Kepler's Laws

Names: ______________________________

I. "The orbits of the planets are ellipses with the Sun at one focus."
II. "A line from the planet to the Sun sweeps out equal areas in equal intervals of time."
III. "A planet's orbital period squared is proportional to its average distance from the Sun cubed: $P^2 \propto a^3$"

Exercise #1: On the ellipse in Fig. 1 are two X's. Confirm that that sum of the distances between the two foci to any point on the ellipse is always the same by measuring the distances between the foci, and the two spots identified with X's. Show your work. (**2 points**)

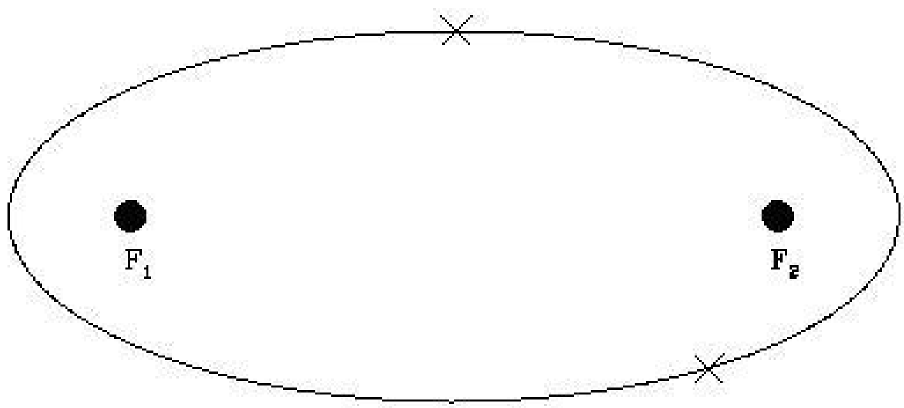

Figure 1: An ellipse with the two foci identified.

Exercise #2: In the ellipse shown in Fig. 2, two points ("P_1" and "P_2") are identified that are not located at the true positions of the foci. Repeat exercise #1, but confirm that P_1 and P_2 are not the foci of this ellipse. (**2 points**)

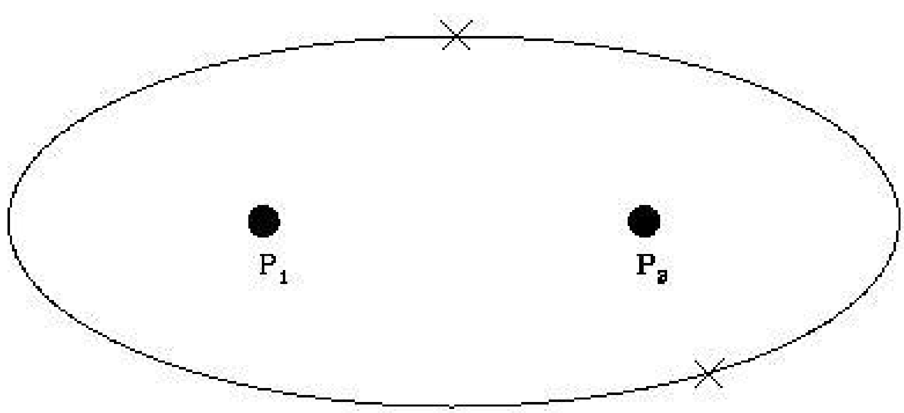

Figure 2: An ellipse with two non-foci points identified.

Exercise #3: Kepler's First Law: Describe the results that are displayed in the right hand panel for this first simulation. (**2 points**).

Describe what the ellipse looks like at 1.35 vs. that at 1.2. Does the sum of the vectors (right hand panel) still add up to a constant? (**3 points**)

Now let's put the Initial Velocity down to a value of 1.0. Run the simulation. What is happening here? The orbit is now a circle. Where are the two foci located? In this case, what is the distance between the focus and the orbit equivalent to? (**4 points**)

Exercise #4: Kepler's Second Law: Describe what is happening here. Does this confirm Kepler's second law? How? When the planet is at perihelion, is it moving slowly or quickly? Why do you think this happens? (**4 points**)

- If R = 1, what does $1/R^2$ = ____________________________?
- If R = 2, what does $1/R^2$ = ____________________________?
- If R = 4, what does $1/R^2$ = ____________________________?

What is happening here? As R gets bigger, what happens to $1/R^2$? Does $1/R^2$ decrease/increase quickly or slowly? (**2 points**)

The equation for the force of gravity has a $1/R^2$ in it, so as R increases (that is, the two objects get further apart), does the force of gravity *felt* by the body get larger, or smaller? Is the force of gravity stronger at perihelion, or aphelion? Newton showed that the speed of a planet in its orbit depends on the force of gravity through this equation:

$$V = \sqrt{(G(M_{\text{sun}} + M_{\text{planet}})(2/r - 1/a))}$$

where "r" is the radial distance of the planet from the Sun, and "a" is the mean orbital radius (the semi-major axis). Do you think the planet will move faster, or slower when it is closest to the Sun? Test this by assuming that r = 0.5a at perihelion, and r = 1.5a at aphelion, and that a=1! [Hint, simply set $G(M_{\text{sun}} + M_{\text{planet}}) = 1$ to make this comparison very easy!] Does this explain Kepler's second law? (**4 points**)

What do you think the motion of a planet in a circular orbit looks like? Is there a definable perihelion and aphelion? Make a prediction for what the motion is going to look like–how are the triangular areas seen for elliptical orbits going to change as the planet orbits the Sun in a circular orbit? Why? (**3 points**)

Now let's run a simulation for a circular orbit by setting the Initial Velocity to 1.0. What happened? Were your predictions correct? (**3 points**)

Exercise 4: Kepler's Third Law: If an asteroid has an average distance from the Sun of 4 AU, what is its orbital period? Show your work. (**2 points**)

In the Third Law simulation, there is a slide bar to set the average distance from the Sun for any hypothetical solar system body. At start-up, it is set to 4 AU. Run the simulation, and confirm the answer you just calculated. Note that for each orbit of the inner planet, a small red circle is drawn on the outer planet's orbit. Count up these red circles to figure out how many times the Earth revolved around the Sun during a single orbit of the asteroid. Did your calculation agree with the simulation? Describe your results. (**2 points**)

If you were observant, you noticed that the program calculated the number of orbits that the Earth executed for you (in the "Time" window), and you do not actually have to count up the little red circles. Let's now explore the orbits of the nine planets in our solar system. In the following table are the semi-major axes of the nine planets. Note that the "average distance to the Sun" (a) that we have been using above is actually a quantity astronomers call the "semi-major axis" of a planet. a is simply one half the major axis of the orbit ellipse. Fill in the missing orbital periods of the planets by running the Third Law simulator for each of them. (**3 points**)

Table 1: The Orbital Periods of the Planets

Planet	a (AU)	P (yr)
Mercury	0.387	0.24
Venus	0.72	
Earth	1.000	1.000
Mars	1.52	
Jupiter	5.20	
Saturn	9.54	29.5
Uranus	19.22	84.3
Neptune	30.06	164.8
Pluto	39.5	248.3

Notice that the further the planet is from the Sun, the slower it moves, and the longer it takes to complete one orbit around the Sun (its "year"). Neptune was discovered in 1846, and Pluto was discovered in 1930 (by Clyde Tombaugh, a former professor at NMSU). How many orbits (or what fraction of an orbit) have Neptune and Pluto completed since their discovery? (**3 points**)

Exercise 5: Binary Star systems. Use the slide bars (or type in the numbers) to set Transverse Velocity = 1.0, Velocity (magnitude) = 0.0, and Direction = 0.0. For now, we simply want to play with the mass ratio. Use the slide bar so that Mass Ratio = 1.0. Click "Ok". This now sets up your new simulation. Click Run. Describe the simulation. What are the shapes of the two orbits? Where is the center of mass located relative to the orbits? What does q = 1.0 mean? Describe what is going on here. (**4 points**)

Ok, now we want to run a simulation where only the mass ratio is going to be changed. Go back to Input and enter in the correct mass ratio for a binary star system with $M_1 = 4.0$, and $M_2 = 1.0$. Run the simulation. Describe what is happening in this simulation. How are the stars located with respect to the center of mass? Why? [Hint: see Fig. 4.6.] (**4 points**)

Go back to Input and now set the Transverse Velocity = 1.3. Run the simulation. Describe what is happening. When do the stars move the fastest? The slowest? Does this make sense? Why/why not? (**4 points**)

In the "Two-Body and Many-Body" simulations window, click on the "Dbl. Star and a Planet" button. Here we simulate the motion of a planet going around the less massive star in a binary system. Click Go. Describe the simulation—what happened to the planet? Why do you think this happened? (**4 points**)

Let's see if you can keep the planet from wandering away from its parent star. Click on the "Settings" window. As you can tell, now that we have three bodies in the system, there are lots of parameters to play with. But let's confine ourselves to two of them: "Ratio of Stars Masses" and "Planet–Star Distance". The first of these is simply the q we encountered above, while the second changes the size of the planet's orbit. The default values of both at the start-up are $q = 0.5$, and Planet–Star Distance = 0.24. Run simulations with $q = 0.4$ and 0.6. Compare them to the simulations with $q = 0.5$. What happens as q gets larger, and larger? What is increasing? How does this increase affect the force of gravity between the star and its planet? (**4 points**)

[**Note: This is very labor intensive, so do this question last (it is optional). I will give you extra credit if you're correct to 3 decimal places!**] See if you can find the value of q at which larger values cause the planet to "stay home", while smaller values cause it to (eventually) crash into one of the stars (stepping up/down by 0.01 should be adequate). (**2 points**)

Ok, reset $q = 0.5$, and now let's adjust the Planet–Star Distance. In the Settings window, set the Planet–Star Distance = 0.1 and run a simulation. Note the outcome of this simulation. Now set Planet–Star Distance = 0.3. Run a simulation. What happened? Did the planet wander away from its parent star? Are you surprised? (**4 points**)