THE CIRCULAR RESTRICTED THREE BODY PROBLEM

JACOBI CONSTANT

$$\ddot{r} = -\frac{\mu_1}{r_1^3} r_1 - \frac{\mu_2}{r_2^3} r_2$$

FRAME COROTATING WITH SECONDARY

$$(\ddot{r})_{COROT} = (\ddot{r})_{INERTIAL} - \Omega \times (\Omega \times r) - 2\Omega \times \dot{r}$$

$$\Omega \equiv \Omega \hat{z}$$

$$\ddot{x} - 2\Omega \dot{y} = \Omega^2 x - \mu_1 \frac{(x+\mu_2)}{r_1^3} - \mu_2 \frac{(x-\mu_1)}{r_2^3}$$

$$\ddot{y} + 2\Omega \dot{x} = \Omega^2 y - \frac{\mu_1 y}{r_1^3} - \frac{\mu_2 y}{r_1^3}$$

$$\ddot{z} = -\frac{\mu_1}{r_1^3} z - \frac{\mu_2}{r_2^3} z$$

$$\ddot{x} - 2\Omega\dot{y} = \Omega^2 x - \mu_1 \frac{(x+\mu_2)}{r_1^3} - \mu_2 \frac{(x-\mu_1)}{r_2^3}$$

$$\ddot{y} + 2\Omega\dot{x} = \Omega^2 y - \frac{\mu_1 y}{r_1^3} - \frac{\mu_2 y}{r_1^3}$$

$$\ddot{z} = -\frac{\mu_1}{r_1^3} z - \frac{\mu_2}{r_2^3} z$$

$$\ddot{x} - 2\Omega\dot{y} = \frac{\partial U}{\partial x} \qquad (\dot{x})$$

$$\ddot{y} + 2\Omega\dot{x} = \frac{\partial U}{\partial y} \qquad (\dot{y})$$

$$\ddot{z} = \frac{\partial U}{\partial z} \qquad (\dot{z})$$

$$U = \frac{\Omega^2}{2}(x^2+y^2) + \frac{\mu_1}{r_1} + \frac{\mu_2}{r_2}$$

$$\dot{x}\ddot{x} + \dot{y}\ddot{y} + \dot{z}\ddot{z} = \frac{\partial U}{\partial x}\dot{x} + \frac{\partial U}{\partial y}\dot{y} + \frac{\partial U}{\partial z}\dot{z} \equiv \frac{dU}{dt}$$

JACOBI CONSTANT!

$$\dot{x}^2 + \dot{y}^2 + \dot{z}^2 + C = 2 \cdot U$$

$$C_J = 2 \cdot U - v^2$$

$$\boxed{C_J = \Omega^2(x^2+y^2) + 2\left(\frac{\mu_1}{r_1} + \frac{\mu_2}{r_2}\right) - v^2}$$

Tisserand Parameter

$$C_J = 2U - v^2$$

$$C_J = \Omega^2(x^2+y^2) + 2\left(\frac{\mu_1}{r_1} + \frac{\mu_2}{r_2}\right) - v^2$$ COROTATIONAL

$$C_J = 2\Omega(x\dot{y} - y\dot{x}) + 2\left(\frac{\mu_1}{r_1} + \frac{\mu_2}{r_2}\right) - v^2$$ INERTIAL

$$\frac{v^2}{2} - \frac{1}{r} = \frac{1}{2a} \rightarrow v^2 = \frac{2}{r} - \frac{1}{a} \qquad (\mu = 1)$$

$$L = r \times \dot{r} \qquad L_z = x\dot{y} - y\dot{x} \qquad L_z = L \cdot \hat{z} = L\cos I$$

$$L^2 = a(1-e^2)$$

$$C_J = 2\overset{1}{\cancel{\Omega}} L\cos I + \cancel{2\left(\frac{\overset{1}{\mu_1}}{r_1} + \frac{\mu_2}{r_2}\right)} - \cancel{\frac{2}{r}} + \frac{1}{a}$$

$\mu_1 = 1 = \Omega$

$\mu_2 \ll \mu_1$

$\mu_2 \to 0$

$$C_J \approx 2\cos I\sqrt{a(1-e^2)} + \frac{1}{a}$$

$$\boxed{\frac{1}{2a} + \sqrt{a(1-e^2)}\cos I \approx \text{constant}}$$