Class 16 - Dust Physics

Planetesimals / Rocky Planets / Giant Planets

These interact differently with the gas.

Bottom up scenario

The type of grain: ISM has

Extinction: rise in extinction for $\lambda \lesssim 0.1 \mu m$

Grains ~ 10^{-12} g (interplanetary dust particles)

spherical grains with bulk density $\rho = 2.5 \, g/cm^{-3}$

How does dust grow?

$\rho = 2.24 \, g/cm^{-3}$ (density of graphite)

b_c = C abundance in the grain

So how far can coagulation go?

Assume that all particle collisions lead to sticking.

$$t_{collide} = \frac{1}{n\sigma\Delta v}$$

n: particle number density.

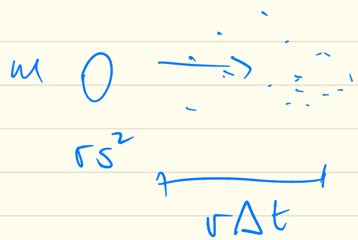
radius s

$$\sigma = \pi s^2$$

number density

$$n = \frac{\varepsilon \rho_{gas}}{m}$$

ε = dust-to-gas ratio



$$\Delta m = \pi s^2 \cdot v\Delta t \cdot n \cdot m_p$$

$$\therefore \boxed{\frac{dm}{dt} = \pi s^2 \cdot v \cdot \varepsilon \rho_{gas}}$$

$$m = \rho_\bullet \cdot \frac{4\pi s^3}{3} \qquad \therefore \rho_\bullet \cancel{4\pi} \cancel{s^2} \frac{ds}{dt} = \cancel{\pi s^2} v \varepsilon \rho_{gas}$$

$$\frac{ds}{dt} = v\varepsilon \cdot \frac{\rho_{gas}}{\rho_\bullet} = \frac{0.1 \cdot 10^{-2} \cdot 10^{-10}}{3} = 10^{-13} \frac{cm}{s}$$

$$\frac{ds}{dt} = v\,\varepsilon \cdot \frac{\rho_{gas}}{\rho_\bullet}$$

$\varepsilon \simeq 10^{-2}$; $\rho_{gas} \simeq 10^{-10}$ g/cm^3 ; $\rho_\bullet \approx 3$ g/cm^3

but v ?

Brownian motion $\rightarrow \frac{1}{2} m v^2 = kT$ for $T = 300$ and $1\,\mu m$ grain, $v \simeq 0.1$ cm/s

$$\frac{ds}{dt} = 10^{-1} \cdot 10^{-2} \cdot 10^{-10} / 3 \simeq \frac{10^{-13}\ \text{cm}}{3\ \text{s}}$$

(convenient factor 3)

$$\frac{ds}{dt} = \frac{10^{-13}}{3 \times 10^7\ \text{s}} \times 10^7 \simeq 10^{-6}\ \frac{\text{cm}}{\text{yr}} = 1\ \text{cm/Myr}$$

Due purely to brownian motion, a μm sized grain would grow to cm-size.

Observations β:

$K(\nu) \propto \nu^{\beta}$ $\qquad\qquad j_\nu = B_\nu \cdot K_\nu$

optically thin in mm $\therefore$

$I = j \rho L \qquad F = j \cdot \rho$

And $j = B_\nu \cdot K_\nu \quad \therefore \boxed{F_\nu \propto B_\nu K_\nu}$

$\therefore$ At Rayleigh-Jeans limit

$B \propto \nu^2$

So $\boxed{F \propto \nu^{2+\beta}}$

considering $F \propto \nu^{\alpha}$, then $\alpha = 2+\beta$

β is a function of grain size