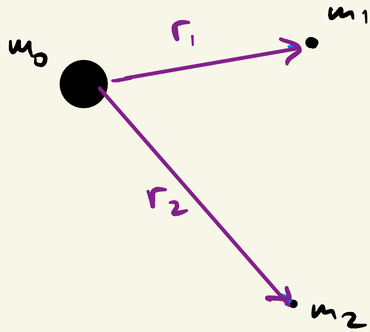
3-BODY PROBLEM



$$\mathcal{U}_i = -\frac{G(m_0+m_i)}{r_i} - \sum_{\substack{j=1\\ j\neq i}}^{N} G m_j \left[\frac{1}{|\vec{r}_j-\vec{r}_i|} - \frac{\vec{r}_i \cdot \vec{r}_j}{r_j^3} \right]$$

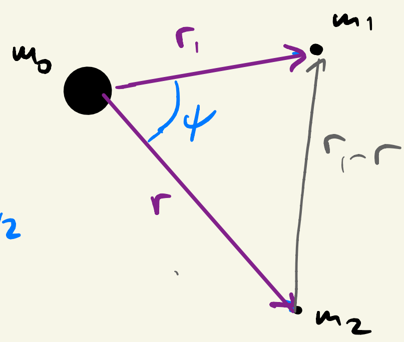
$m_2 = 0$

$$\mathcal{U}_1 = -\frac{G(m_0+m_1)}{r_1}$$ 2-BODY MOTION

$$\mathcal{U}_2 = -\frac{Gm_0}{r_2} - Gm_1 \left[\frac{1}{|r_1-r_2|} - \frac{\vec{r}_2 \cdot \vec{r}_1}{r_1^3} \right]$$

$$H = -\frac{Gm_0}{2a} - Gm_1 \left[\frac{1}{|r_1-r_2|} - \frac{\vec{r}_2 \cdot \vec{r}_1}{r_1^3} \right]$$

$$H_{PERT} = -Gm_1\left[\frac{1}{|r_1 - r|} - \frac{\vec{r}\cdot\vec{r}_1}{r_1^3}\right]$$



$$|r_1 - r| = \left(r_1^2 + r^2 - 2rr_1\cos\psi\right)^{1/2}$$

$$\frac{1}{|r_1 - r|} = \frac{1}{\left(r_1^2 + r^2 - 2rr_1\cos\psi\right)^{1/2}}$$

$$\sum_{n=0}^{\infty} P_n(x)t^n = \frac{1}{\sqrt{1-2xt+t^2}}$$

LEGENDRE POLYNOMIALS

$$\frac{1}{|r_1 - r|} = \frac{1}{r_1}\,\frac{1}{\sqrt{1 - 2(r/r_1)\cos\psi + (r/r_1)^2}}$$

$t = \frac{r}{r_1}$

$x = \cos\psi$

$$\frac{1}{|r_1 - r|} = \frac{1}{r_1}\sum_{n=0}^{\infty}\left(\frac{r}{r_1}\right)^n P_n(\cos\psi)$$

$$H_{PERT} = -Gm_1 \left[\frac{1}{r_1} \sum_{n=0}^{\infty} \left(\frac{r}{r_1}\right)^n P_n(\cos\psi) - \frac{\vec{r}\cdot\vec{r}_1}{r_1^3} \right]$$

$$\vec{r}\cdot\vec{r}_1 = r_1 r \cos\psi$$

$$= r_1 r P_1(\cos\psi)$$

$$P_0(x) = 1$$

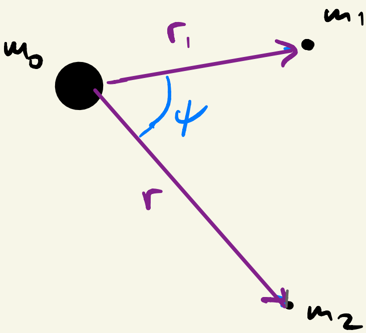
$$P_1(x) = x$$

$$\frac{\vec{r}\cdot\vec{r}_1}{r_1^3} = \frac{r}{r_1^2} P_1(\cos\psi)$$

$$H_{PERT} = -Gm_1 \left\{ \frac{1}{r_1}\left[P_0(\cos\psi) + \cancel{\frac{r}{r_1} P_1(\cos\psi)} + \sum_{n=2}^{\infty}\left(\frac{r}{r_1}\right)^n P_n(\cos\psi)\right] - \cancel{\frac{r}{r_1^2} P_1(\cos\psi)} \right\}$$

$$= -\frac{Gm_1}{r_1}\left[1 + \sum_{n=2}^{\infty}\left(\frac{r}{r_1}\right)^n P_n(\cos\psi)\right]$$

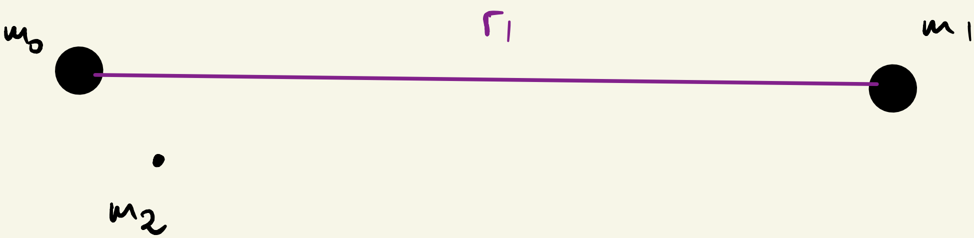
$$H_{PERT} = -\frac{Gm_1}{r_1} \sum_{n=2}^{\infty}\left(\frac{r}{r_1}\right)^n P_n(\cos\psi)$$



$$H_{PERT} = -\frac{Gm_1}{r_1}\sum_{n=2}^{\infty}\left(\frac{r}{r_1}\right)^n P_n(\cos\psi)$$

$$\approx -\frac{Gm_1}{r_1}\left(\frac{r}{r_1}\right)^2 P_2(\cos\psi)$$

$r \ll r_1$

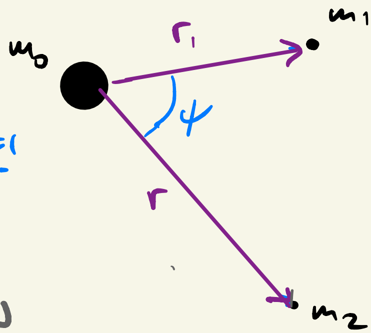


$$H_{PERT} = -\frac{Gm_1}{r_1}\sum_{n=2}^{\infty}\left(\frac{r}{r_1}\right)^n P_n(\cos\psi)$$

KEEP ONLY THE $n=2$ TERM ; $P_2(x) = \frac{1}{2}\left(3x^2 - 1\right)$

$$H_{PERT} \approx -\frac{Gm_1}{r_1}\left(\frac{r}{r_1}\right)^2 P_2(\cos\psi)$$

$$H_{PERT} \approx -\frac{Gm_1}{2}\frac{r^2}{r_1^3}\left[3\cos^2\psi - 1\right]$$



$$\cos\psi = \frac{\vec{r}\cdot\vec{r}_1}{r r_1} = \frac{x x_1 + y y_1 + z z_1}{r r_1}$$

IF THE ORBITS ARE KEPLERIAN

$$\frac{x}{r} = \cos\Omega\cos(\omega+f) - \sin\Omega\sin(\omega+f)\cos I$$

$$\frac{y}{r} = \sin\Omega\cos(\omega+f) + \cos\Omega\sin(\omega+f)\cos I$$

$$\frac{z}{r} = \sin(\omega+f)\sin I$$

$$\cos\psi = \frac{\vec{r}\cdot\vec{r}_1}{r r_1} = \frac{x x_1 + y y_1 + z z_1}{r r_1}$$

$$\frac{x}{r} = \cos\Omega\cos(\omega+f) - \sin\Omega\sin(\omega+f)\cos I$$

$$\frac{y}{r} = \sin\Omega\cos(\omega+f) + \cos\Omega\sin(\omega+f)\cos I$$

$$\frac{z}{r} = \sin(\omega+f)\sin I$$

$$\cos\psi = \frac{x x_1}{r r_1} + \frac{y y_1}{r r_1} + \frac{z z_1}{r r_1} =$$

$\Omega_1 = \omega_1 = 0$

$f_1 = M_1$

$$= \left[\cos\Omega\cos(\omega+f) - \sin\Omega\sin(\omega+f)\cos I\right] \times$$

$$\left[\cancelto{1}{\cos\Omega_1}\cos(\cancel{\omega_1}+f_1) - \cancel{\sin\Omega_1\sin(\omega_1+f_1)}\cancelto{1}{\cos I_1}\right]$$

$$+$$

$$\left[\sin\Omega\cos(\omega+f) + \cos\Omega\sin(\omega+f)\cos I\right] \times$$

$$\left[\cancel{\sin\Omega_1\cos(\omega_1+f_1)} + \cancelto{1}{\cos\Omega_1}\sin(\cancel{\omega_1}+f_1)\cancelto{1}{\cos I_1}\right]$$

$$+ \cancel{\left[\sin(\omega+f)\sin I\right]\left[\sin(\omega_1+f_1)\cancelto{0}{\sin I_1}\right]}$$

$\cos\psi \quad (I_1 = \Omega_1 = \omega_1 = 0)$

$$= [\cos\Omega \cos(\omega+f) - \sin\Omega \sin(\omega+f) \cos I] \cos M_1$$

$$+ [\sin\Omega \cos(\omega+f) + \cos\Omega \sin(\omega+f) \cos I] \sin M_1$$

$$H_{PERT} \approx -\frac{G m_1}{2} \frac{r^2}{r_1^3} \left[3\cos^2\psi - 1\right]$$

θ	J
M	E
ω	L
Ω	L_z

$$\frac{\partial H}{\partial \theta} = -\dot{J} \neq 0$$