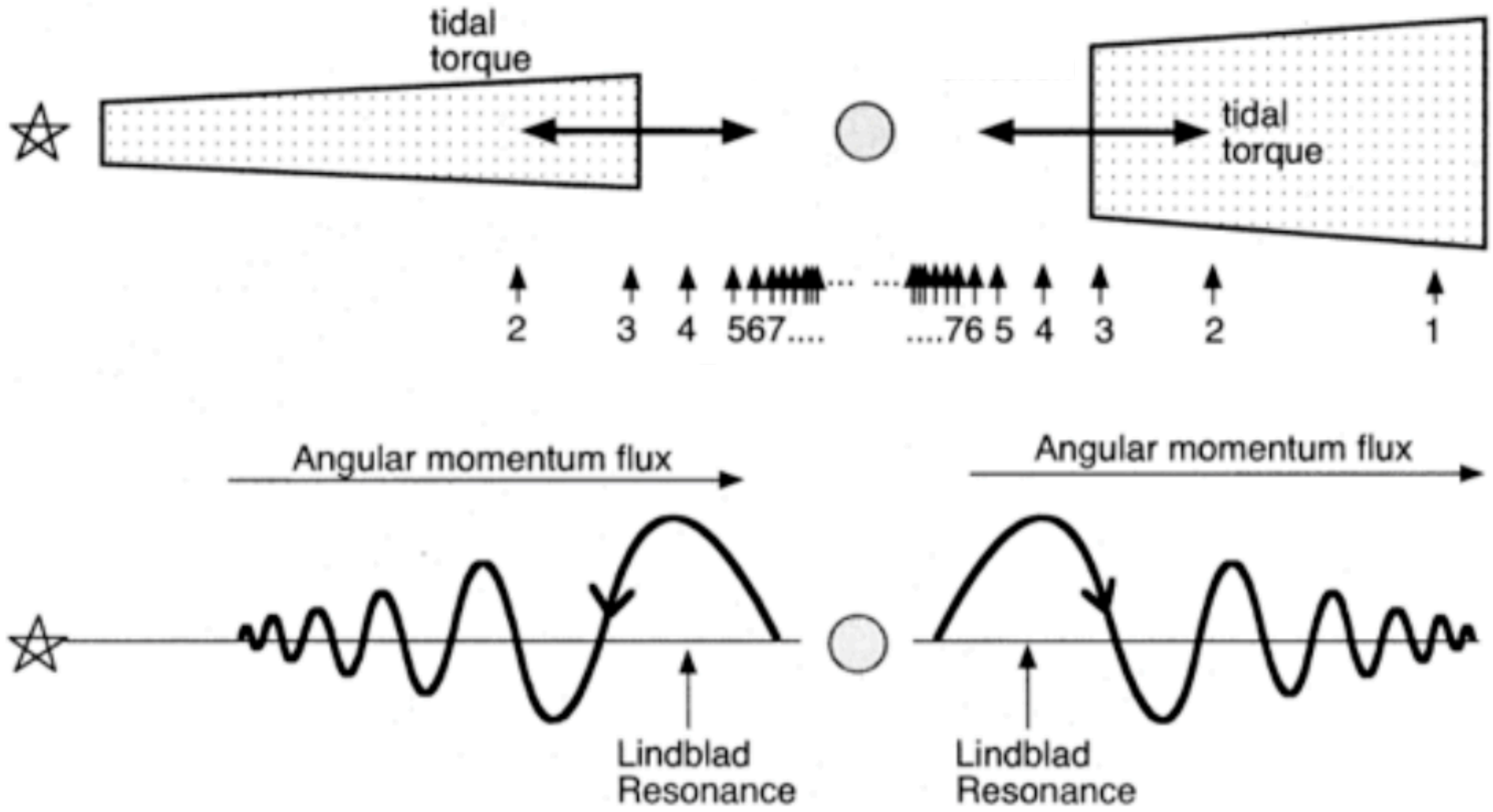
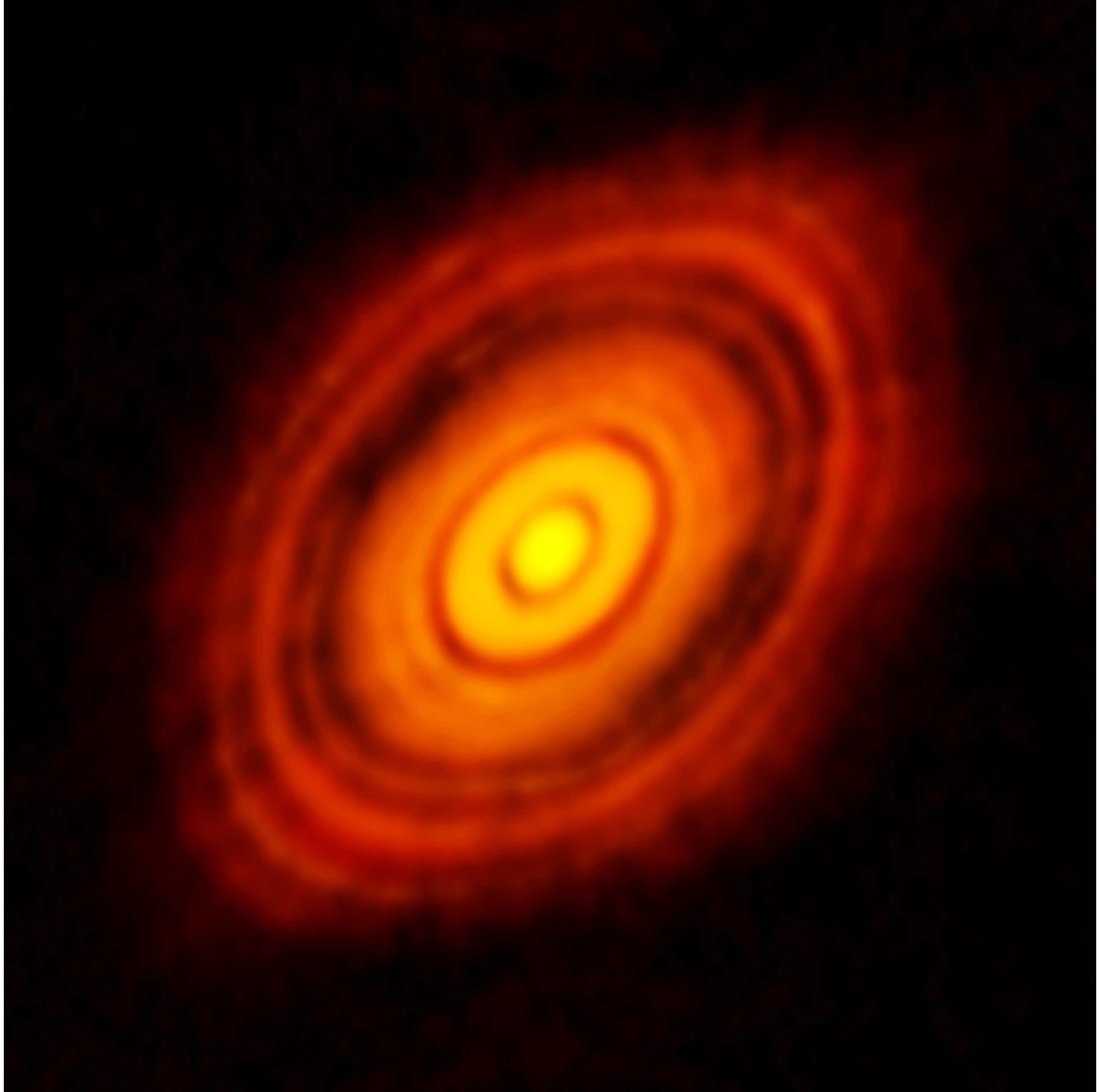
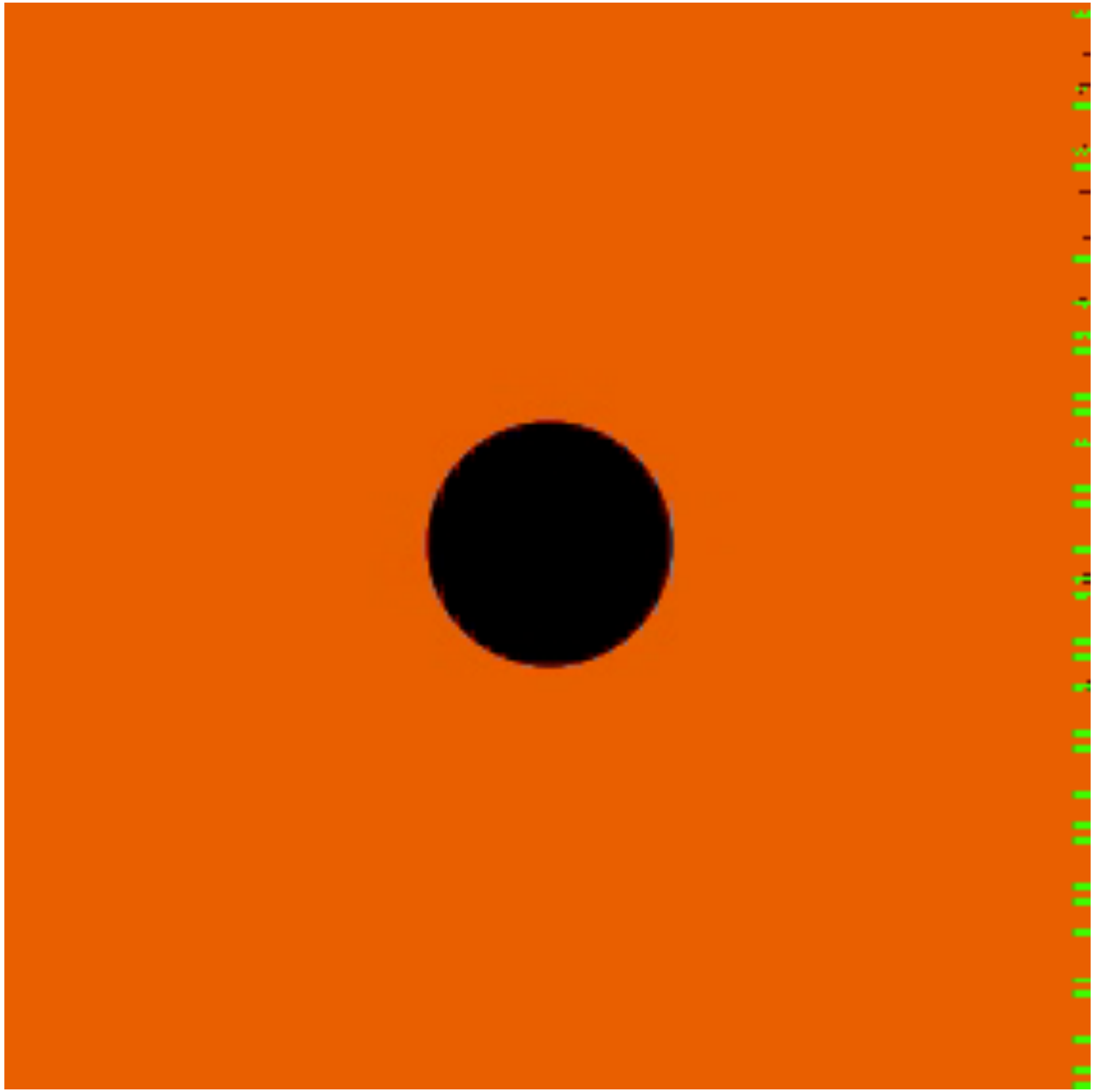
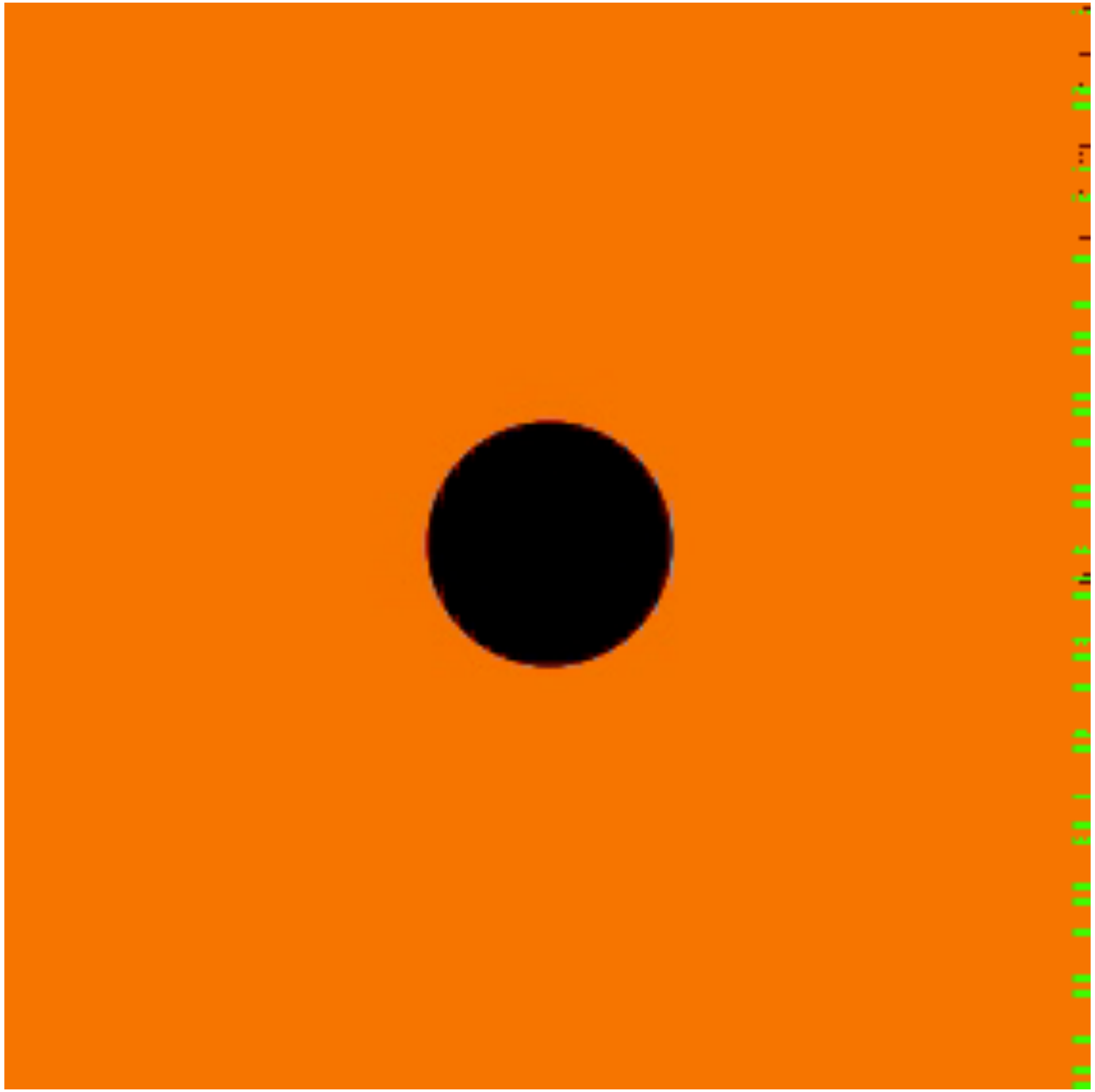

tidal
torque
tidal
torque
2
3
4
567....
....76
5
4
3
2
1
Angular momentum flux
Angular momentum flux
Lindblad
Resonance
Lindblad
Resonance



# Gap Opening



# Type I to Type II transition



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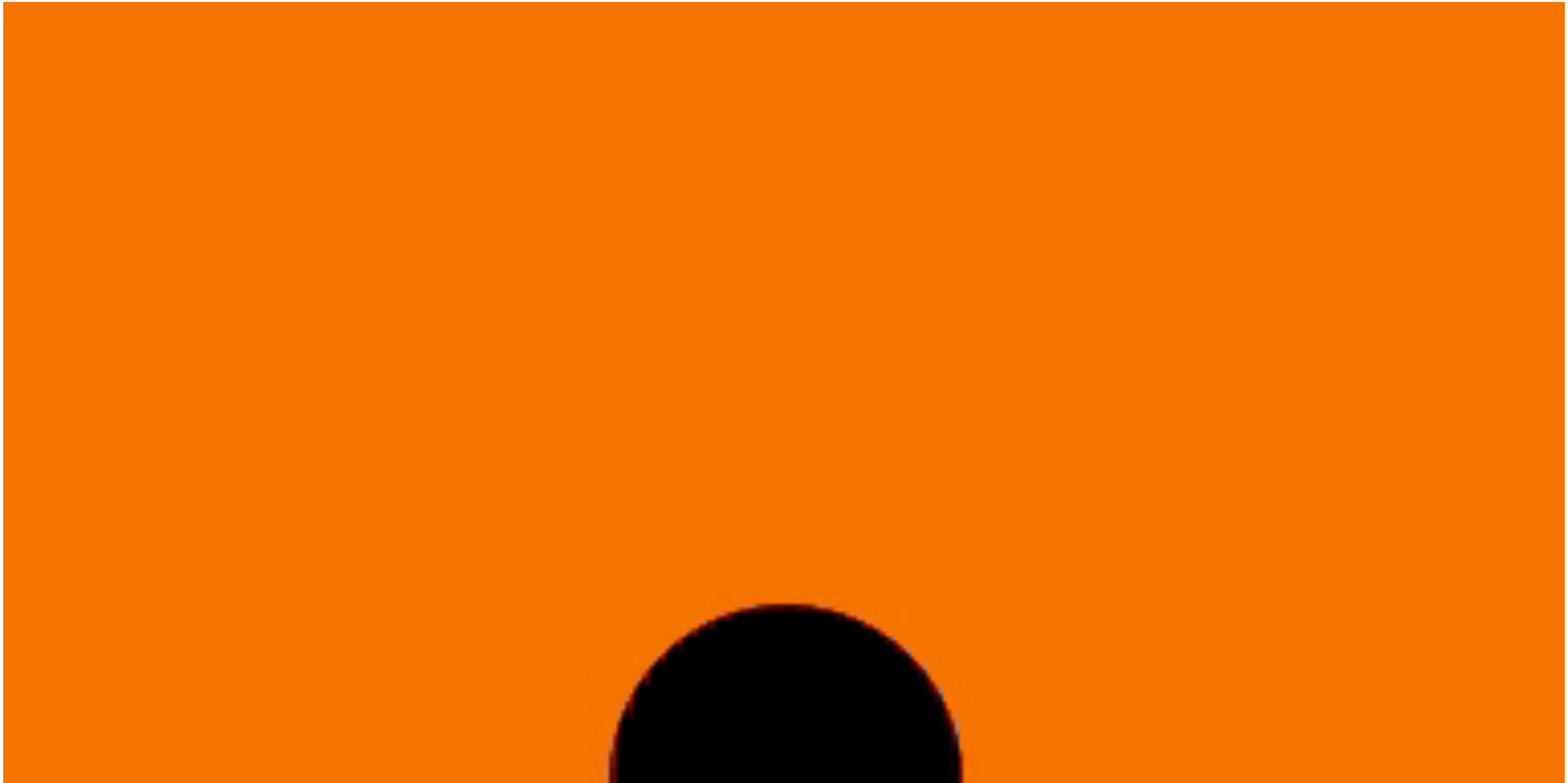
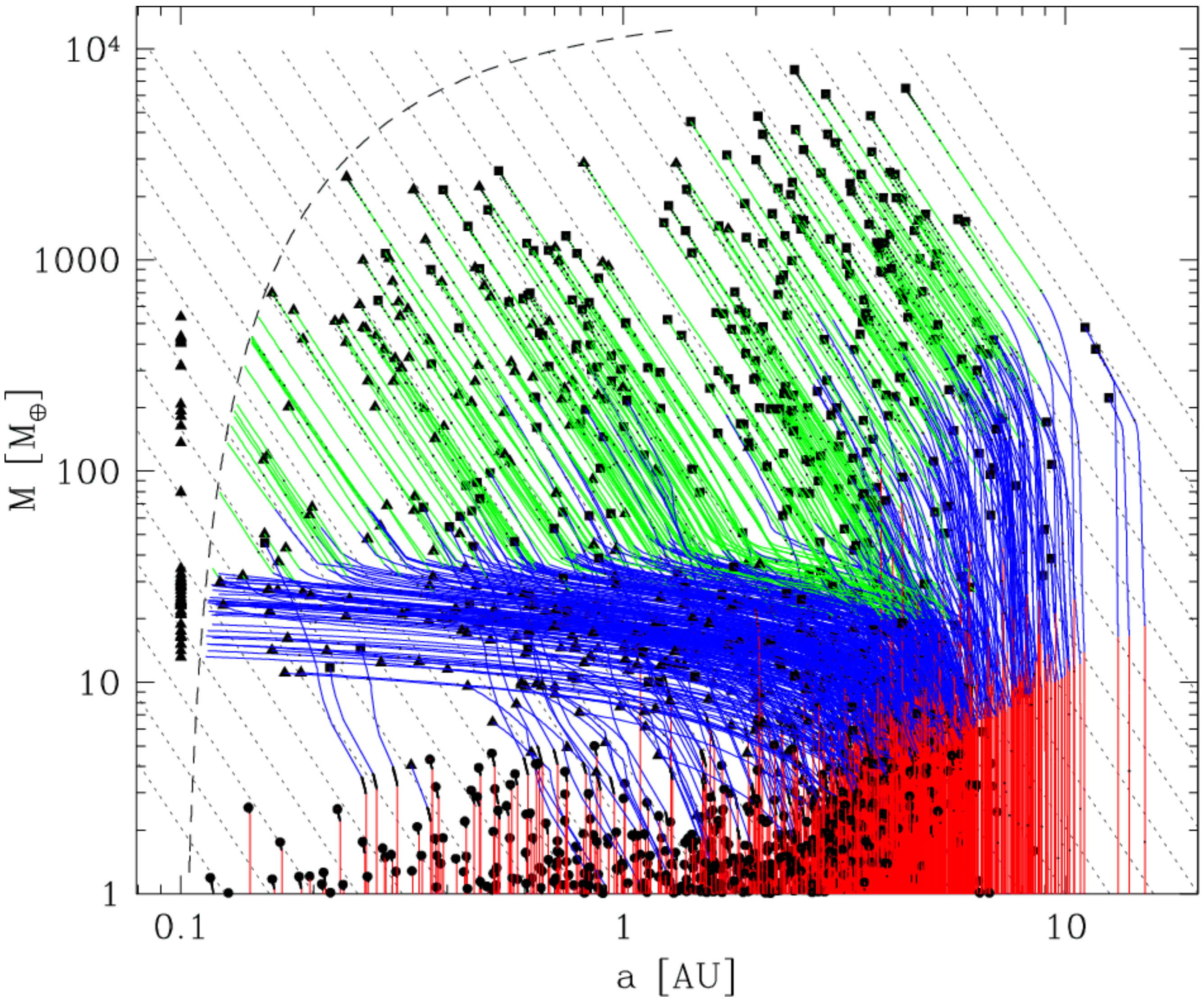
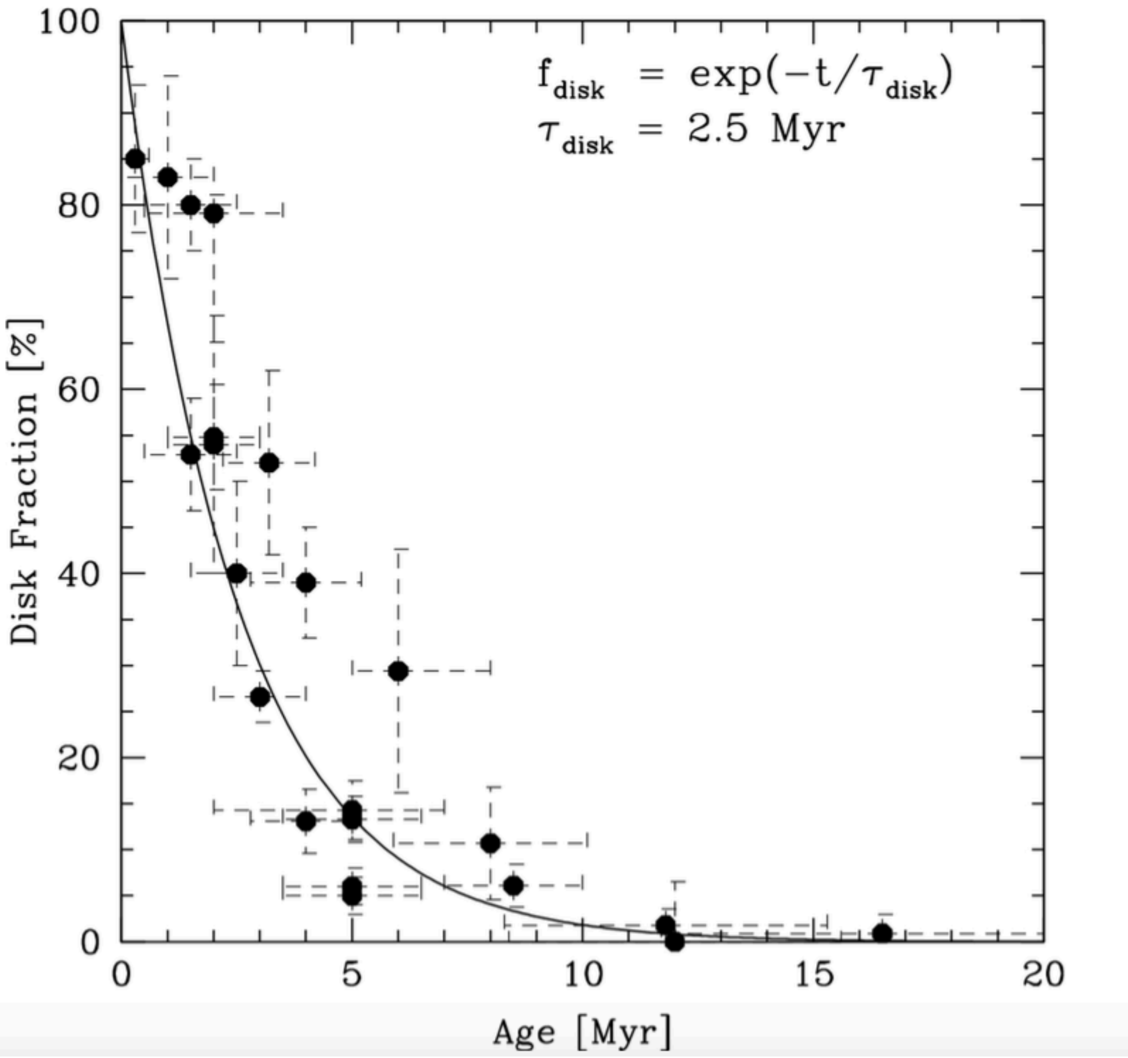


# Planet Population Synthesis



# Disk lifetime




Disks dissipate with an e-folding time of 2.5 Myr