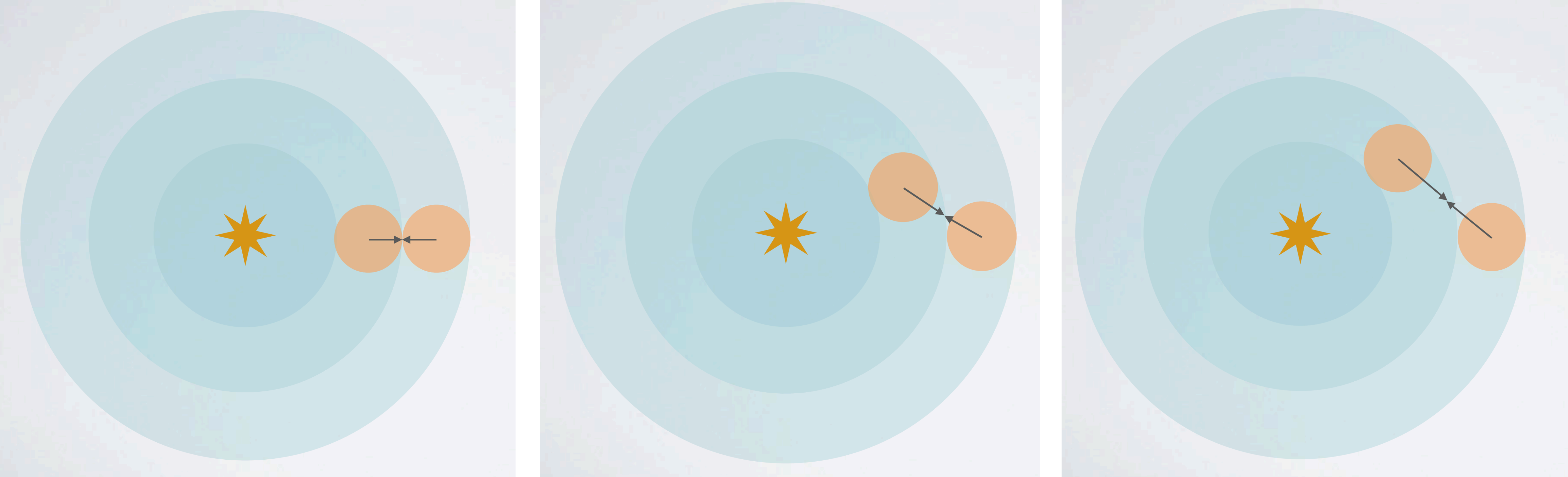
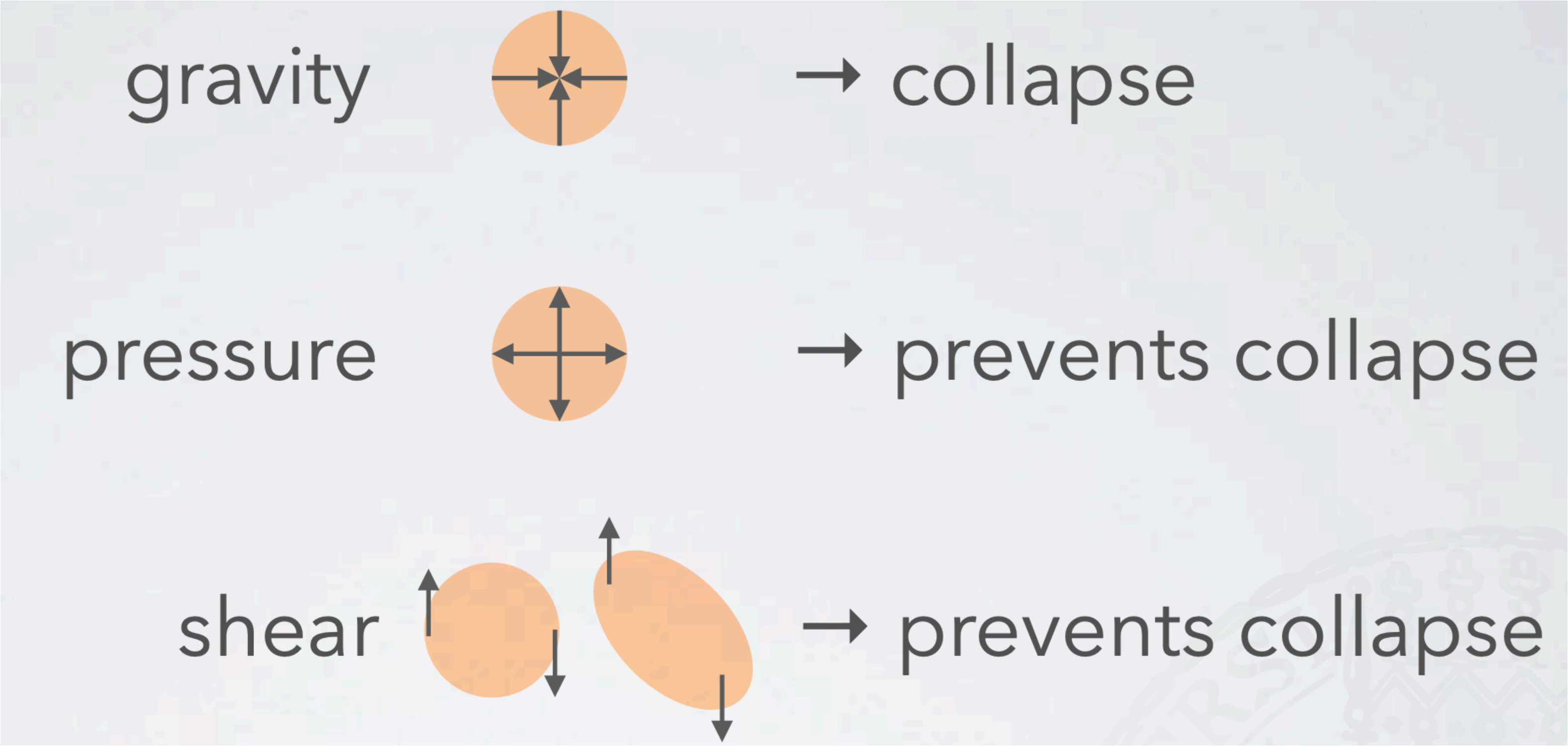
Class 23 – Apr 23rd, 2020

Self gravity



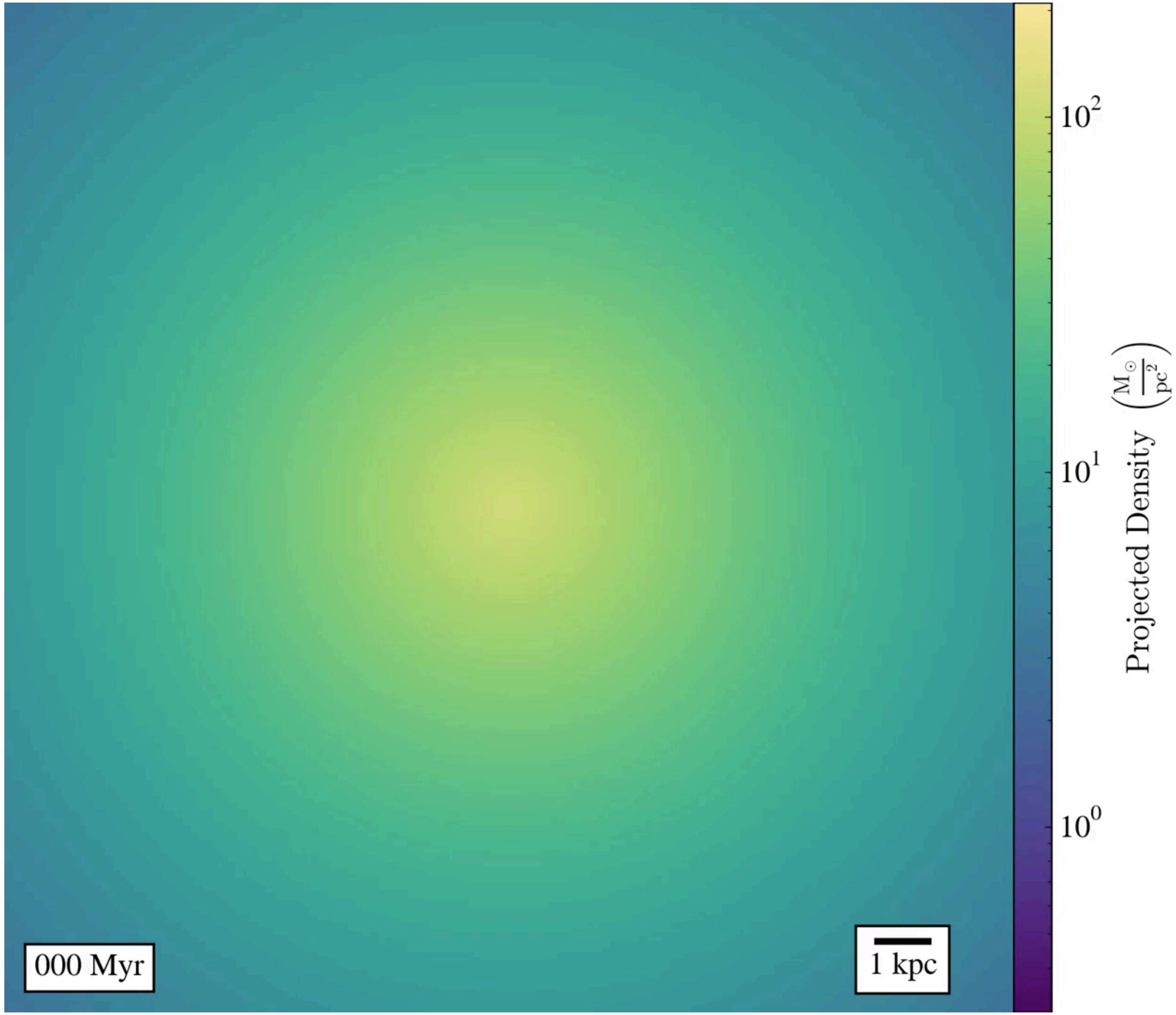
Self gravity

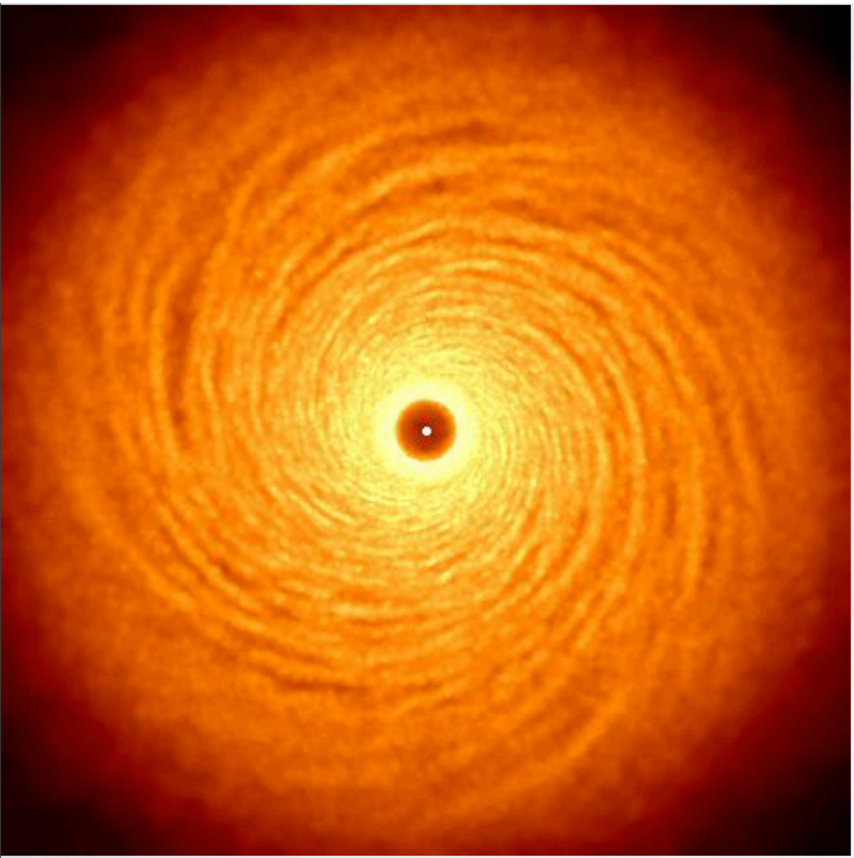


gravity → collapse

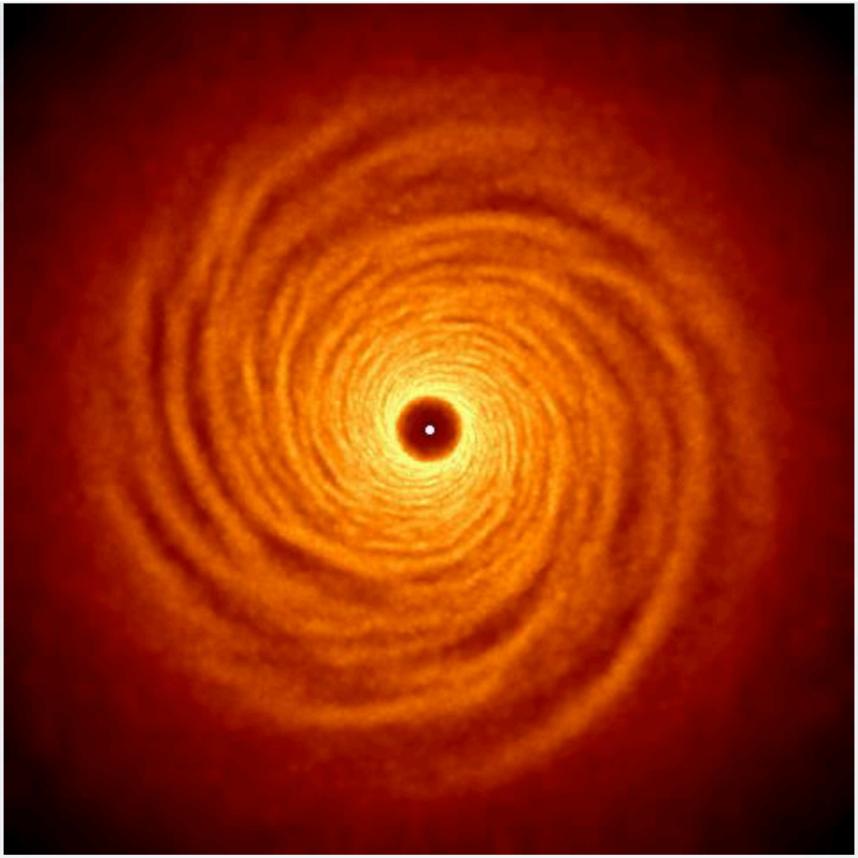
pressure → prevents collapse

shear → prevents collapse

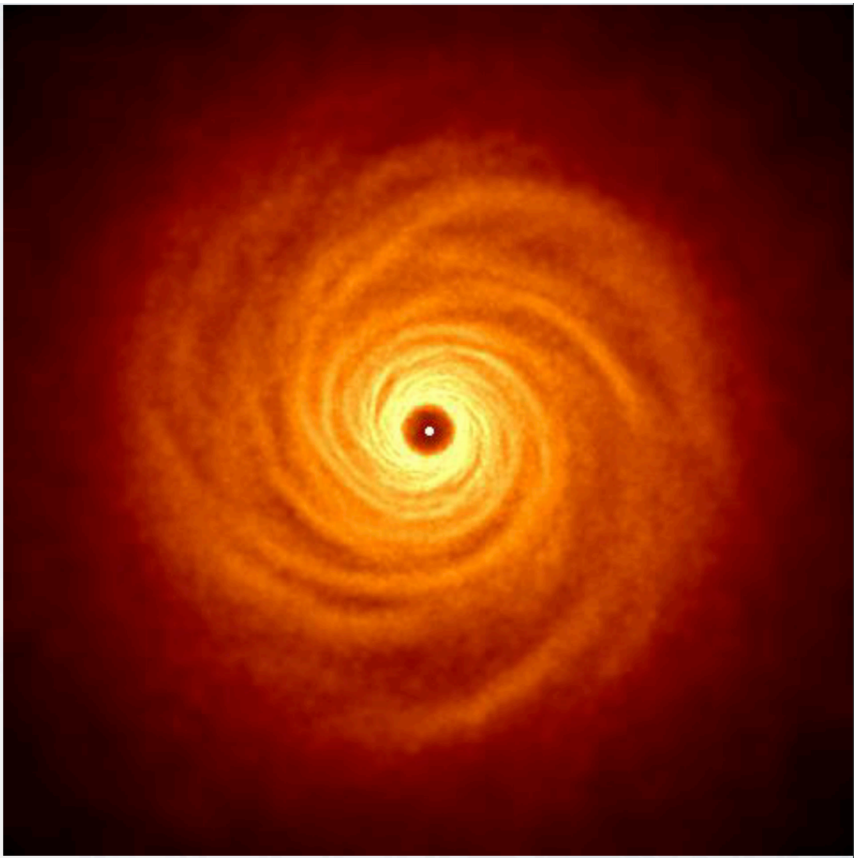

000 Myr
1 kpc
10^2
10^1
10^0
Projected Density $\left(\frac{M_\odot}{pc^2}\right)$



$M_{\mathrm{disk}}/M_\star = 0.05$

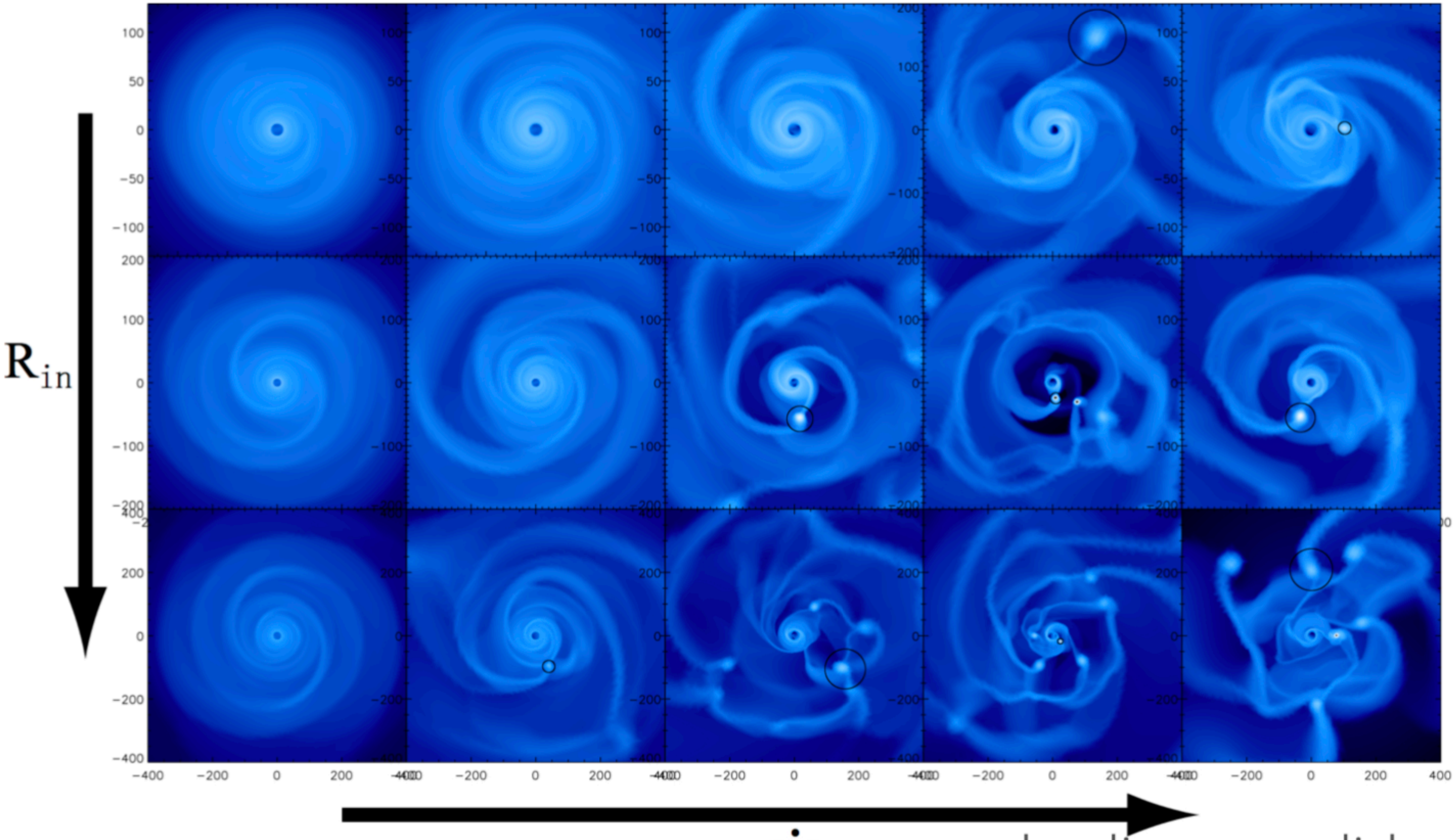


$M_{\mathrm{disk}}/M_\star = 0.1$



$M_{\mathrm{disk}}/M_\star = 0.25$

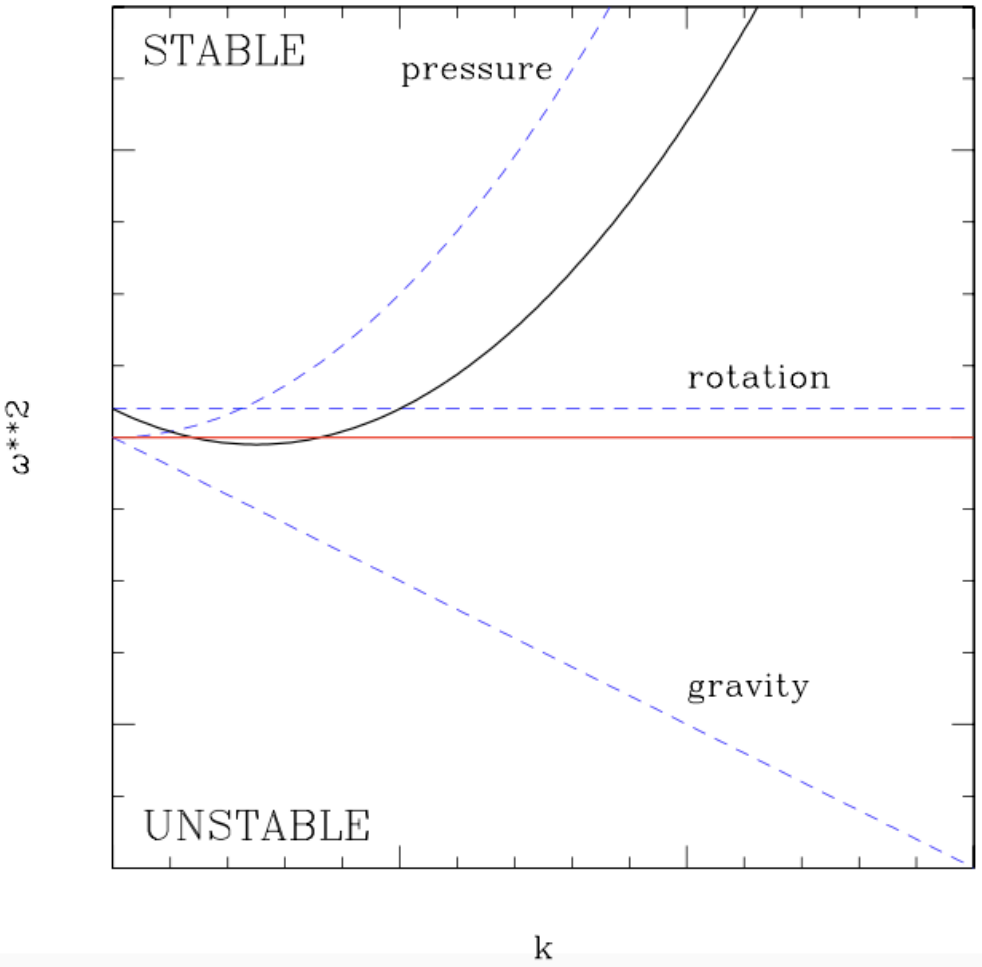
– Disk Irradiation and Clump Migration, Accretion, and Tidal Destruction –



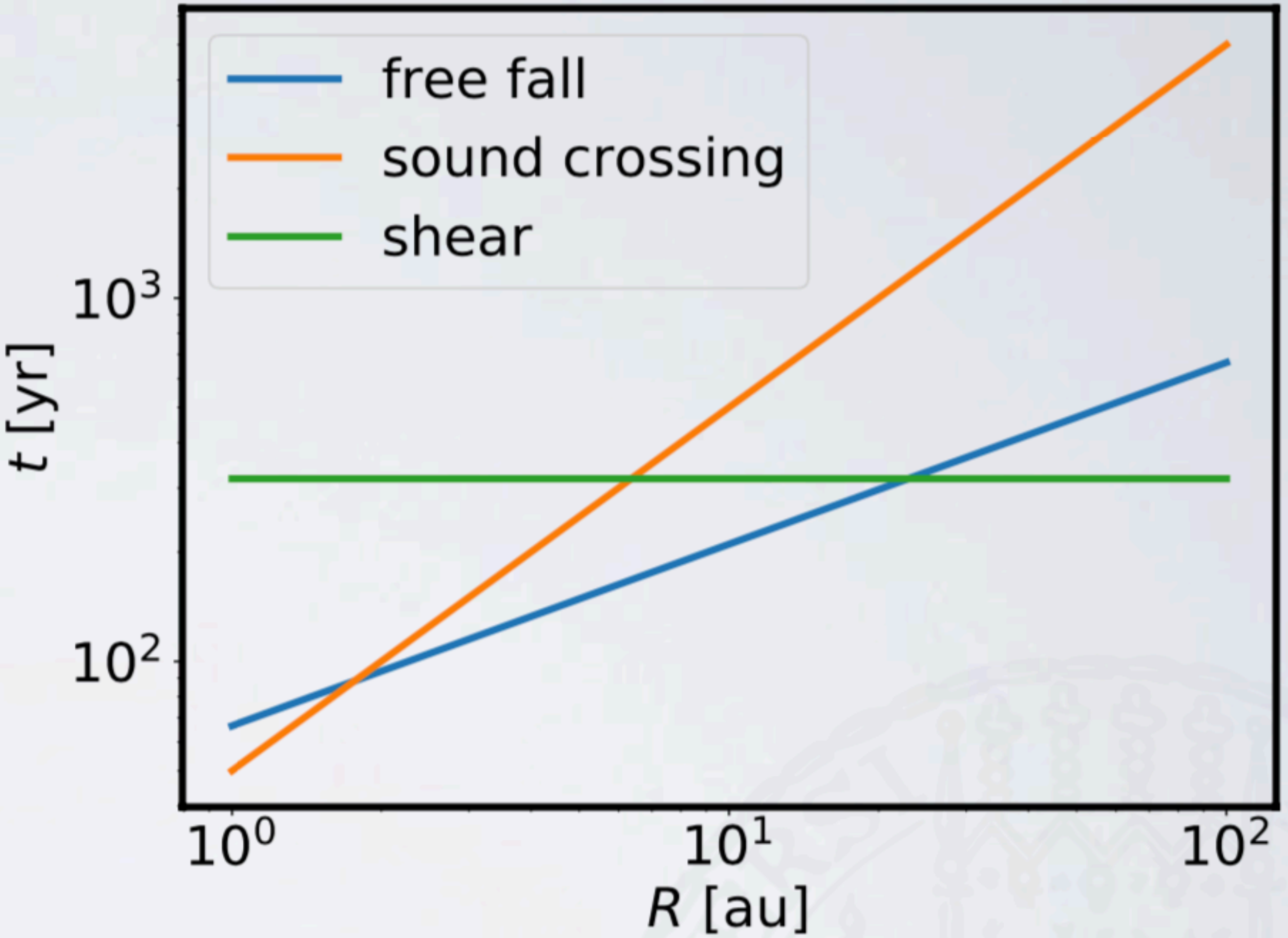
$\dot{M}_{in}$= mass loading onto disk

FIG. 1.— The disk surface density distribution at the end of the simulations (time shown in Table 1) with different infall rates (increasing from $3\times10^{-6}M_\odot\ yr^{-1}$ on the left to $3\times10^{-4}M_\odot\ yr^{-1}$ on the right) and infall radii (65 AU, 100 AU, 200 AU from the top to bottom). The black circle labels the Hill radius of the selected clump if the disk fragments.

Gravitational Instability



$$\begin{array}{c} t_{\mathrm{ff}} \\ \uparrow \\ \pi\sqrt{\dfrac{R^3}{8\,\mathrm{G}\,M}} \end{array} < \begin{cases} t_{\mathrm{sound}} = \dfrac{2\,R}{c_{\mathrm{s}}} \\ t_{\mathrm{shear}} = \dfrac{2}{\Omega} \end{cases}$$

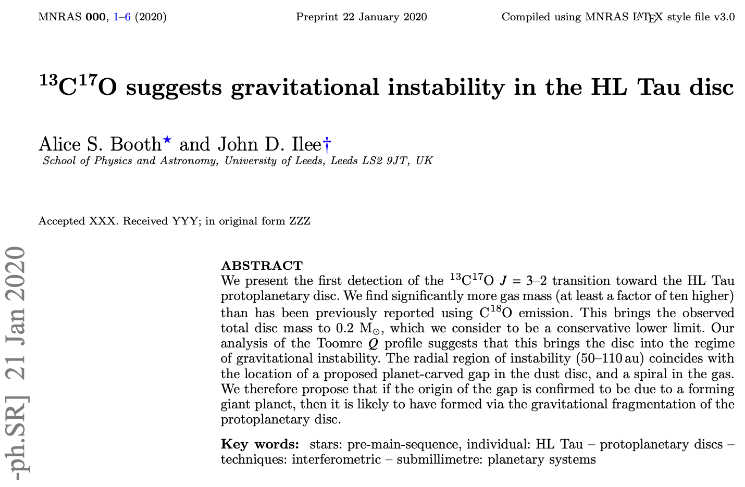


$$Q \lesssim 1 \Leftrightarrow \frac{M_{\mathrm{disk}}}{M_\star} \gtrsim \frac{H}{r}$$

$$\Rightarrow Q = \frac{c_{\mathrm{s}}\,\Omega}{\Sigma\,\mathrm{G}\,\pi} \lesssim 1$$

massive disk needed, possibly present during Class 0 phase

Measured Q values?



MNRAS **000**, 1–6 (2020) Preprint 22 January 2020 Compiled using MNRAS LaTeX style file v3.0

$^{13}C^{17}O$ suggests gravitational instability in the HL Tau disc

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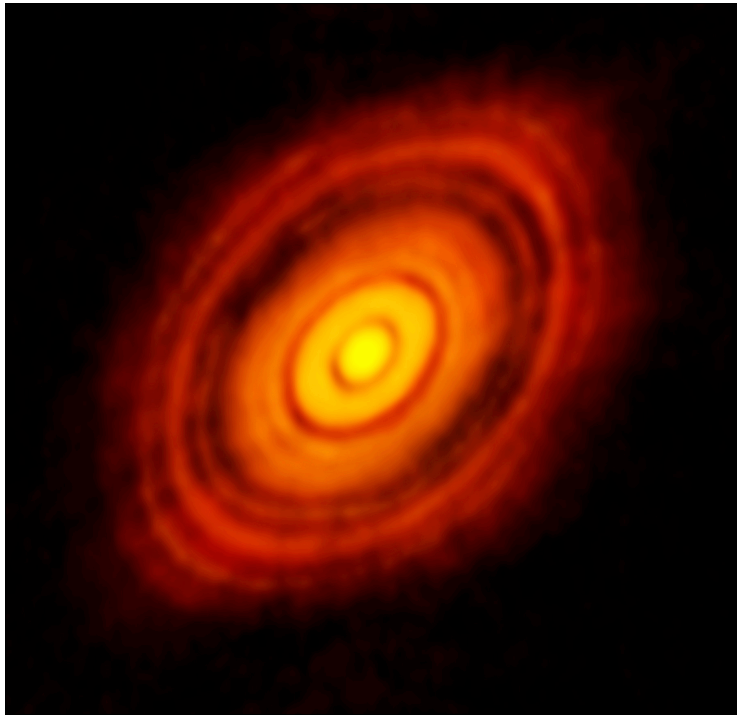
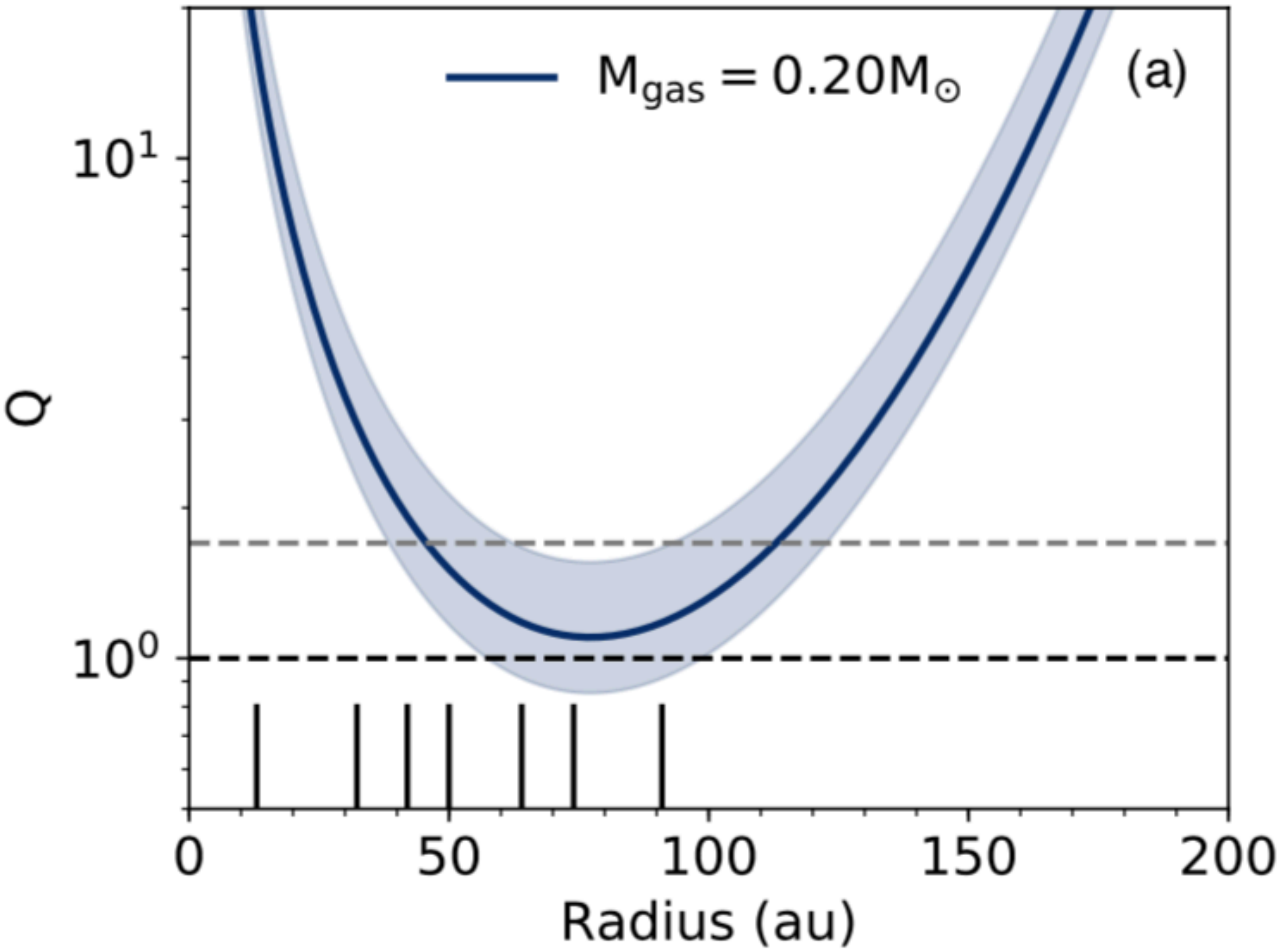
Accepted XXX. Received YYY; in original form ZZZ

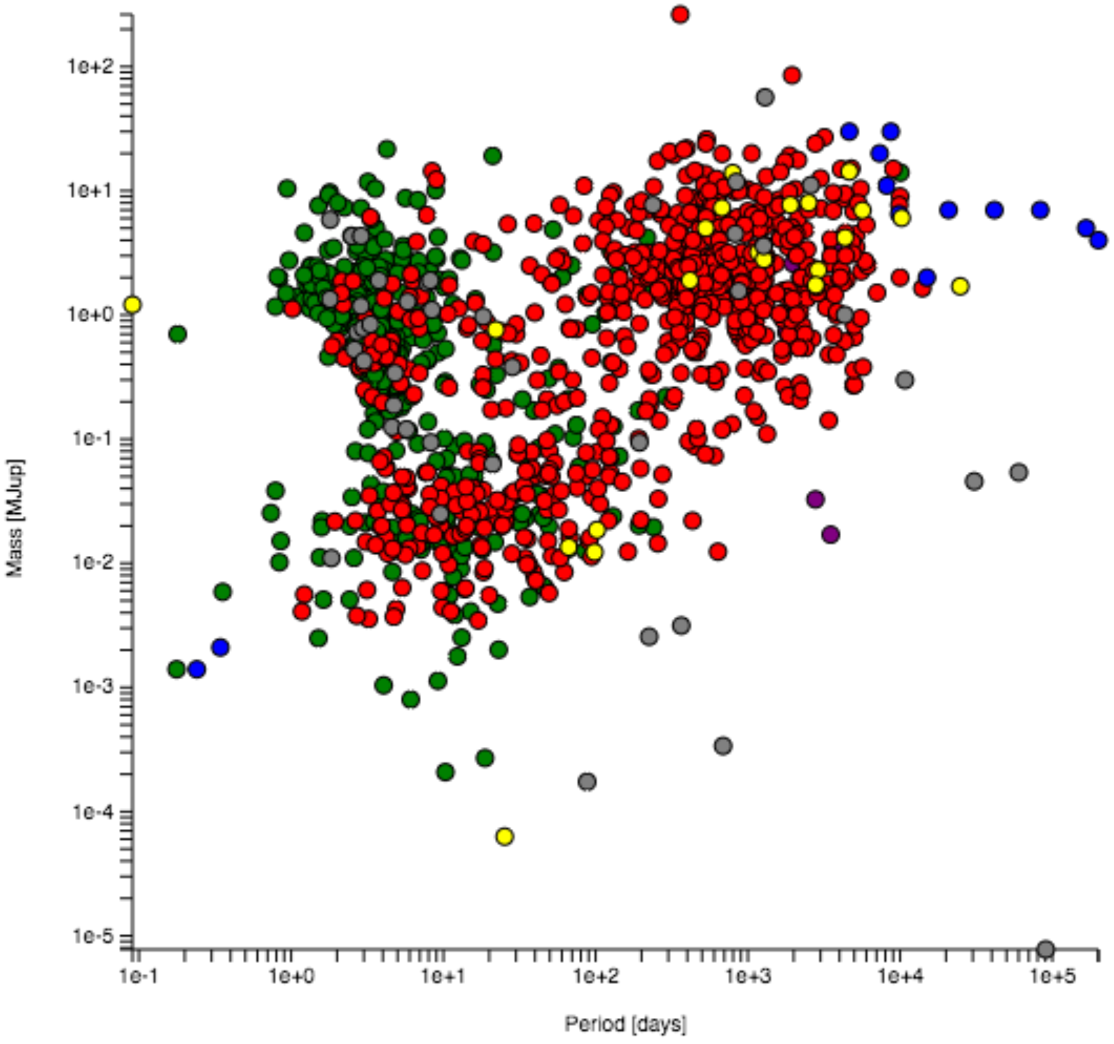
ABSTRACT
We present the first detection of the $^{13}C^{17}O$ $J = 3$–2 transition toward the HL Tau protoplanetary disc. We find significantly more gas mass (at least a factor of ten higher) than has been previously reported using $C^{18}O$ emission. This brings the observed total disc mass to 0.2 $M_\odot$, which we consider to be a conservative lower limit. Our analysis of the Toomre Q profile suggests that this brings the disc into the regime of gravitational instability. The radial region of instability (50–110 au) coincides with the location of a proposed planet-carved gap in the dust disc, and a spiral in the gas. We therefore propose that if the origin of the gap is confirmed to be due to a forming giant planet, then it is likely to have formed via the gravitational fragmentation of the protoplanetary disc.

Key words: stars: pre-main-sequence, individual: HL Tau – protoplanetary discs – techniques: interferometric – submillimetre: planetary systems





Exoplanets



Transit

Radial Velocity

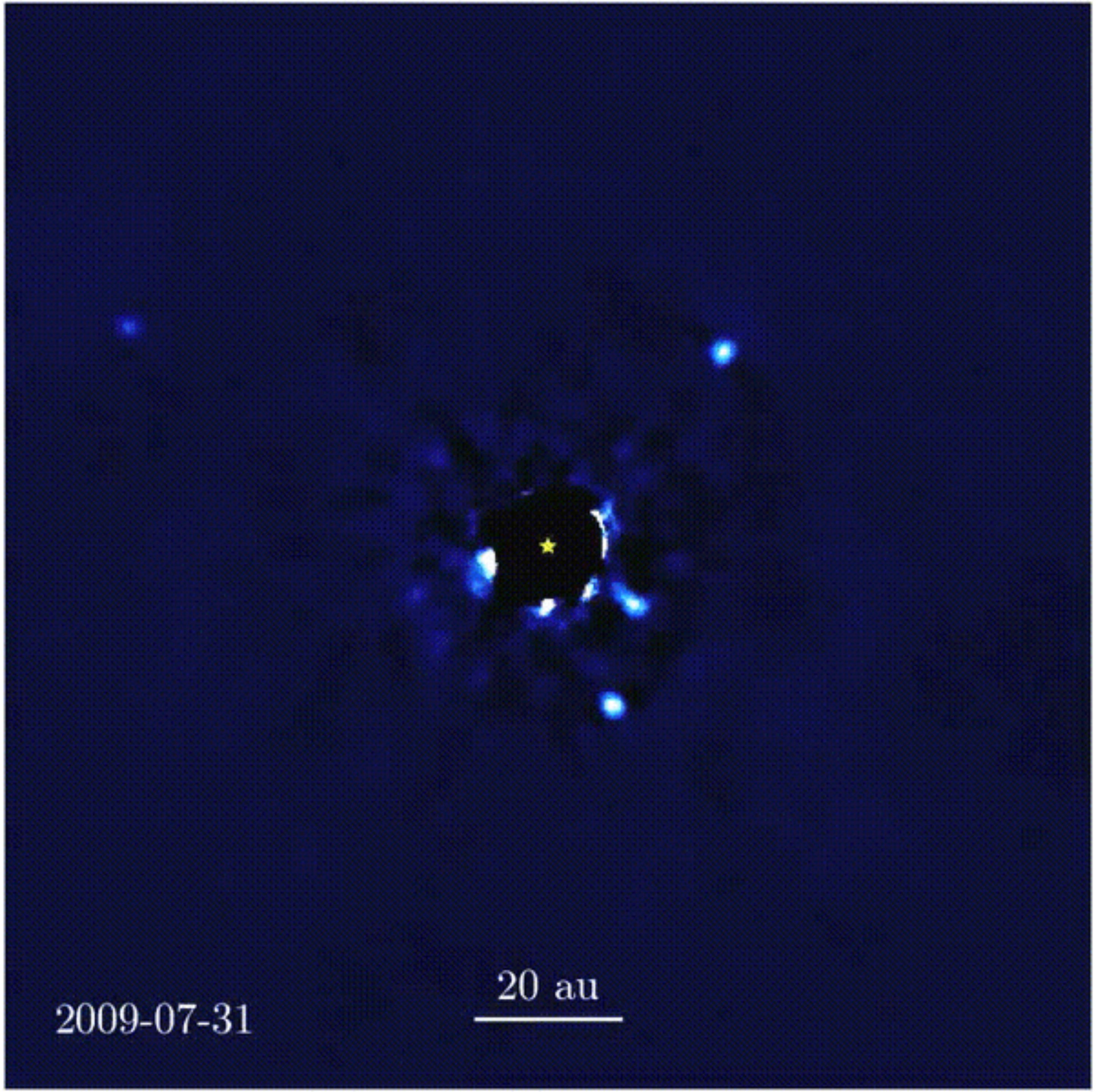
Microlensing

Direct Imaging

Pulsar Timing

Other Method

(Solar System /
Transit Time Variation)


20 au
2009-07-31

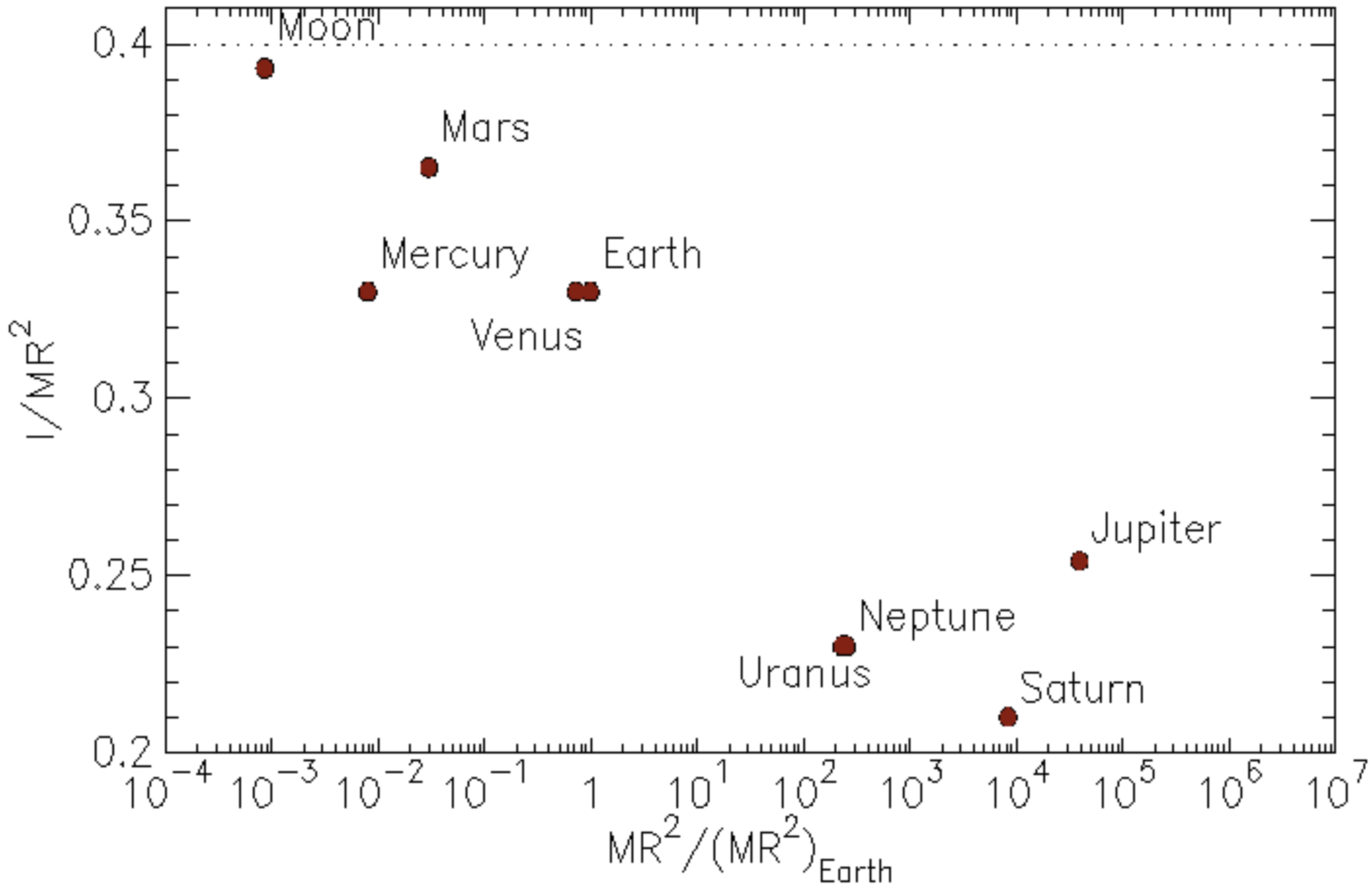
Potential of oblate bodies

Newton’s second theorem

“A spherically symmetric body affects external objects as if all its mass was concentrated in its center”

But planets are not spherically symmetric

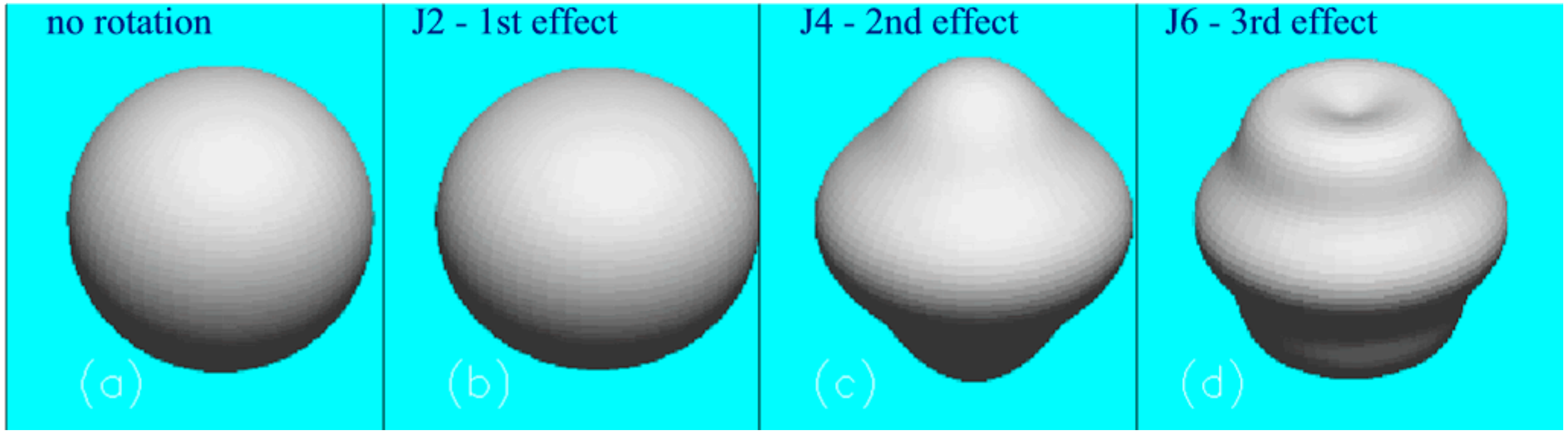
Deviation from Homogeneity



Oblateness caused by rotation

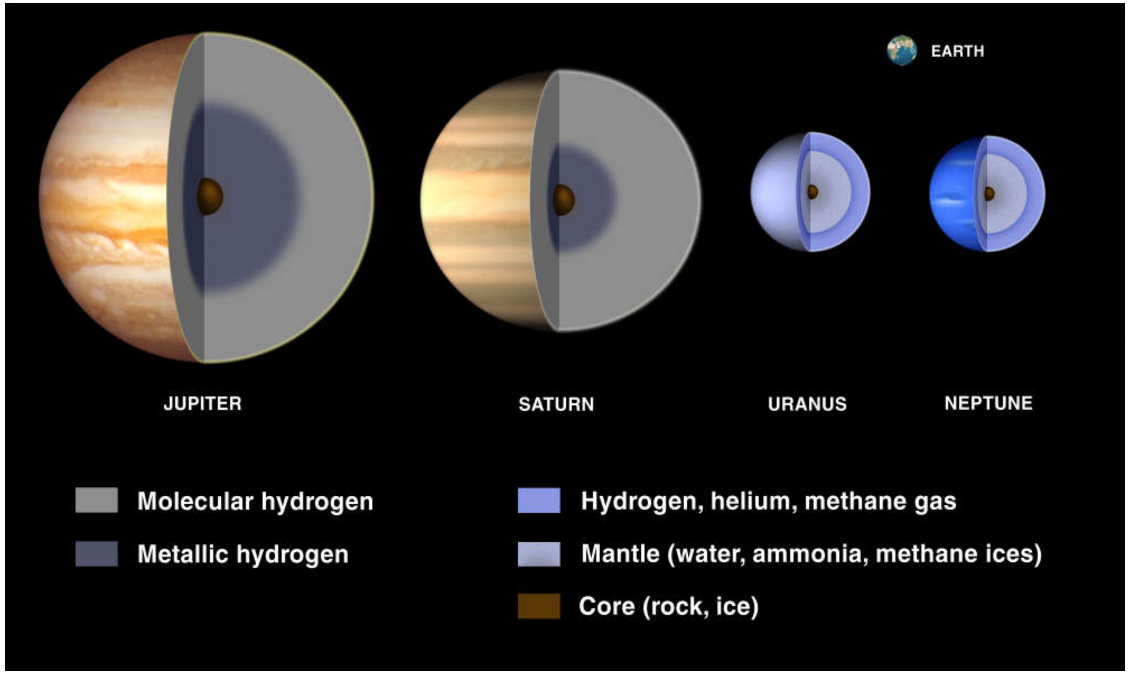
Gravitational Potential

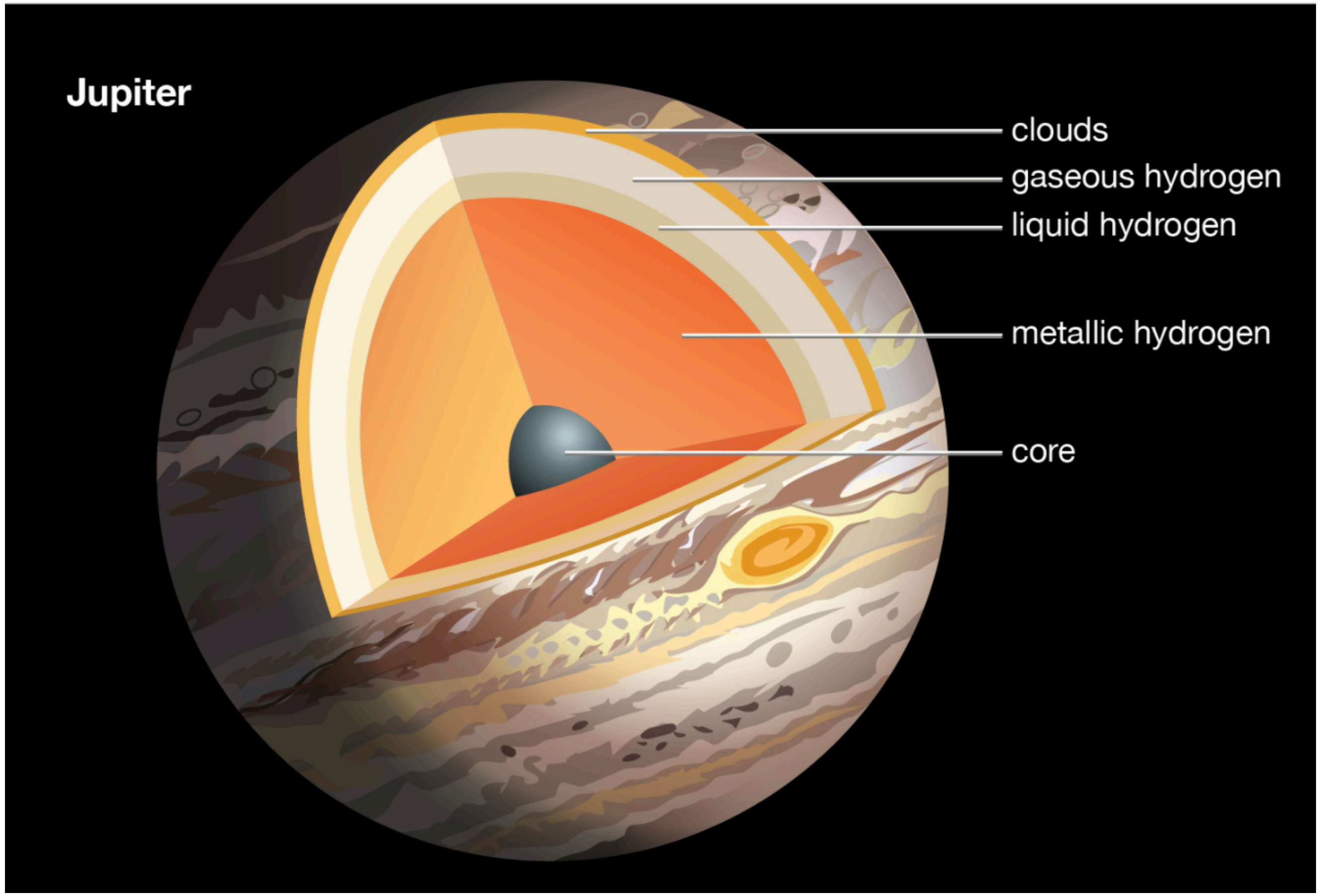
$$\Phi_g(r,\phi,\theta) = -\frac{GM}{r}\left[1 - \sum J_n P_n(\cos\theta)\left(\frac{R}{r}\right)^n\right]$$

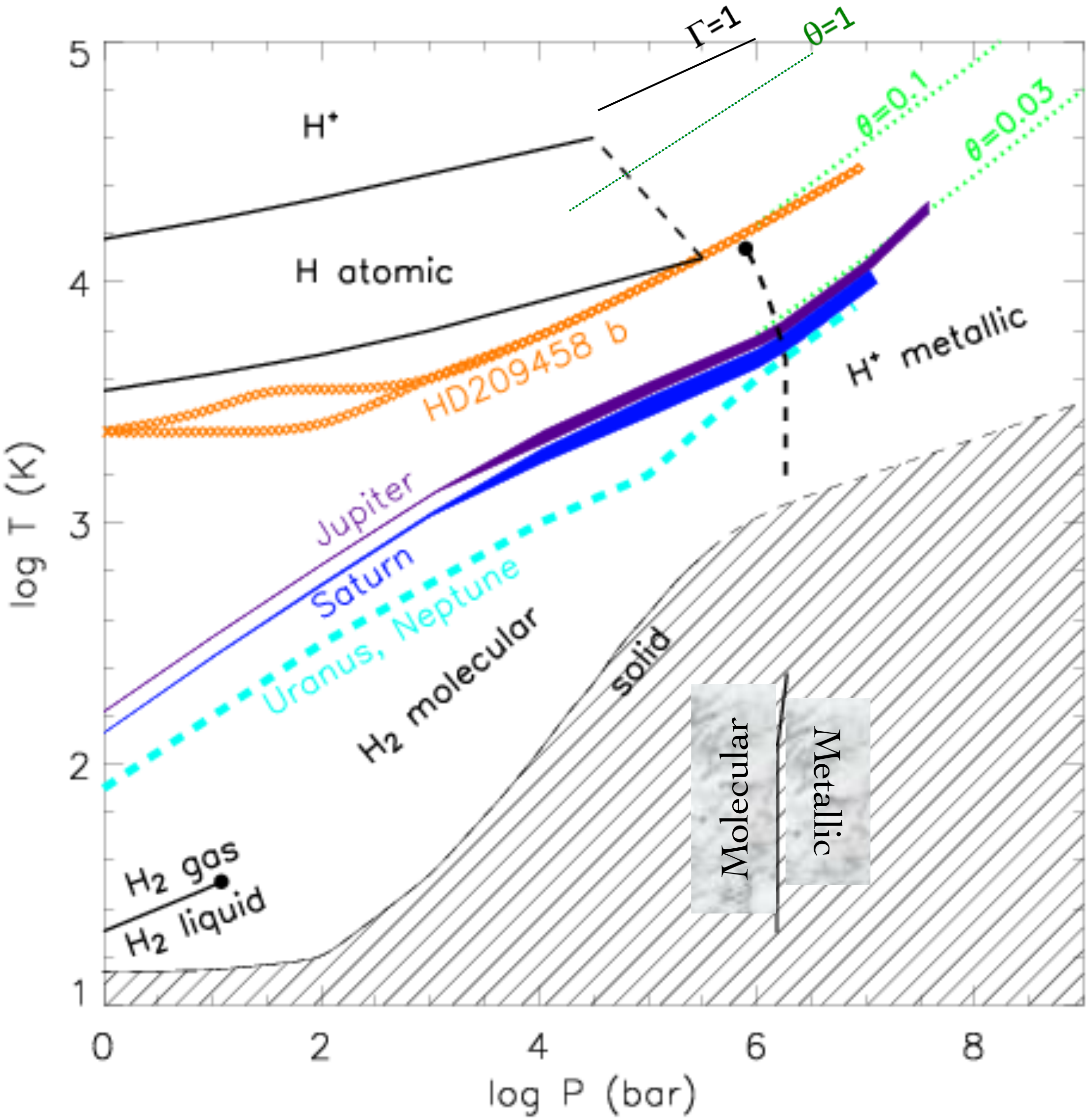


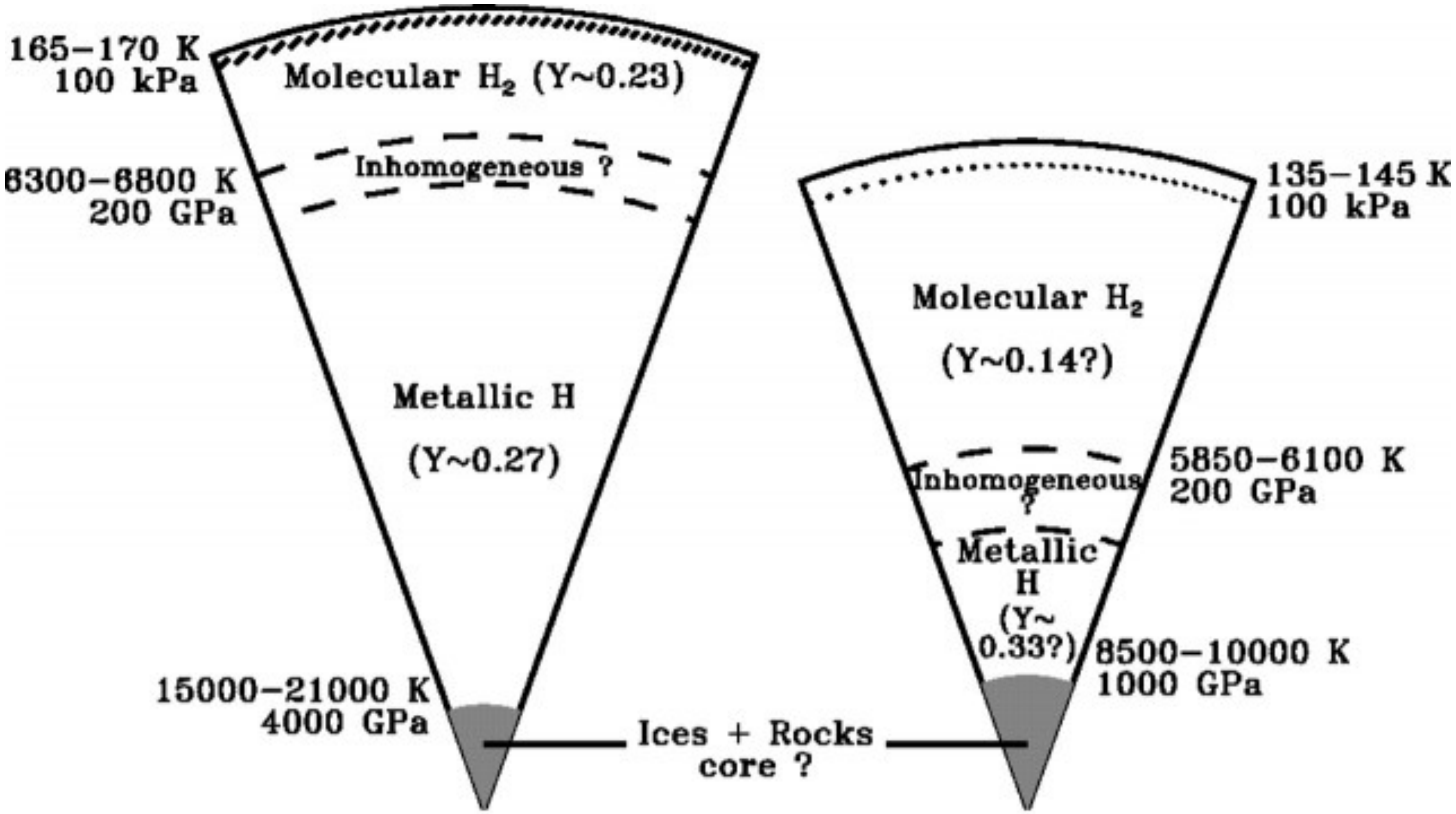
	J_2 (x10^{-6})	J_4 (x10^{-6})	J_6 (x10^{-6})
Jupiter	14696.4+/-0.2	587+/-2	34+/-5
Saturn	16290.7+/-0.3	936+/-3	86+/-9

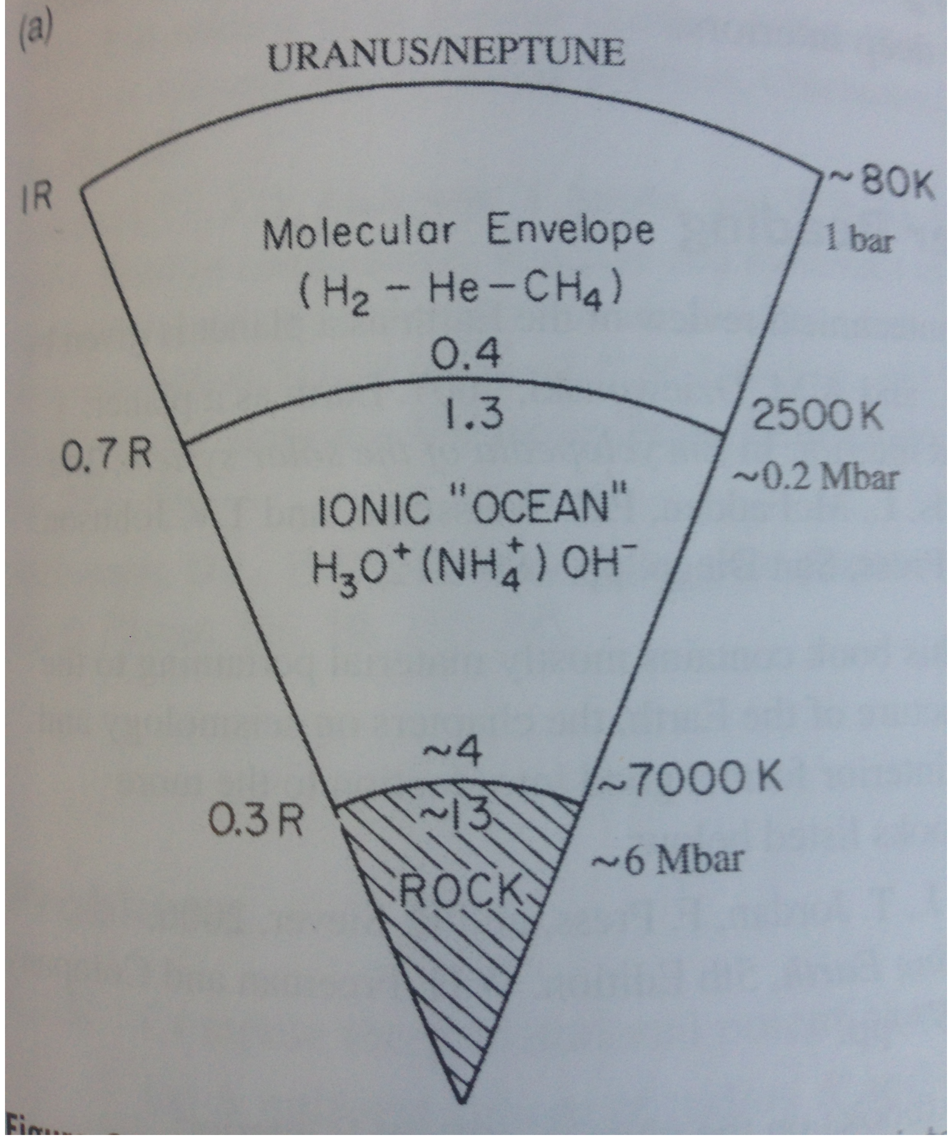
Interior of Giant Planets






$\Gamma=1$
$\theta=1$
$\theta=0.1$
$\theta=0.03$
H^+
H atomic
HD209458 b
H^+ metallic
Jupiter
Saturn
Uranus, Neptune
H_2 molecular
solid
Molecular
Metallic
H_2 gas
H_2 liquid
log T (K)
1
2
3
4
5
0
2
4
6
8
log P (bar)


165–170 K
100 kPa
Molecular H_2 (Y~0.23)
6300–6800 K
200 GPa
Inhomogeneous ?
Metallic H
(Y~0.27)
15000–21000 K
4000 GPa
Ices + Rocks
core ?
135–145 K
100 kPa
Molecular H_2
(Y~0.14?)
5850–6100 K
200 GPa
Inhomogeneous
?
Metallic
H
(Y~
0.33?)
8500–10000 K
1000 GPa


(a)
URANUS/NEPTUNE
1R
~80K
1 bar
Molecular Envelope
$(H_2 - He - CH_4)$
0.4
1.3
0.7 R
2500 K
~0.2 Mbar
IONIC "OCEAN"
$H_3O^+ (NH_4^+) OH^-$
~4
~13
0.3 R
~7000 K
~6 Mbar
ROCK

Confidential manuscript submitted to *Geophysical Research Letters*

Comparing Jupiter interior structure models to *Juno* gravity measurements and the role of a dilute core

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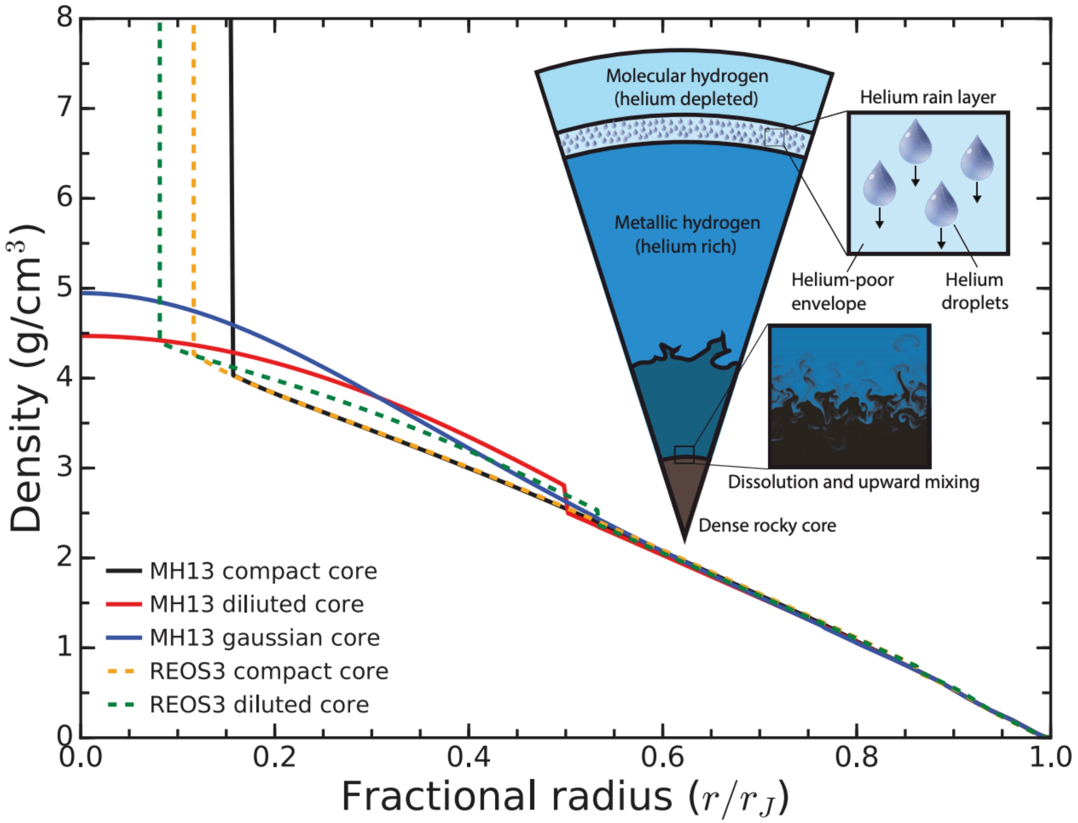
[9]LESIA, Observatoire de Paris, PSL Research University, CNRS, Sorbonne Universits, UPMC Univ. Paris 06, Univ. Paris Diderot, Sorbonne Paris Cit, 5 place Jules Janssen, 92195 Meudon, France
[10]JPL, Pasadena, CA, 91109, USA
[11]NASA/GSFC, Greenbelt, MD, 20771, USA
[12]SwRI, San Antonio, TX, 78238, USA

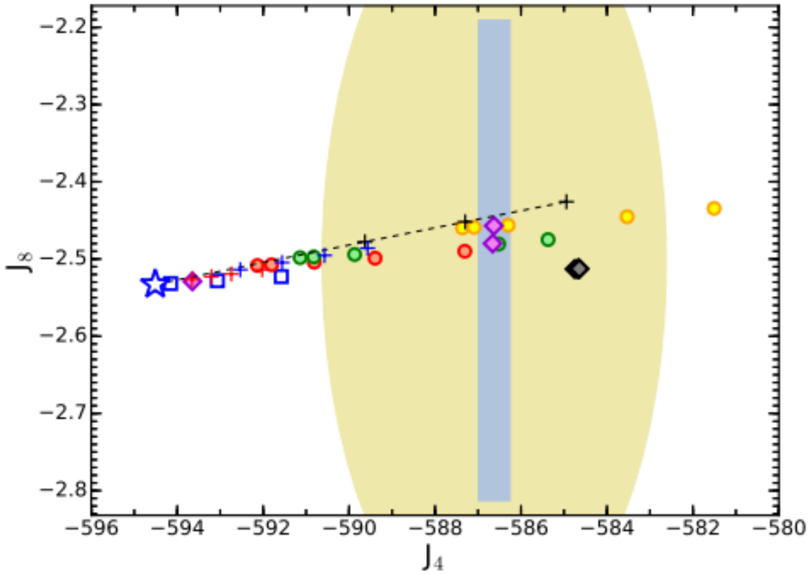
2.3 Dilute Core

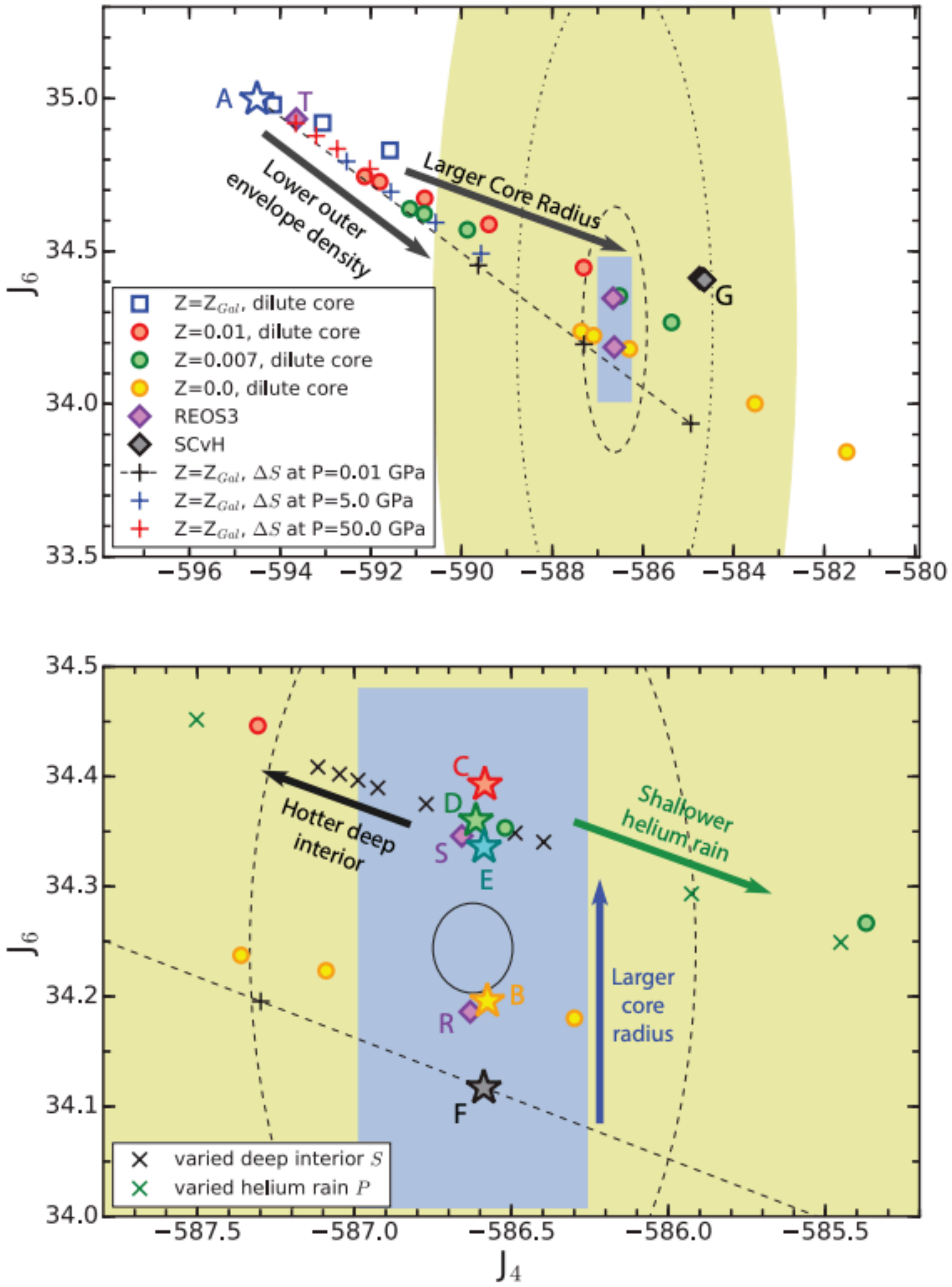
The thermodynamic stability of various material phases in giant planet interiors has been assessed using DFT-MD calculations [*Wilson and Militzer*, 2012a,b; *Wahl et al.*, 2013; *Gonzalez et al.*, 2013]. These calculations suggest that at the conditions at the center of Jupiter, all likely abundant dense materials will dissolve into the metallic hydrogen-helium envelope. Thus, a dense central core of Jupiter is expected to be presently eroded or eroding. However, the redistribution of heavy elements amounts to a large gravitational energy cost and the efficiency of that erosion is difficult to assess [see *Guillot et al.*, 2004]. It was recently shown by *Vazan et al.* [2016], that redistribution of heavy elements by convection is possible, unless the initial composition gradient is very steep. Some formation models suggest that a gradual distribution of heavy elements is an expected outcome, following the deposition of planetesimals in the gaseous envelope [*Lozovsky et al.*, 2017]. The formation of a compositional gradient could lead to double-diffusive convection [*Chabrier and Baraffe*, 2007; *Leconte and Chabrier*, 2013] in Jupiter's deep interior, which could lead to a slow redistribution of heavy elements, even on planetary evolution timescales.



In a selection of the models presented here, we consider Jupiter's "core" to be a region of the planet in which Z is enriched by a constant factor compared to the envelope region exterior to it. This means that the model core is a diffuse region composed largely of the hydrogen-helium mixture. In fact, this configuration is not very different from the internal structure derived by *Lozovsky et al.* [2017] for proto-Jupiter. Given the current uncertainty in the evolution of a dilute core, we consider models with core in various degrees of expansion, $0.15 < r/r_J < 0.6$. In a few models, we also test the importance of the particular shape of the dilute core profile by considering a core with a Gaussian Z profile instead. Fig. 1 demonstrates the density profiles resulting from these different assumptions about the distribution of core heavy elements.

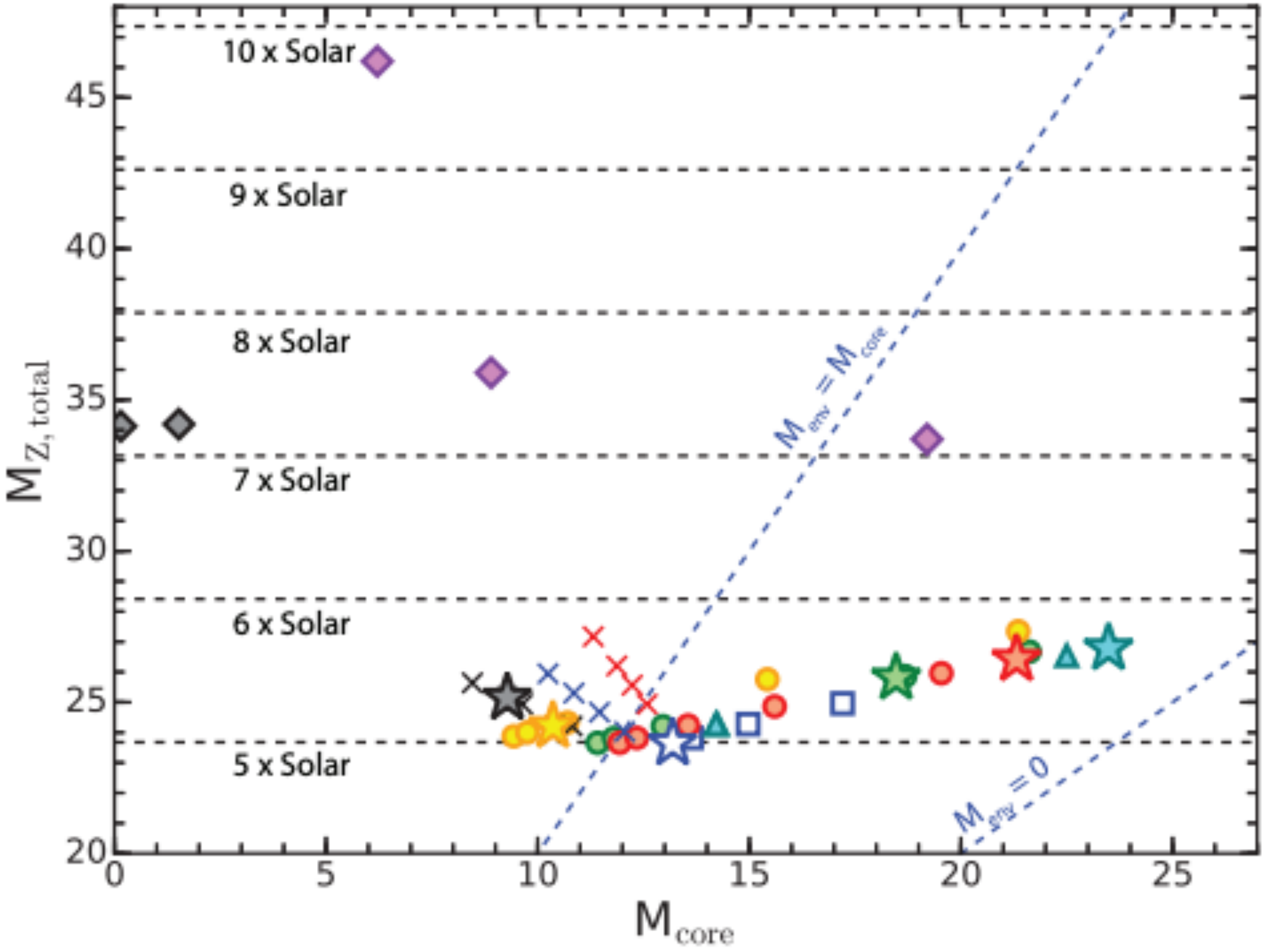
Fig. 2 shows the effect of increasing the radius of the dilute core on J_4 and J_6. Starting with the MH13 reference model with $r/r_J = 0.15$ (Model A), the core radius is increased incrementally to $r/r_J \sim 0.4$, above which the model becomes unable to fit J_2. Therefore, considering an extended core shifts the higher order moments towards the *Juno* values, but is unable to reproduce J_4, even considering a large dynamical contribution to J_n. Supplementary Fig. S1 shows a similar trend for J_8, although the relative change in J_8 with model parameters compared to the observed value is less significant than for J_4 and J_6.

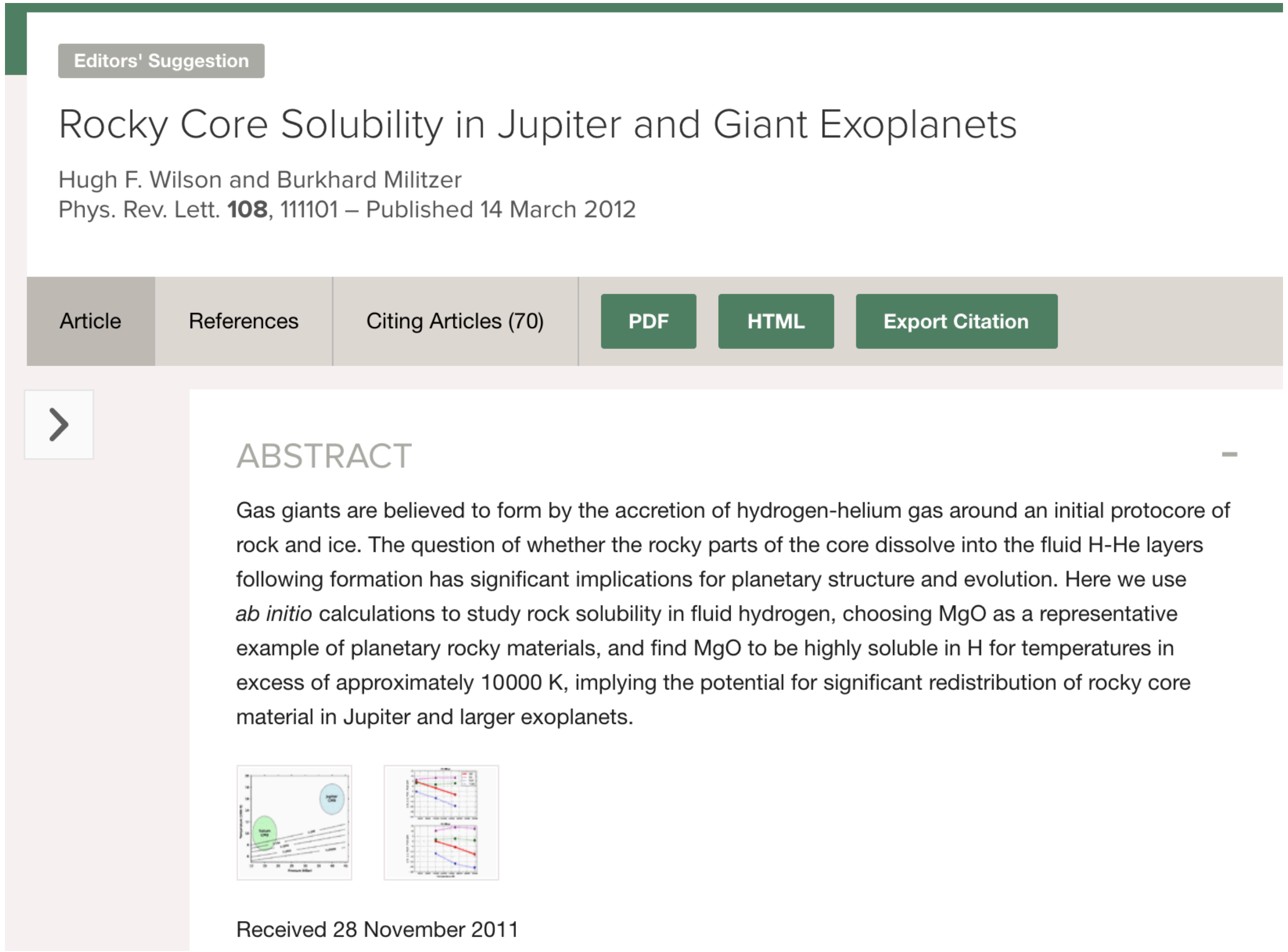




Key Points:

- Precise gravity measurements allow better predictions of interior structure and core mass.
- Juno's gravity measurements imply an increase in the abundance of heavy elements deep in the planet, at or inside its metallic region.
- The inferred structure includes a dilute core, expanded to a significant fraction of Jupiter's radius.


10 x Solar
9 x Solar
8 x Solar
7 x Solar
6 x Solar
5 x Solar
$M_{env} = M_{core}$
$M_{env} = 0$
45
40
35
30
25
20
0
5
10
15
20
25
$M_{Z,\, total}$
M_{core}



Rocky Core Solubility in Jupiter and Giant Exoplanets

Hugh F. Wilson and Burkhard Militzer

Phys. Rev. Lett. **108**, 111101 – Published 14 March 2012

ABSTRACT

Gas giants are believed to form by the accretion of hydrogen-helium gas around an initial protocore of rock and ice. The question of whether the rocky parts of the core dissolve into the fluid H-He layers following formation has significant implications for planetary structure and evolution. Here we use *ab initio* calculations to study rock solubility in fluid hydrogen, choosing MgO as a representative example of planetary rocky materials, and find MgO to be highly soluble in H for temperatures in excess of approximately 10000 K, implying the potential for significant redistribution of rocky core material in Jupiter and larger exoplanets.

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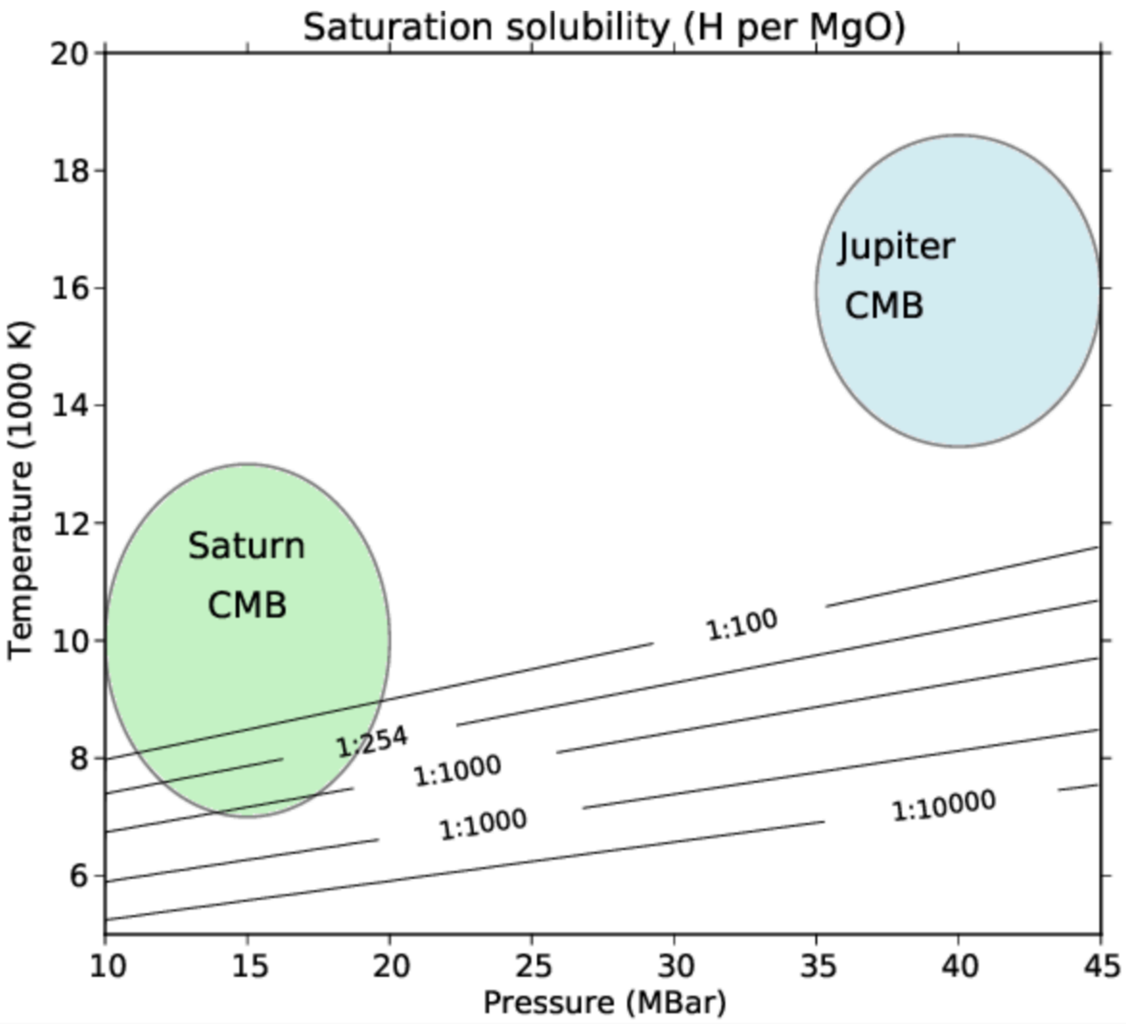

ICARUS **62,** 4–15 (1985)

Cosmochemistry and Structure of the Giant Planets and Their Satellites[1]

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Received January 4, 1985

Harold Urey made many contributions to a wide range of scientific areas and in the latter part of his career he played a pioneering role in the development of planetary science as a recognizable scientific discipline. Although you have to go back to his original papers to appreciate fully the power and originality of his work, perhaps his biggest influence was through the semipopular book *The Planets, Their Origin and Development* (Urey, 1952), which I turned to in seeking inspiration for this presentation.

I wish to consider first the main points of this book (Urey, p. 223, 1952):

> The principal conclusions of this book are: (1) The earth and the other terrestrial planets were formed at much lower temperatures than were generally thought to be the case up to the present time. (2) A more uniform distribution of iron throughout the silicate phases of the earth existed in the past than exists now and the iron core of the earth has been formed at least partly during geologic time.

After reading this, it is natural to ask: How could he have been so wrong?! Our current understanding of Earth formation suggests a hot beginning and prompt core formation, contemporaneous with accretion (see, for example, Elsasser, 1963; Safronov, 1969; Wetherill, 1972; Kaula, 1980; Stevenson, 1981, 1983). But on reflection, the question of whether Urey's conclusions were correct is not relevant to his main contribution to planetary science. To understand this, I quote from the preface of the same work (p. x):

> The data on which our discussions are based are often inadequate and of insufficient certainty to make unqualified conclusions possible: in fact almost every important point must be qualified with adverbs and adjectives expressing uncertainty, approximation, tentativeness, and so forth. Often only possible conclusions are indicated. Hence modifications of these conclusions must be expected. It may be asked whether discussion under such circumstances is justified or advisable. But are the densities of the terrestrial planets not the same? Is Trumpler's value for the radius of Mars correct? Does the moon really have a nonequilibrium bulge? And so on. Perhaps tentative conclusions will stimulate further investigation of these points by enough people so that their work will be checked and real agreement secured. It is hoped that the present study will stimulate some interest of this kind.

What Urey is doing is giving a license to study ill-posed problems. By ill-posed, I mean that the issue can be stated with precision but the knowledge needed to settle the issue is not yet entirely adequate. Urey's willingness to study ill-posed problems made this kind of science more respectable—a very important legacy, since planetary science would be dreadfully dull if we all worked only on well-posed problems. The important lesson that we learn is this: *It is not so important that you get the right answer; it is most important that you ask the right question.*

Many of the problems of cosmochemistry and planetary structure that I wish to discuss here are ill-posed. Stimulated by the lesson that Urey's work teaches us, I have chosen to avoid a conventional review

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[1] The Urey lecture of the Division for Planetary Sciences of the American Astronomical Society given in Kona, Hawaii, 12 October 1984. Contribution Number 4169 of the Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena.

0019-1035/85 $3.00

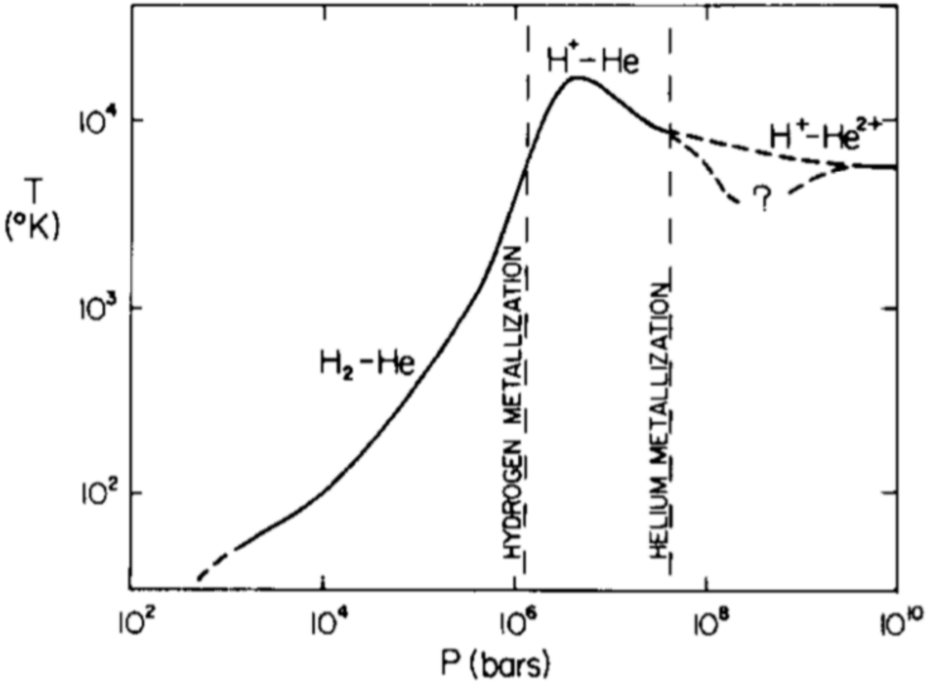
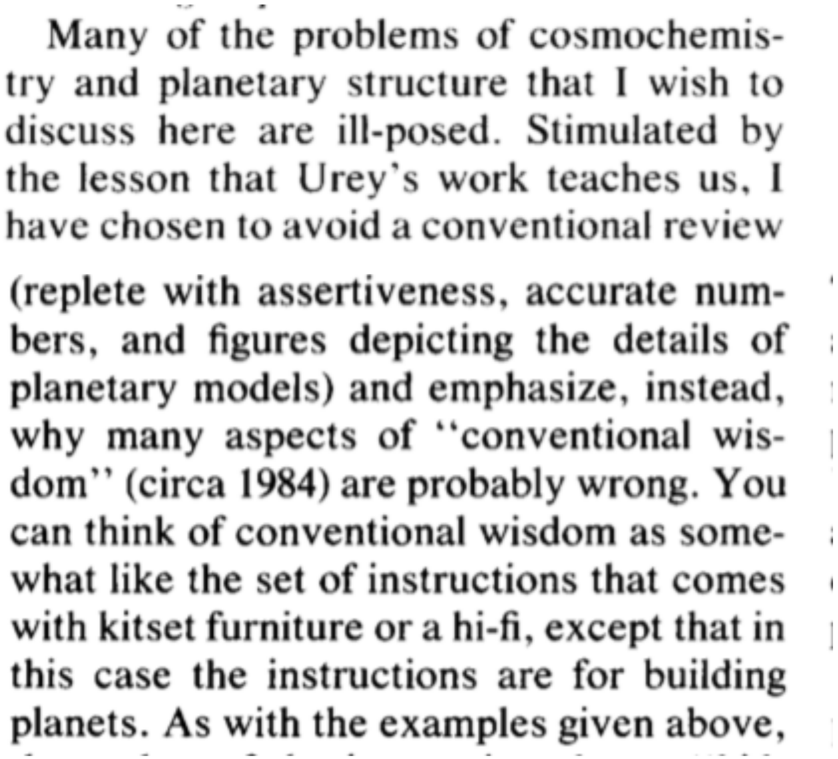


FIG. 1. Critical temperature for hydrogen–helium as a function of pressure. Above this temperature, hydrogen and helium mix in all proportions. The peak critical temperature occurs where hydrogen and helium are electronically very different: metal and tight-binding insulator, respectively. This region is relevant to the giant planets.



Many of the problems of cosmochemistry and planetary structure that I wish to discuss here are ill-posed. Stimulated by the lesson that Urey's work teaches us, I have chosen to avoid a conventional review (replete with assertiveness, accurate numbers, and figures depicting the details of planetary models) and emphasize, instead, why many aspects of "conventional wisdom" (circa 1984) are probably wrong. You can think of conventional wisdom as somewhat like the set of instructions that comes with kitset furniture or a hi-fi, except that in this case the instructions are for building planets. As with the examples given above,

I begin with a consideration of the component classes of materials from which the Solar System was constructed: ''gas,'' ''ice,'' and ''rock.'' Gas refers primarily to hydrogen and helium, constituents which would never condense under conditions encountered during or after Solar System formation. Ice refers primarily to the volatile forms of O, C, and N, the next three most abundant elements after hydrogen and helium. The most important molecules in this class are H_2O, CH_4, NH_3, CO, N_2, and possibly CO_2. They vary widely in volatility and their relative abundances depend on chemical processes before and during Solar System formation, some of which are discussed further below. The rock component is essentially everything else, but primarily silicates (rich in Mg, Si, and O) and iron (as metal, oxide, sulfide, or substituting for Mg in the silicates).

TABLE I

	Conventional wisdom	Reality
1.	Planetary constituents can be categorized as ''gas,'' ''ice,'' or ''rock'' with well established relative abundances. The relative incorporation of these components into planets and satellites reflect the environment of formation.	There exist physical and chemical processes which can greatly modify the categorization and relative abundance of these constituents, especially the ''ice'' component.
2.	The three constituent classes are still meaningful at high temperature and pressure.	The usual distinctions between ''gas,'' ''ice,'' and ''rock'' are meaningless at high temperature and pressure.
3.	Gravity acts to put the more dense constituents at the centers of planets.	Gravity is not usually able to drive the more dense constituents to the planetary center.
4.	Planetary interiors are adiabatic.	Global adiabaticity is unlikely because of compositional gradients.

solubility likely. The mixing arises because high pressure tends to diminish or eliminate the well-defined bonding categories of conventional chemistry. The van der Waals bonding is irrelevant, hydrogen bonding is eliminated, covalency and ionicity often lack clear distinction as the electrons delocalize, and everything is tending toward the democracy of metallization. High temperatures also play a role by guaranteeing a substantial negative contribution to the free energy of mixing (~*kT* ln 2 per atom in magnitude).

versial. However, I want to focus on the much lower pressure regime where hydrogen and helium are most dissimilar. At *P* ~ 1–10 Mbar, pure hydrogen is undergoing metallization but pure helium is still an insulator with a large band gap. Not surprisingly, noble gases are highly insoluble in metals because of their very different electronic environments, and there is no reason to expect that the solution of helium in metallic hydrogen is an exception. Detailed

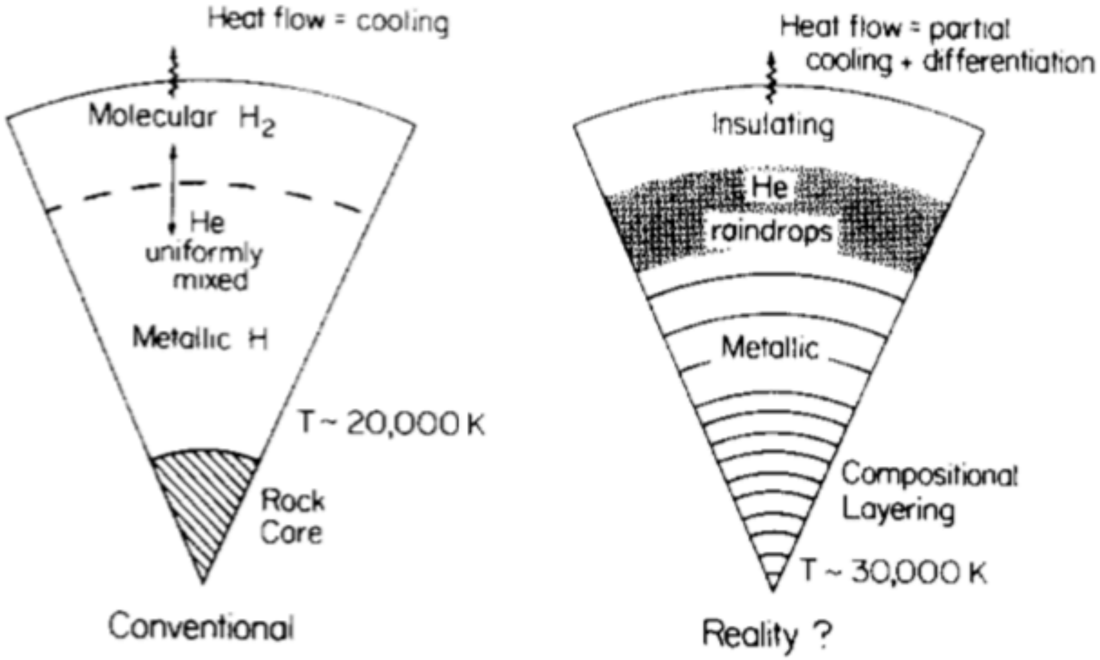


FIG. 3. Comparison of interior models for Jupiter according to the conventional view and in reality (similar to Table 1).

If we proceed one step further in complexity and add silica to form the SiO_2–H_2O–H_2 ternary system then we again encounter a tendency toward complete mixing (except in the irrelevant limit of very high SiO_2 mole fraction) for temperature as low as $\sim$2000°K and $P \gtrsim 10$ kbar (Nakamura, 1974). This is not surprising in view of the earlier work indicating complete mixing of SiO_2 and H_2O is possible in these conditions (Kennedy *et al.*, 1962).

creasing pressure and temperature is responsible for this mixing ability. An important implication of this result (assuming it is applicable to silicates in general) concerns the accretion of a rock/ice body in the presence of hydrogen. In the early, cooler stages there is a well-defined solid or liquid surface overlain by a hydrogen-rich atmosphere, but as the accretion proceeds, the increasing temperature and pressure cause the atmosphere and protoplanet to "merge"—the material becomes supercritical. This transition to supercriticality, wherein atmosphere and interior are joined by a continuous thermodynamic path, can occur for bodies as small as Mars (Stevenson, 1984c).

If mixing is so common then how can a planet have a core? Models of Jupiter and

If Urey's book is any guide, a lot of what is said in this paper is wrong. Perhaps that is the most important lesson! The exercise is a valuable one, however, and I can find no better words to express my confidence in the worthiness of this endeavor than the

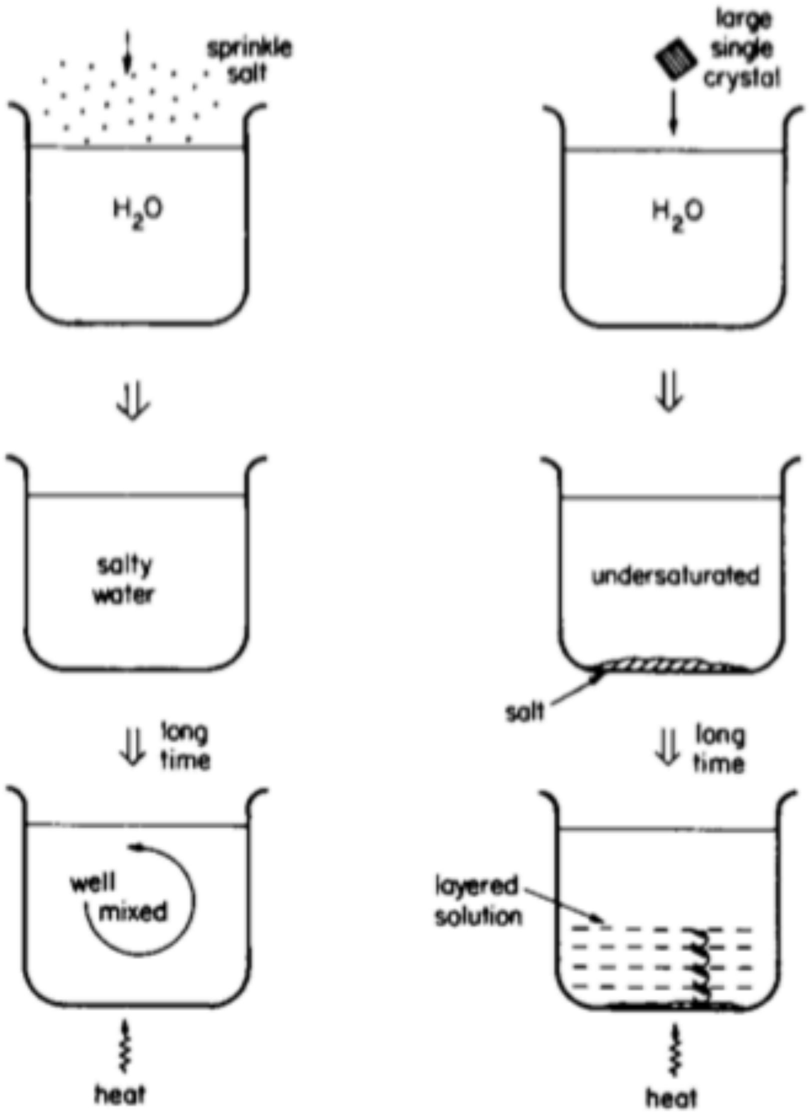
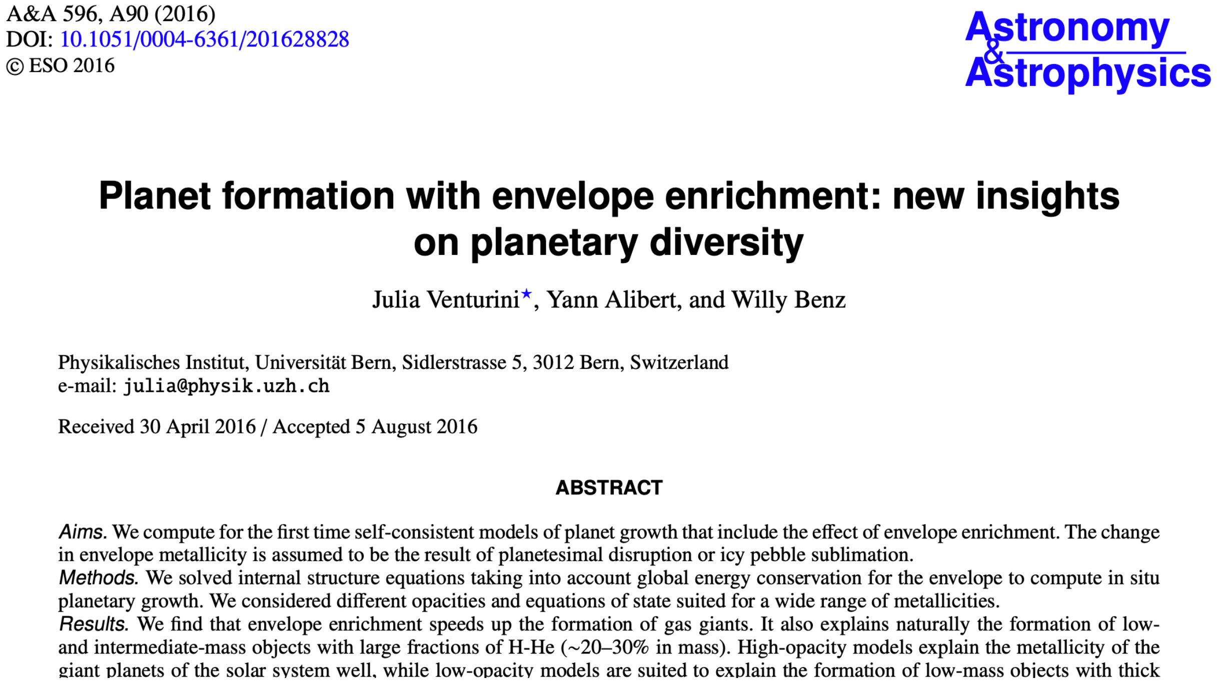


FIG. 2. Cartoon illustrating how settling under gravity can be difficult even when favored by density differences. However, it is achieved if density anomalies are added "catastrophically" as in the addition of a large, single crystal.



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Planet formation with envelope enrichment: new insights on planetary diversity

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ABSTRACT

Aims. We compute for the first time self-consistent models of planet growth that include the effect of envelope enrichment. The change in envelope metallicity is assumed to be the result of planetesimal disruption or icy pebble sublimation.
Methods. We solved internal structure equations taking into account global energy conservation for the envelope to compute in situ planetary growth. We considered different opacities and equations of state suited for a wide range of metallicities.
Results. We find that envelope enrichment speeds up the formation of gas giants. It also explains naturally the formation of low- and intermediate-mass objects with large fractions of H-He (~20–30% in mass). High-opacity models explain the metallicity of the giant planets of the solar system well, while low-opacity models are suited to explain the formation of low-mass objects with thick

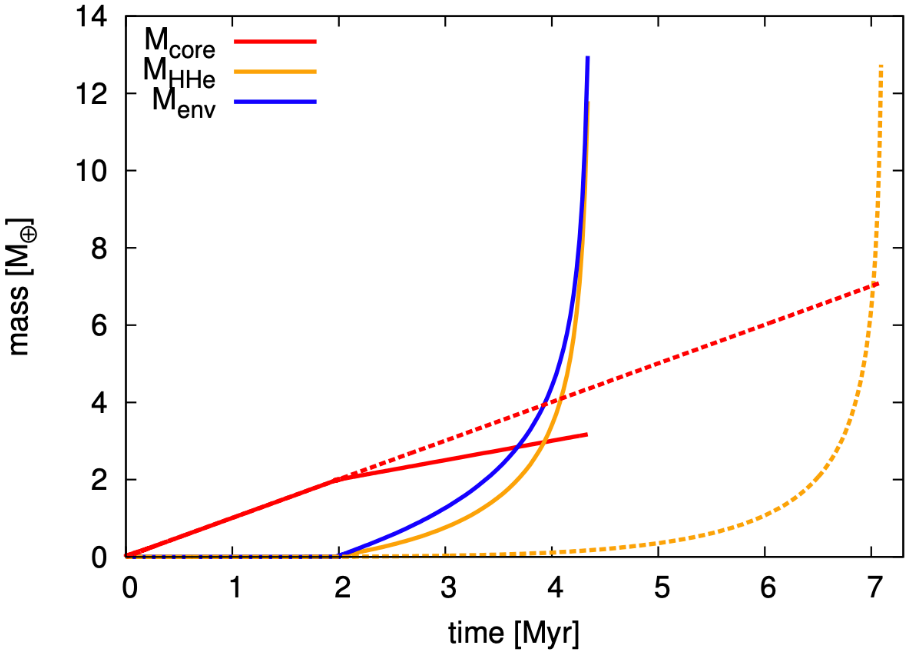

Fig. 2. Planet growth assuming $\dot{M}_Z = 10^{-6}\ M_\oplus$/yr for the enriched and non-enriched cases. Solid lines: enriched case with $M_{\rm thresh} = 2\ M_\oplus$ and $\beta = 0.5$. Dashed lines: non-enriched case (H-He envelope).