

# Class 14 - Active disks Dec 2013

3/7/20

Accretion equations (active disk)

Quenching of the MRI

other processes

- {
  - Photoevaporation
  - winds
  - other turbulence mechanisms? Gravitational turbulence?
  - Disk dispersal
  - Magnetospheric accretion

Viscous heating

$$2 \rho v \Sigma^2$$

$$S = u_{i,j} + u_{j,i}$$

$$= \left( \frac{\partial u_j}{\partial x_i} \right)^2 = \Sigma^2 + r \frac{du}{dr} = \Sigma(1+\xi) \approx \eta \Sigma$$

$$2 \rho v \Sigma^2 \eta^2 = \frac{7}{4} \rho v \Sigma^2 \text{ integrate in } z$$

$$\frac{7}{4} \Sigma v \rho \Sigma^2 \text{ (viscous heating)}$$

half travels up and half travels down

$$\tau_d^4 = \frac{7}{8} \Sigma v \rho \Sigma^2$$

$$\text{Active disk } \tau T_d^4 = \frac{9}{8} \nu \Sigma r^2 \quad \nu \Sigma = \frac{\dot{m}}{3\pi}$$

$$T_d^4 = \frac{9}{8} \frac{\dot{m} r^2}{3\pi \sigma} \quad \boxed{T_d^4 = \frac{3 \dot{m} r^2}{8\pi \sigma}} = \frac{3 G M \dot{m}}{8\pi \sigma r^3}$$

Concept: For accretion due to turbulence, viscosity deposits heat locally.  
Show diagram of brandenburg.

### Vertical temperature structure

Discuss the process. The energy is being deposited locally on the bottom, it will either travel upwards radiatively or via convection. The entropy gradient is adiabatic. Radiation



Viscous heating is concentrated near midplane

x Heat is transported upward radiatively

- x In an active disk the temperature is hotter in the midplane.
- x In a passive disk the temperature is colder in the midplane.

$$F = - \frac{16\sigma T^3}{3k_r \rho} \frac{dT}{dz}$$

$$F \cdot \rho dz = - \frac{16\sigma}{3k_r} T^3 dT \quad ; \quad F = \sigma T_{\text{eff}}^4 \text{ (const)}$$

$$\sigma T_{\text{eff}}^4 \cdot \int_0^\infty \rho dz = - \frac{16\sigma}{3k_r} \int_{T_c}^{T_{\text{eff}}} T^3 dT \quad \Rightarrow T_{\text{eff}}^4 \cdot \frac{\Sigma}{2} = - \frac{16}{3k_r} \left[ \frac{T_{\text{eff}}^4 - T_c^4}{K} \right]$$

$$\nabla \cdot T_c^4 \gg T_{\text{eff}}^4$$

$$T_{\text{eff}}^4 \cdot \frac{\varepsilon}{2} = \frac{4}{3k_r} [T_c^4 - T_{\text{eff}}^4] \quad \approx \frac{\varepsilon k}{2}$$

$$T_c^4 = \frac{3}{4} \approx T_{\text{eff}}^4$$

\$T\_c \approx T^{1/4} \text{ Teff}\$

Usually, for an active disk  $T_{\text{eff}}^4 \approx \frac{9}{4} \varepsilon \Omega^2$

decreases strongly with distance. Active heating important in inner disk.

$$T_c^4 \approx \frac{3}{4} \approx T_{\text{eff}}^4 + T_{\text{inr}}^4 / 2$$

Only one type of disk with clear active accretion: FU Orionis disks.

Accretion outburst up to  $\dot{m} \approx 10^4 M_\odot/\text{yr}$

10-100 yr

## MRI dead zones

Disk evolution equations for active disk

$$v = \eta g H$$

$$c_s^2 = \frac{kT}{\mu m_H}$$

$$\rho = \frac{1}{\sqrt{2\pi}} \frac{\Sigma}{H}$$

$$H = \frac{c_s}{\Omega}$$

$$\tau_c^4 = \frac{3}{4} \tau_{\text{eff}}^4$$

$$\tau = \frac{1}{2} \Sigma K_R$$

$$v\Sigma = \frac{m}{3\pi}$$

$$\tau_{\text{eff}}^4 = \frac{9}{8} v\Sigma R^2$$

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{\Gamma} \frac{\partial}{\partial r} \left[ r^{1/2} \frac{\partial}{\partial r} (v\Sigma r^{1/2}) \right]$$

## Quenching of the MRI

magnetic Reynolds number

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) ; \quad Re = \frac{\mathbf{v} \times \mathbf{B}}{\eta \nabla \times \mathbf{B}} = \frac{v B}{\eta B/L} = \frac{v \cdot L}{\eta} \quad (\text{similar to Reynolds number})$$

For  $\text{Re}_m < 1$

$$\frac{UL}{\eta} < 1 \quad ; \quad U = U_A; \quad L = \lambda_{\text{max}} \propto H; \quad$$

$$\eta_{\text{SPitzer}} \approx 2.3 \times 10^2 \frac{1}{\alpha} \sqrt{T} \frac{\text{cm}^2}{\text{s}} \quad ; \quad \alpha \equiv \text{ionization fraction}$$

$$\eta = \frac{c^2}{4\pi \sigma T_c} \quad (\sigma \Rightarrow \text{conductivity})$$

$$\text{from} \quad U_A \propto \sqrt{2} \cdot c_s \quad ( \quad \alpha \equiv \frac{B_\phi B_r}{4\pi \rho c^2} \approx \frac{\omega^2}{c_s^2} \quad )$$

$$\text{Re} = \frac{U_A H}{\eta} = \alpha^{1/2} \frac{c_s^2}{\eta \sqrt{2}}$$

$$\approx 1.4 \times 10^{12} \alpha \left( \frac{\alpha}{10^{-2}} \right)^{1/2} \left( \frac{R}{1 \text{ AU}} \right)^{3/2} \left( \frac{T_c}{300 \text{ K}} \right)^{1/2} \left( \frac{M_\oplus}{M_\odot} \right)^{-1/2}$$

$$\alpha_{\text{critical}} \approx 10^{-12}$$

very low. One ion in  $10^{12}$  neutrals is sufficient to couple the field to the disk.

Ionization state :

Solve equation

$$\frac{n_{\text{ion}} ne}{n} = 2 \frac{g_{\text{ion}}}{g} \left( \frac{2 \pi m_e k_B T}{h^2} \right)^{3/2} \exp \left( - \frac{\chi}{k_B T} \right)$$

( $\chi$ )  $\rightarrow$  ionization energy

Observations

$$\kappa = 8.6 \times 10^{-5} \frac{\text{eV}}{\text{n}}$$

$$\sim 10^4 \text{ eV/n}$$

$$\frac{\chi}{k_B T} = \frac{5}{10^{-4}} = 5 \times 10^4 \text{ n} \quad \text{too high ionization.}$$

$$\Delta V \propto \sqrt{v_{\text{ions}}^2 + 2 \frac{k_B T_{\text{kin}}}{M M_p}}$$

$$\sqrt{r} \sim \sqrt{v_{\text{visc}}} \Rightarrow \frac{\frac{D_r w}{w} \frac{B_z B_\phi}{B_z^2 + B_\phi^2}}{\varepsilon r} \sim \frac{3 v}{2 r}$$

$$v = \alpha g_1 t \quad \omega = \frac{c}{D_r} \frac{B_z^2}{B_z^2 + B_\phi^2}$$

$$\frac{\sigma_{r,\text{in}}}{\sigma_{r,\text{out}}} \sim \frac{B_z^s B_\phi^s}{B_z^c B_\phi^c} \left( \frac{h}{r} \right)^{-1} \quad (\text{not needed})$$

## Wind launching

at some point above, atmosphere becomes magnetically dominated.  
Must have  $\nabla \times B = 0$  so that acceleration is finite. The field becomes free-free.

$$\rho v^2 = \frac{B^2}{8\pi} \quad \text{mag energy} = \text{kinetic energy} \quad (\text{Alfvén surface})$$

Beyond Alfvén surface gas inertia bends lines, rotate in spiral.

Analysis of centrifugal wind:  $\frac{D_r w}{w} \Rightarrow ?$  (why?)

Discuss turbulence and winds, with a WE DON'T KNOW

## Phase separation