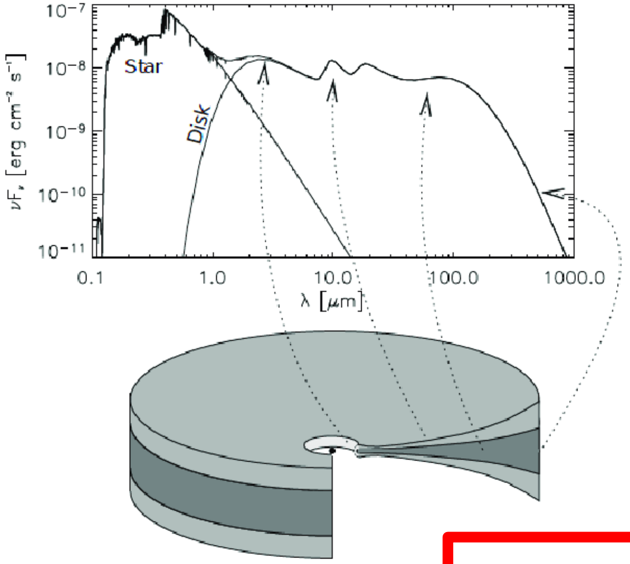
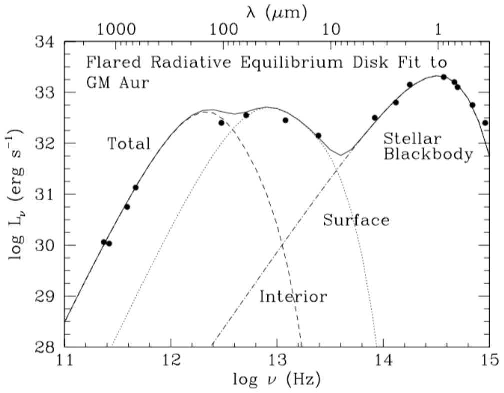
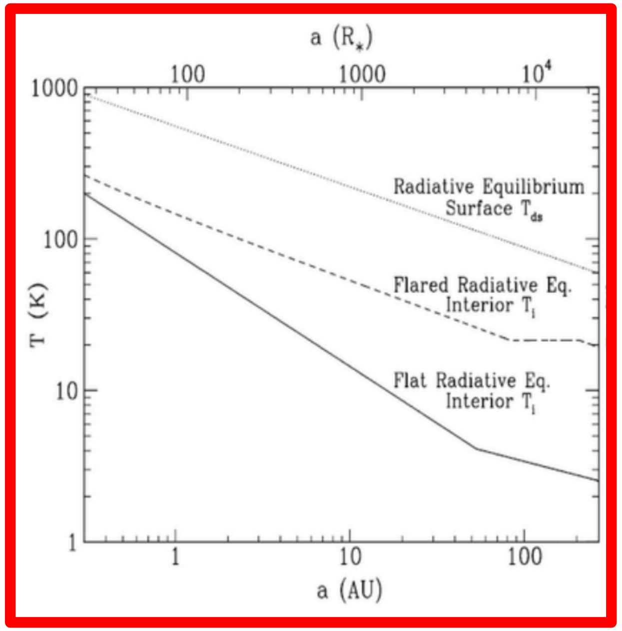
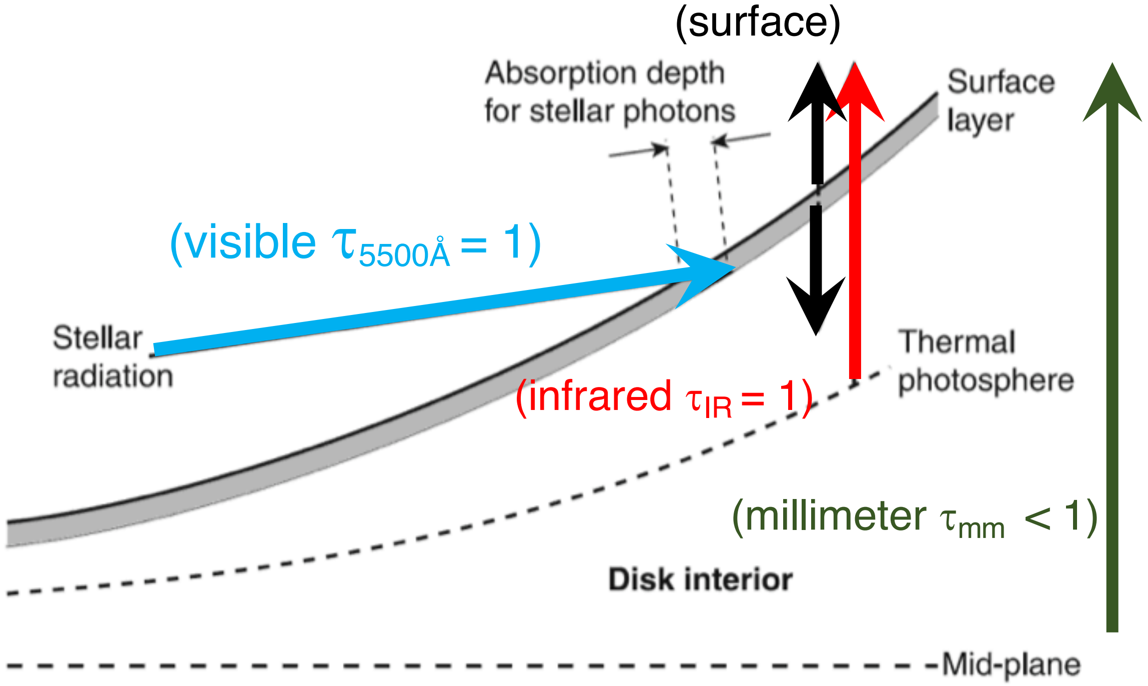
Class 11 – Feb 27^{th}, 2020

The Chiang-Goldreich model for Disk Temperature



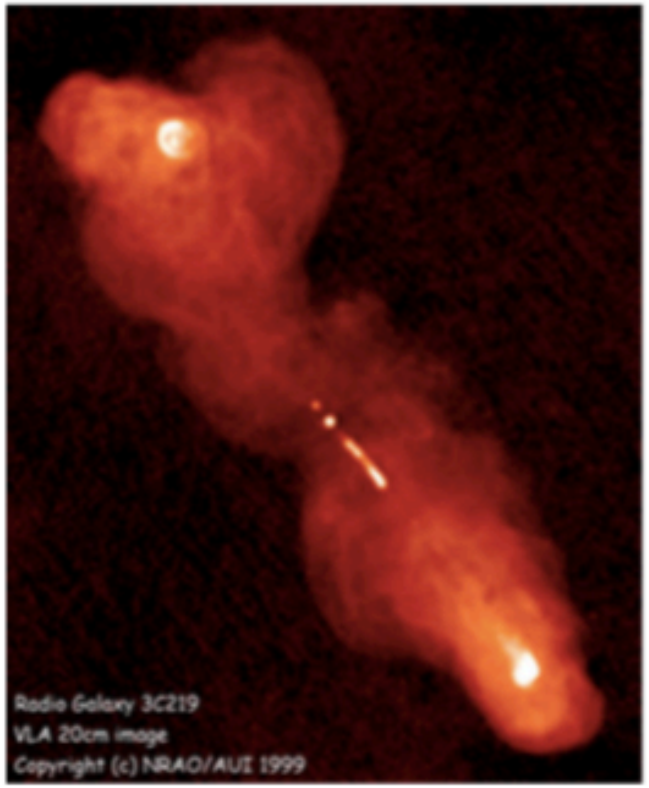






Chiang & Goldreich 1997

Accretion



300 Kp

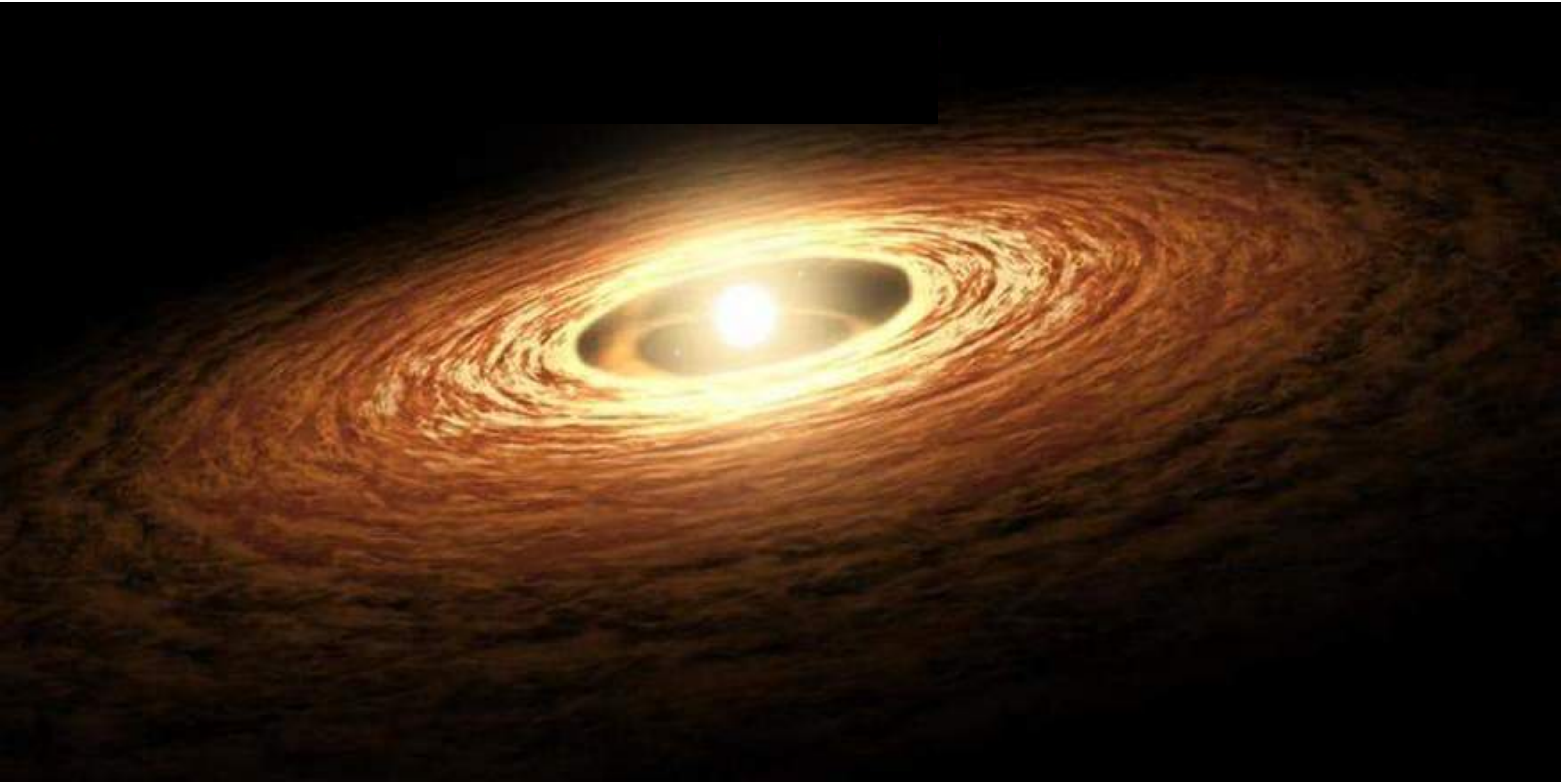
Accretion onto a central compact object is believed to power some of the most energetic phenomena in the universe

Black hole accretion
(Lynden-Bell 1969)

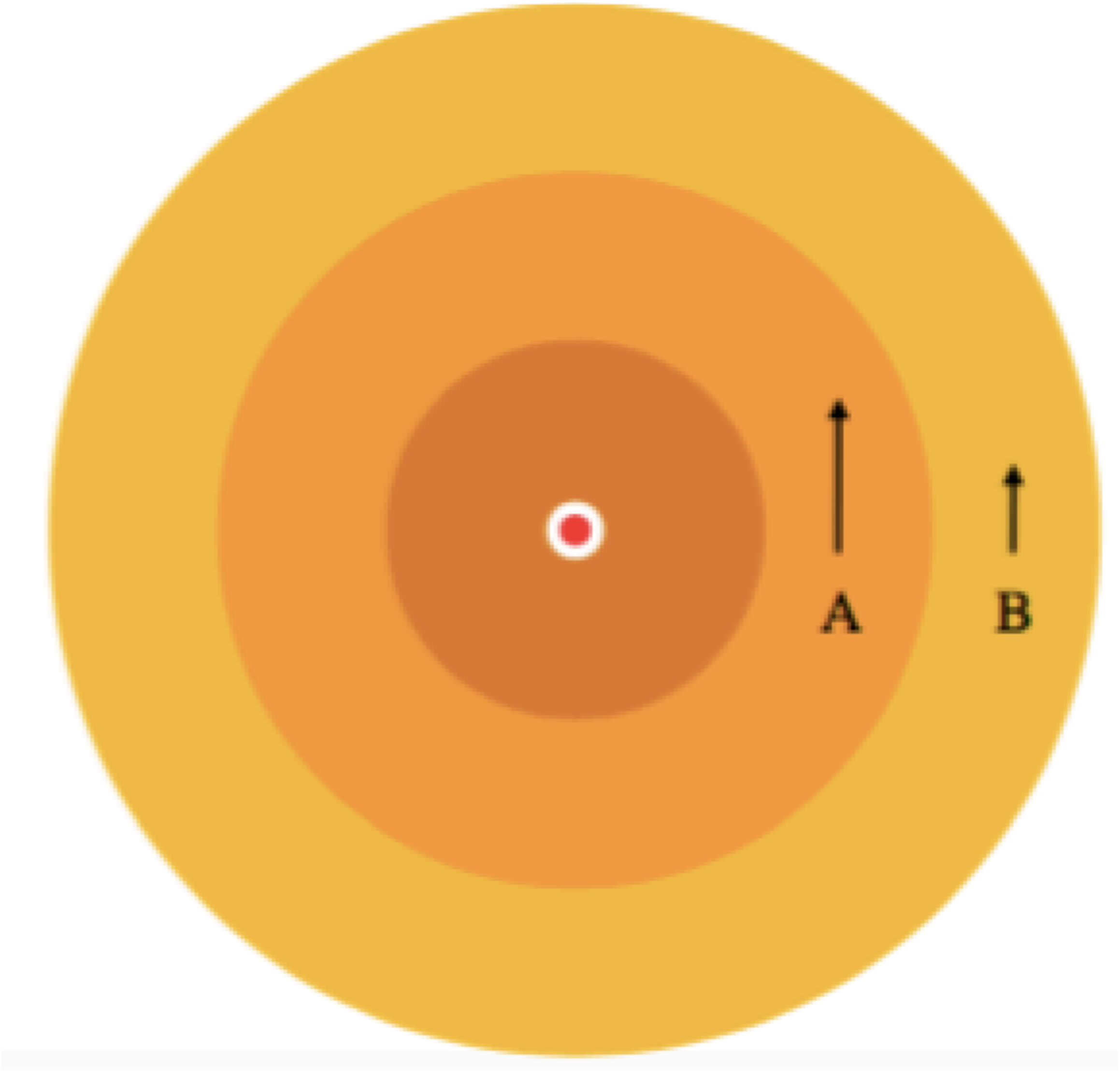
- **Central mass ~10^8-10^{10} M_{sun}**
- **Accretion rate ~1 M_{sun}/yr**
- **Total luminosity ~10^{47} L_{sun}**

"The central problem of nearly 30 years of accretion disk theory has been to understand how they accrete."

Balbus & Hawley (1998)



Viscous friction:
Outward transport of angular momentum



$$\nabla\times\mathbf{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial\theta} - \frac{\partial A_\theta}{\partial z}\right)\hat{\mathbf{e}}_r + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\mathbf{e}}_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\hat{\mathbf{e}}_z, \tag{C.3}$$

$$\nabla^2\psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{\partial^2\psi}{\partial z^2}. \tag{C.4}$$

C.2 The Navier–Stokes equation in cylindrical coordinates

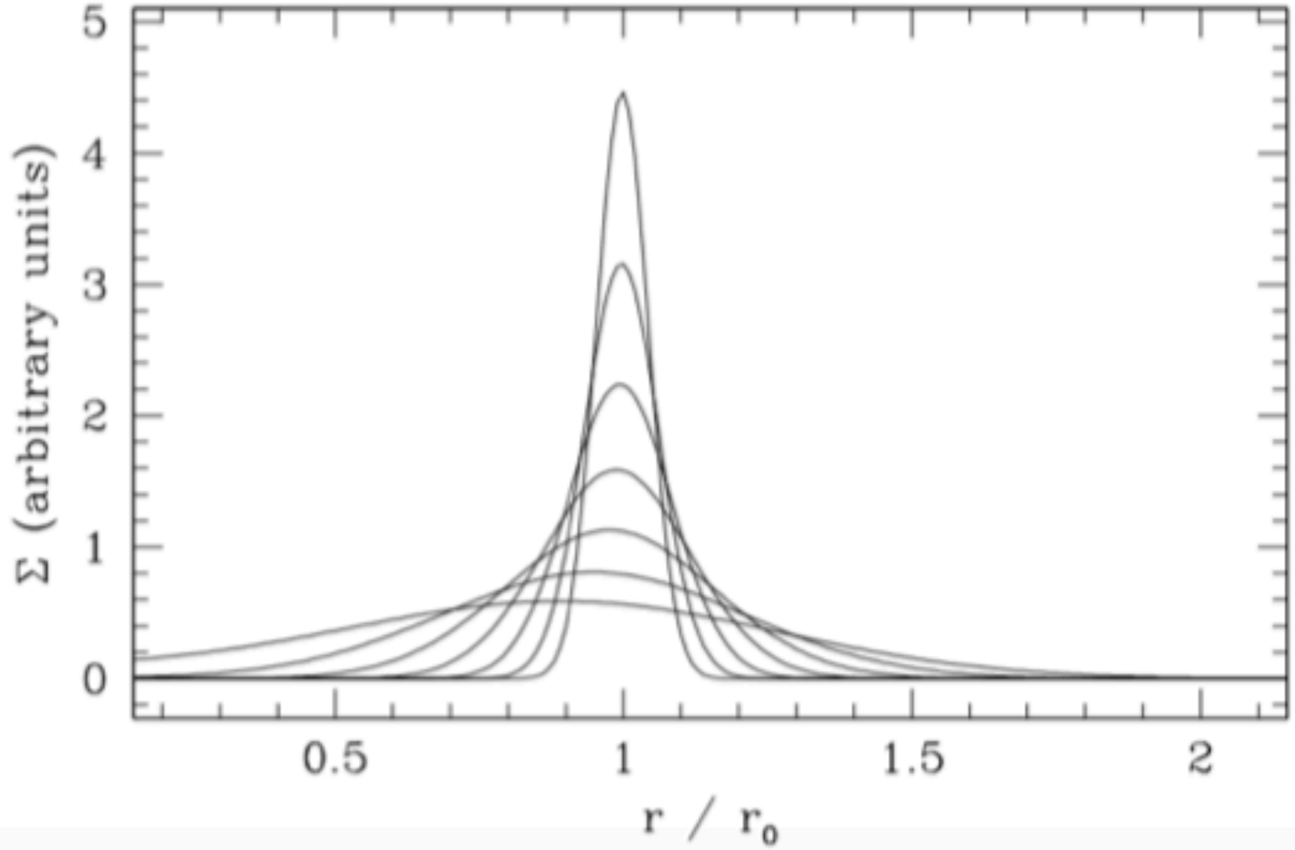
The components of the Navier–Stokes equation (5.10) in cylindrical coordinates are

$$\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2} + \frac{\partial^2 v_r}{\partial z^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta} - \frac{v_r}{r^2}\right) + F_r, \tag{C.5}$$

$$\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial\theta} + \nu\left(\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{1}{r}\frac{\partial v_\theta}{\partial r} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta} - \frac{v_\theta}{r^2}\right) + F_\theta, \tag{C.6}$$

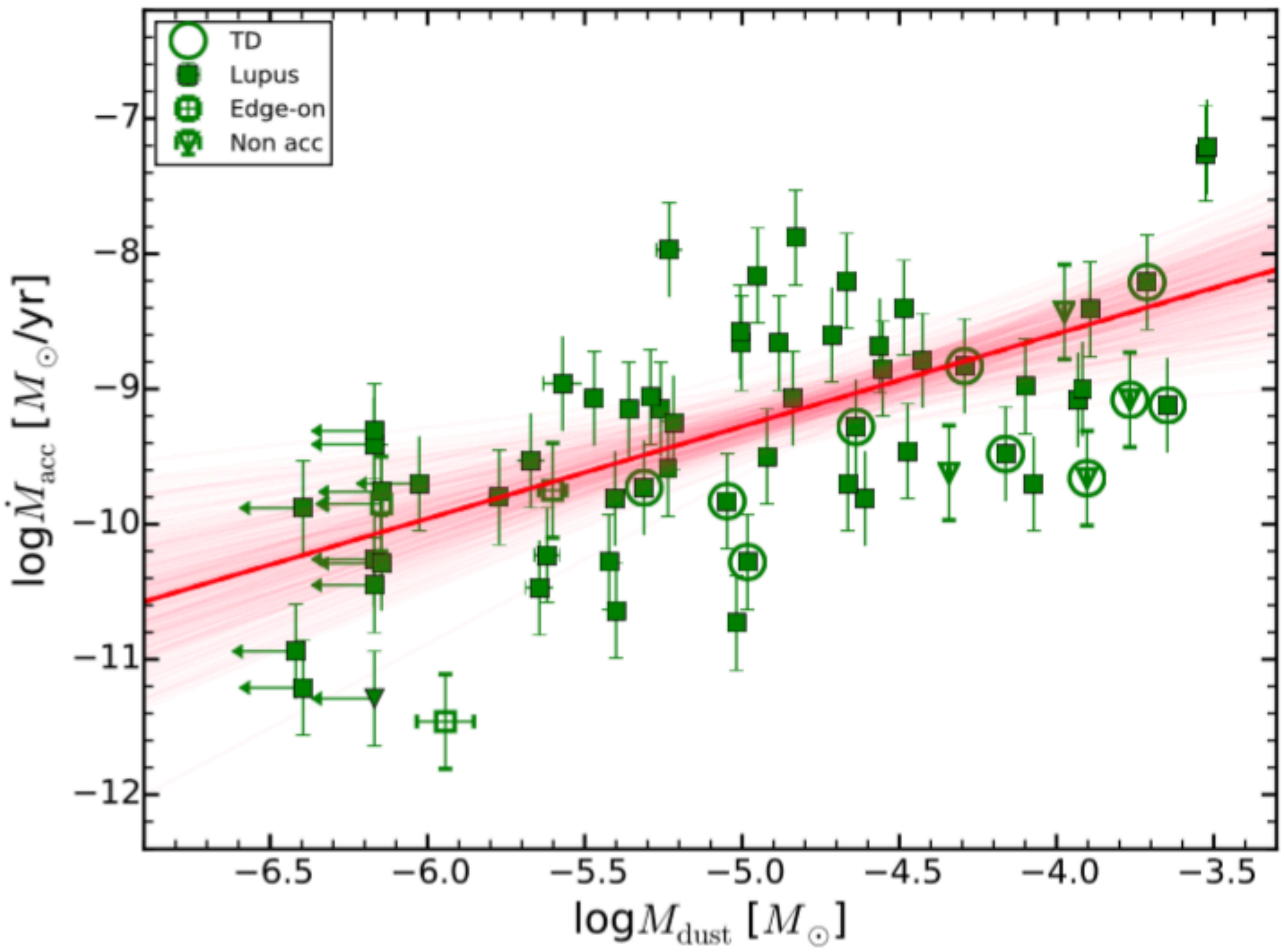
Analytical solution

$$\Sigma(x,\tau) = \frac{m}{\pi r_0^2}\frac{1}{\tau}x^{-1/4}\exp\left[-\frac{(1+x^2)}{\tau}\right]I_{1/4}\left(\frac{2x}{\tau}\right)$$



Initial condition: Dirac Delta.
A negligible amount of matter flows outwards, carrying the angular momentum.

Mass Accretion Rates



Manara et al 2016

The Shakura-Sunyaev model 1973