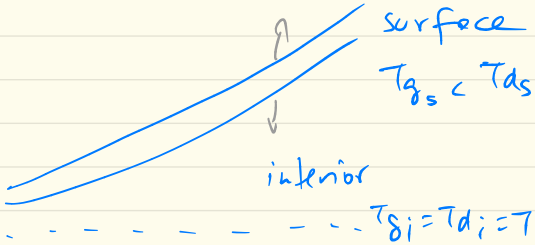
Class 10 – Chiang-Goldreich model

2/25/2020

The Chiang-Goldreich model is the standard model for passive disks. In essence it balances heating and cooling.



(dust at the surface is heated directly by starlight and is not in equilibrium with the gas).
(half the flux is radiated away, and half heats up the disk).

Balancing heating and cooling for a dust grain at the optical surface:

Heating : $\pi s^2 F_*$ — s: grain size ; F_* stellar flux

Cooling : $4\pi s^2 \sigma T_d^4 \varepsilon_d$ — T_d: grain temperature ; ε_d: dust emissivity

$$\boxed{T_{dust} = \left(\frac{1}{\varepsilon_d}\right)^{1/4} \left(\frac{R_*}{2r}\right)^{1/2} \cdot T_*}$$

The emissivity is $$\varepsilon_\lambda = \begin{cases} 1 & \text{if } \lambda \leq 2\pi s \\ \left(\frac{2\pi s}{\lambda}\right)^{\beta} & \text{if } \lambda > 2\pi s \end{cases}$$

where β is a free parameter of order unity.

If $\lambda > 2\pi s$, then $\varepsilon \propto \lambda^{-\beta}$, and according to Wien's displacement law, the peak wavelength correlates with the temperature. $\lambda_{peak}^{-1} \propto T$.

Normalizing by the stellar temperature

$$\frac{\varepsilon_{dust}}{\varepsilon_\star} = \left(\frac{T_{dust}}{T_{star}}\right)^\beta$$

and the stellar emissivity $\varepsilon_\star = 1$ given that the star is a blackbody. Solving for T_{dust}

$$\boxed{T_{dust} = \left(\frac{R}{2r}\right)^{\frac{2}{4+\beta}} \cdot T_\star}$$

For $\beta=1$, $T_{dust} \propto r^{-2/5}$. The temperature is higher than blackbody because the absorption in optical is higher than the emissivity in infrared.

Interior temperature

The disk interior is irradiated by half of the flux that strikes the surface

$$F_i \simeq \frac{F_{surf}}{2} \simeq \left(\frac{\alpha}{4}\right)\left(\frac{R_\star}{r}\right)^2 \sigma T_\star^4$$

with the grazing angle given by $\alpha \sim r \frac{d}{dr}\left(\frac{H}{r}\right)$

There are four possible regimes for capturing and re-processing this flux, depending on optical depth for absorption and emission

Capture	Emission
Optically thick	Optically thick
Optically thin	Optically thin

The regime of optically thin for absorption and optically thick for emission is discarded as $\tau_{absorption} > \tau_{emission}$. We are left with three regimes

1- Optically thick absorption and optically thick emission

Heating: $\left(\frac{\alpha}{4}\right)\left(\frac{R}{r}\right)^2 \sigma T_*^4$ Cooling: $S = \sigma T_i^4$

Solving for the interior temperature T_i:

$$\boxed{T_i = \left(\frac{\alpha}{4}\right)^{1/4}\left(\frac{R_*}{r}\right)^{1/2} T_*}$$

2- Optically thick absorption and optically thin emission

The heating is the same, but now the cooling is optically thin

Heating: $\left(\frac{\alpha}{4}\right)\left(\frac{R}{r}\right)^2 \sigma T_*^4$ Cooling: $S \cdot \tau = \underbrace{(\varepsilon_i \cdot \sigma T_i^4)}_{\text{source function}} \cdot \underbrace{(\kappa \Sigma)}_{\text{optical depth}}$

Solving for the interior temperature T_i:

$$T_i = \left(\frac{1}{\varepsilon_i \tau}\right)^{1/4}\left(\frac{\alpha}{4}\right)^{1/4}\left(\frac{R_*}{r}\right)^{1/2} T_*$$

Using the same emissivity model as used for the dust layer, $\varepsilon_i = \varepsilon_* \cdot (T_i/T_*)^\beta$. Solving for T_i:

$$T_i = \left(\frac{\alpha}{4\tau}\right)^{\frac{1}{4+\beta}}\left(\frac{R_*}{r}\right)^{\frac{2}{4+\beta}} T_*$$

Again, for $\beta = 1$;

$$\boxed{T_i = \left(\frac{\alpha}{4\tau}\right)^{1/5} \left(\frac{R_*}{r}\right)^{2/5} T_*}$$

3 - Optically thin absorption and optically thin emission

Now both the absorption and emission are non-black body

Heating: $\tau_s F_s$ Cooling: $\tau_i F_i$

$$F_i = \frac{\tau_s}{\tau_i} F_s$$

$$T_i = \left(\frac{\tau_s}{\tau_i}\right)^{1/4} \left(\frac{\alpha}{4}\right)^{1/4} \cdot \left(\frac{R_\odot}{r}\right)^{1/2} T_\odot$$

$$\boxed{T_i = \left(\frac{\alpha \varepsilon_s}{4 \varepsilon_i}\right)^{1/4} \left(\frac{R_\odot}{r}\right)^{1/2} T_\odot}$$

The temperature at a given radius is given by the maximum of these three.

Spectrum

Given these temperatures, the spectrum is computed

$$L_\nu = \text{Area} \times \text{Flux} = 8\pi^2 \nu \int_{r_0}^{r_1} \left[\int_{-\infty}^{\infty} B_\nu(T)\, e^{-\tau}\, d\tau \right] r\,dr$$

A numerical solution is required.