Class 18 - Planetesimal formation

- → Roche density; planetesimals.
- → Barriers to growth.

Class 17 learning objective: meter-size barrier (a tough nut to crack).

Balance selfgravity and tide. A planetesimal has internal cohesion larger than the solar tide. What's their size?

Roche limit:

$$\frac{GM_p}{R_p^2} > \frac{2GM_*}{r^3} R_p$$

$$\frac{\frac{4\pi}{3}\rho_d \cdot \cancel{R}^3}{\cancel{R}^2} > \frac{2M_*}{r^3} \cdot \cancel{R_p}$$

$$\rho_d > \frac{3}{2\pi} \cdot \frac{M_*}{r^3} \approx \frac{M_*}{2r^3} \qquad \boxed{\rho_d > \frac{M_*}{2r^3}}$$

$$\rho_d \simeq 0.5 \left(\frac{M_*}{M_\odot}\right) M_\odot \left(\frac{r}{AU}\right)^{-3} \frac{1}{AU^3}$$

$$\boxed{\rho_d \simeq 10^{-7} \frac{g}{cm^3} \left(\frac{M_*}{M_\odot}\right)\left(\frac{r}{1AU}\right)^{-3}} \quad 10^3 \times \text{Typical MMSN}$$

Settling limited by diffusion

Gas: $H = \frac{c_s}{\Omega}$; Similarly, grains: $H_d = \frac{v_{rms}}{\Omega}$

Parametrize $v_{rms} = \sqrt{\delta} \cdot c_s$

δ shown to be related to α and St $\delta = \frac{\alpha}{\alpha + St}$

$$H_d = \sqrt{\delta} \, \frac{c_s}{\Omega} = \sqrt{\delta} \cdot H = \sqrt{\frac{\alpha}{\alpha + St}} \cdot H$$

$$\boxed{H_d = \sqrt{\frac{\alpha}{\alpha + St}} \cdot H}$$

Limits

$\alpha = 0 \quad \rightarrow \quad H_d = 0$ (needs turbulence to stir)

$St = 0 \quad \rightarrow \quad H_d = H$ (small particles do not settle)

$St \rightarrow \infty \quad \rightarrow \quad H_d = 0$ (decoupled particles settle efficiently).

Backreaction and Streaming Instability

$$\frac{\partial u}{\partial t} = \dots + \frac{\rho_d}{\rho_g} \frac{(v-u)}{\tau}$$

if $\varepsilon = \frac{\rho_d}{\rho_g} > 1$; the dust becomes dynamically dominant.

$$\frac{\partial v}{\partial t} = \dots - \frac{(v-u)}{\tau}$$

System:

$$\frac{\partial \rho_d}{\partial t} + \nabla(\rho_d \cdot v_d) = 0 \quad ; \quad \nabla \cdot u = 0$$

$$\frac{\partial v_d}{\partial t} + (v_d \cdot \nabla) v_D = -\Omega^2 r - \frac{1}{\tau}(v-u)$$

$$\frac{\partial v_u}{\partial t} + (u \cdot \nabla) u = -\Omega^2 r + \frac{\varepsilon}{\tau}(v-u)$$

Nakagawa-Sekiya-Hayashi (NSH) equilibrium

streaming Instability produces: objects in the 10-100 km-range.