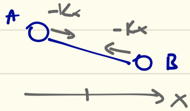
Magnetorotational Instability

Consider now that the gas parcels are connected by a tether with a restoring force $-Kx$



$$\frac{\partial u_x}{\partial t} - 2\Omega u_y = -Kx$$

$$\frac{\partial u_y}{\partial t} + (2-q)\Omega u_x = -Ky$$

$$x - 2\Omega y + Kx = 0$$
$$\ddot{y} + (2-q)\Omega x + Ky = 0$$

Consider the Lagrangian displacement $x(t) = \bar{x} + \xi(t)$, with $\bar{x} = 0$ and $\xi = X_0 e^{i\omega t}$

$$\dot{x} = i\omega\xi \qquad \ddot{x} = -\omega^2\xi$$

$$-\xi_x\omega^2 - 2\Omega\xi_y i\omega + K\xi_x = 0$$
$$-\xi_y\omega^2 + (2-q)\Omega\xi_x i\omega + K\xi_y = 0$$

$$\boxed{\omega^4 - (2K + \kappa^2)\omega^2 + K(K - 2q\Omega^2) = 0}$$

Bi-quadratic equation Solution $\omega^2 = \frac{(2\kappa + k^2)^2 \pm \sqrt{4\kappa(\kappa - 2q\Omega^2)}}{2}$

Condition for instability

$$\boxed{\kappa - 2q\,\Omega^2 < 0}$$

If $\kappa \Rightarrow 0$, the it is simply $q>0$, i.e, the angular velocity decreasing outward That is satisfied in Keplerian disks

Magnetic field $\equiv$ spring

$$\frac{\partial B'}{\partial t} = -(u\nabla)B + (B\nabla)u - \cancel{B(\nabla u)}$$

$$\delta B = B_z K \xi$$

$$\frac{(B\nabla)B}{\mu_0\rho} \quad \frac{\bar{B}\,\partial B}{\mu_0\rho} = \frac{B_z K B'\xi}{\mu_0\rho} = -\frac{K^2 B^2}{\mu_0\rho}\xi = -(K\,v_A)^2\xi$$

Equivalence between MRI and springs The magnetic tension behaves EXACTLY like a restoring force, provided $\kappa \equiv (k\,v_A)^2$ So, the instability condition is

$$\boxed{(k\,v_A)^2 - 2q\,\Omega^2 < 0}$$

Discuss weak and strong field limit

Show dispersion relation.

Critical wavelength

$$k v_A = 2q\Omega \quad \therefore \quad k = \sqrt{3}\,\frac{\Omega}{v_A}$$

$$\lambda_{crit} = \frac{2\pi}{k} = 2\pi \cdot \sqrt{3} \cdot \frac{v_A}{\Omega} \quad cH$$

Weak field instability

$\Omega \sim 10^{-8}$; $H = 10^{12}$;

$$v_A = \sqrt{\frac{2}{\beta}} \cdot c_s$$

$$v_A = \frac{B}{\sqrt{4\pi\rho}} ;$$

$\beta = 100$

$v_A \sim 0.1\,c_s$

$v_A \sim 100\,m/s$

$\sim 10^4\,cm/s$

$$B = v_A \cdot \sqrt{4\pi\rho}$$

$\rho \sim 10^{-11}\,g/cm^3$

$$B = v_A \cdot \sqrt{4\pi\rho}$$

$=$

$$\therefore B = 10^4 \cdot 10^{-5} \sim \underline{\underline{10^{-1}\,G}}$$

$$\rho = 10^{-8}\,\frac{Kg}{m^3}$$

weak field.

$$= 10^{-8} \cdot \frac{10^3\,g}{(10^2\,cm)^3} = \frac{10^3}{10^6}$$

$$10^{-8} \times 10^{-3} = 10^{-11}\,g\,cm^{3}$$