Class 17 - Meter Size barrier   4/2/20

Class structure

- The planetesimal hypothesis
- How to build planetesimals

Last class: ISM starts from sub-μm sized grains; disks have up to cm-sized grains. So grain growth occurs.

Coagulation alone by brownian motion would lead to growth of 1 cm/Myr.

× Drag
× Fragmentation
× Friction time and Stokes number
× The meter-size barrier

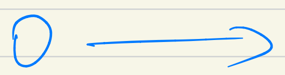
Coagulation and fragmentation balance.

Velocity? Drag matters

Aerodynamic drag $\frac{dv}{dt} = -\frac{v}{t_{friction}}$

Epstein (ballistic) drag → free molecular flow

Cross section σ



σ ρ

Volume in time $\Delta t \rightarrow \quad dV = \sigma v \Delta t$

In the reference frame of the object, the gas is travelling ballistically with speed $-v$.

Transferring all the momentum,

$$F_d = \frac{dp}{dt} = \rho dV \cdot \frac{dv}{dt} = -\rho \pi s^2 v \vec{v}$$

Valid up to $\lambda = \frac{1}{n \sigma_{coll}} = \frac{1}{10^{13} cm^3 \times 10^{15} cm} \approx 1m$

Integrate with Maxwell Boltzmann distrib. of velocities

Substitute v for MB $\quad \int_0^\infty n(v) v^2 dv$

$$F = -\frac{4\pi}{3} \rho s^2 v_{TH} \vec{v}$$

$$v_{TH} = \sqrt{\frac{8kT}{\pi \mu m_H}} = \sqrt{\frac{8}{\pi}} c_s$$

SKIP

Front side $\Rightarrow \Delta t = \frac{d}{v+v_{TH}}$ Back side $\Rightarrow \Delta t = \frac{d}{v-v_{TH}}$

$\Delta P = 2 m v_{TH}$

Scaling

$\left(\frac{\Delta P}{\Delta t}\right)_{front} + \left(\frac{\Delta P}{\Delta t}\right)_{back} = v_{TH} \cdot \left[\frac{1}{(v+v_{TH})} - \frac{1}{(v-v_{TH})}\right]$

$\underline{v_{TH} \cdot v}$

Divide by $m = \rho_\bullet \frac{4\pi}{3} s^3$

$$f = \frac{F}{m} = -\frac{\frac{4\pi}{3} \rho s^2 v_{TH} v}{\rho_\bullet \frac{4\pi}{3} s^3} \rightarrow \boxed{f = -v_{TH} \cdot \frac{\rho}{s \rho_\bullet} \cdot v}$$

Effect of drag on dust dynamics

Friction timescale

$$t_{friction} = \frac{v}{\dot{v}} = \frac{}{F} = \frac{1 \bullet}{\rho_g v_{TH}}$$

$\lnot$ `` v_T

Drag force becomes

$$\boxed{\frac{dv}{dt} = -\frac{v}{t_{friction}}}$$

Stokes number $St = \Omega t_{friction}$

At 1AU

$1\mu m \rightarrow 4s$ ← small dust very well coupled

$1m \rightarrow 0.2yr$

Dimensionless friction time!

$$t_{friction} \cdot \Omega \equiv St$$

Stokes drag:

$$F_D = -\frac{C_D}{2} \pi s^2 \rho v \vec{v}$$

$C_D \simeq \frac{24}{Re}$ low Rey

$C_D \simeq 0.44$ high

Settling:

Match F_{grav} and F_{drag}

$$\Omega^2 z = \frac{v_{TH} \cdot \rho}{s \rho_\bullet} \cdot v$$

$$\boxed{v = \frac{\rho_\bullet s}{\rho v_{TH}} \cdot \Omega^2 z}$$

$\rho \sim 10^{-10}\ g/cm^{-3}$

$z \sim 10^{11}\ cm$

$v_{TH} \sim 10^5\ cm/s$

$v \approx 0.06$ cm/s

$\tau = z/v \approx 10^5$ yr

With coagulation

$$\frac{dm}{dt} = \pi s^2 \cdot v_{settle}\, \varepsilon\, \rho_{gas}$$

v_{settle}

$$\frac{dm}{dt} = \frac{3}{4} \frac{\Omega^2 \varepsilon}{v_{TH}} \cdot z \cdot m \qquad \frac{dz}{dt} = -\frac{\rho_\bullet}{\rho} \cdot \frac{s}{v_{TH}} \cdot \Omega^2 z$$

Show plot of Schraner

The meter-site barrier.

Fragmentation barrier

Fragmentation threshold for silicate particles $\rightarrow \Delta v \approx 1$ m/s.

Fragmentation - limited growth.

$$v_p = \frac{v_g}{\sqrt{1+St}} \qquad v_p^2 = \frac{v_g^2}{1+St}$$

$$\therefore v_p^2 - v_g^2 \simeq v_g^2 \left[\frac{1}{1+St} - 1\right]$$

$$\simeq v_g^2 \left[\frac{1 - 1 - St}{1+St}\right]$$

$$\simeq v_g^2 \, St$$

$$v^2 = \alpha \, St \, c_s^2 \quad ; \quad St = \frac{\rho_\circ s_\circ}{\rho H}$$

$$v_f^2 = \alpha \cdot \frac{\rho_\bullet s}{\rho H} c_s^2 \qquad c = \Omega H$$

$$\rho = \frac{\Sigma}{\sqrt{2\pi} \cdot H} \quad \rightarrow \quad s_{max} \simeq \frac{4}{3\pi} \frac{\Sigma}{\alpha \rho_m} \frac{\Delta v_f^2}{c_s^2}$$

$$s_{max} \simeq \frac{10^3 \, g/cm^2}{10^{-2} \times 3} \times \frac{(10^2)^2}{10^{10}} = \frac{10^{3+4+2-10}}{3}$$

$$= 1 \text{ m}$$

Boulders are the maximum that something can grow by coagulation, and that's assuming no compactification.

Drift barrier

Gas rotates slightly sub-keplerian

$$\frac{\partial u}{\partial t} - \frac{u_\phi^2}{r} = -\frac{GM}{r^2}$$ $\therefore$ in equilibrium

$$\Omega = \frac{GM}{r^3} = \Omega_K$$

with pressure

$$\frac{\partial u}{\partial t} - \frac{u_\phi^2}{r} = -\frac{GM}{r^2} - \frac{1}{\rho}\frac{\partial}{\partial r}p$$

$$\Omega^2 r = \Omega_K^2 r + \frac{1}{\rho}\frac{\partial}{\partial r}p$$

$$\Omega^2 = \Omega_K^2 + \frac{1}{r}\frac{c_s^2}{\rho c_s^2}\frac{\partial}{\partial r}p$$

$$= \Omega_K^2 + \frac{c_s^2}{r^2}\frac{\partial \ln p}{\partial \ln r}$$ $c = \Omega_K H$

$$\Omega^2 = \Omega_K^2 + \frac{\Omega_K^2 H^2}{r^2}\frac{\partial \ln p}{\partial \ln r}$$

$$\Omega^2 = \Omega_K^2\left[1 + \left(\frac{H}{r}\right)^2\frac{\partial \ln p}{\partial \ln r}\right]$$

$$\Omega = \Omega_K \left[1 + \left(\frac{H}{r}\right)^2 \frac{\partial \ln p}{\partial \ln r} \right]^{1/2}$$

$\therefore \; h \ll 1$

$$\Omega = \Omega_K \left(1 + \frac{1}{2}\left(\frac{H}{r}\right)^2 \frac{\partial \ln p}{\partial \ln r} \right)$$

$$\boxed{\Omega = \Omega_K (1 - \eta)} \qquad \boxed{\eta = -\frac{1}{2}\left(\frac{H}{r}\right)^2 \frac{\partial \ln p}{\partial \ln r}}$$

$$u = \Omega r = u_K (1 - \eta) \qquad u_K = \Omega_K \cdot r$$

$\eta \approx 10^{-3}$

small particles $St \ll 1$

$$u_{wind} = v_K - v_g = \eta v_K \approx \frac{1}{2}\left(\frac{H}{r}\right)^2 \cdot \Omega r \qquad c = \Omega H$$

$$= \frac{1}{2}\left(\frac{H}{r}\right) \cdot c_s$$

$$\approx \frac{1}{2} h \cdot c_s \approx 50 \; m/s$$

$$50 \frac{m}{s} = 50 \times \frac{10^{-3}\,km}{10^{-3}\,h} \simeq 100 \text{ miles}/h$$

$$\ddot{r} - r\dot{\phi}^2 = -\frac{v_k^2}{r} - \frac{\dot{r}}{\tau}$$

$$r\ddot{\phi} + 2\dot{r}\dot{\phi} = -\frac{r\dot{\phi}}{\tau} - \frac{u}{\tau}$$

steady state: $\ddot{r} = 0$; $\dot{\phi} = \Omega + u_{gas}$

$$\boxed{\dot{r} = -\frac{2St_s \eta v_k}{(1+St^2)}}$$

$$\frac{\partial \dot{r}}{\partial \tau} = \frac{2\eta v_k (1+\tau_s^2) - 2\tau_s \eta v_k\, 2\cdot\tau_s}{(1+\tau_s^2)^2} = 0$$

$$\cancel{2}\cancel{\eta}v_k (1+\tau_s^2) - \cancel{2}\tau_s \cancel{\eta}\cancel{v_k} \cdot 2\tau_s = 0$$

$$1+\tau_s^2 - 2\tau_s^2 = 0 \quad \therefore \boxed{St = 1}$$

$\boxed{St = 1}$ fastest drift

size $\tau = \frac{\rho_\bullet s}{\rho_{gas} c_s}$ $\therefore St = \Omega\tau = \frac{\rho_\bullet s}{\rho_{gas} \cdot H}$

$$S = \frac{\rho_g}{\rho_\bullet} \cdot H = \frac{10^{-10} \times 10^{12}}{1} \approx 10^2 cm \simeq 1m$$

again 1m

$$\boxed{\dot{r} = -\frac{2 St_s \eta v_K}{(1 + St^2)}}$$

small grains $St \ll 1$ $\rightarrow$ $\dot{r} = -2\eta v_K \frac{St}{(1+St^2)}$

$$\simeq -2\eta v_K \cdot St$$

In large particles $St \gg 1$

$\dot{r} = 0$ they don't feel the gas drag anymore

$$t_{drift} = \frac{r}{\dot{r}} = \frac{(1 + St^2)}{2 St \eta \Omega}$$

for $St = 1$

$$t_{drift, max} = \frac{1}{2\eta\Omega} = \frac{T}{4\pi\eta} \qquad \eta \simeq 10^{-3}$$

$$\simeq 10^2 T$$

$\boxed{\text{At } 1\,AU \rightarrow 100 \text{ yrs}}$!!!

This is the fastest constraint EVER imposed on planet formation.

Meter-size barrier

Once m-sized objects are built, they are lost to the star.

Conclusion: Planetesimal formation must be rapid.

Radial redistribution is likely to occur.