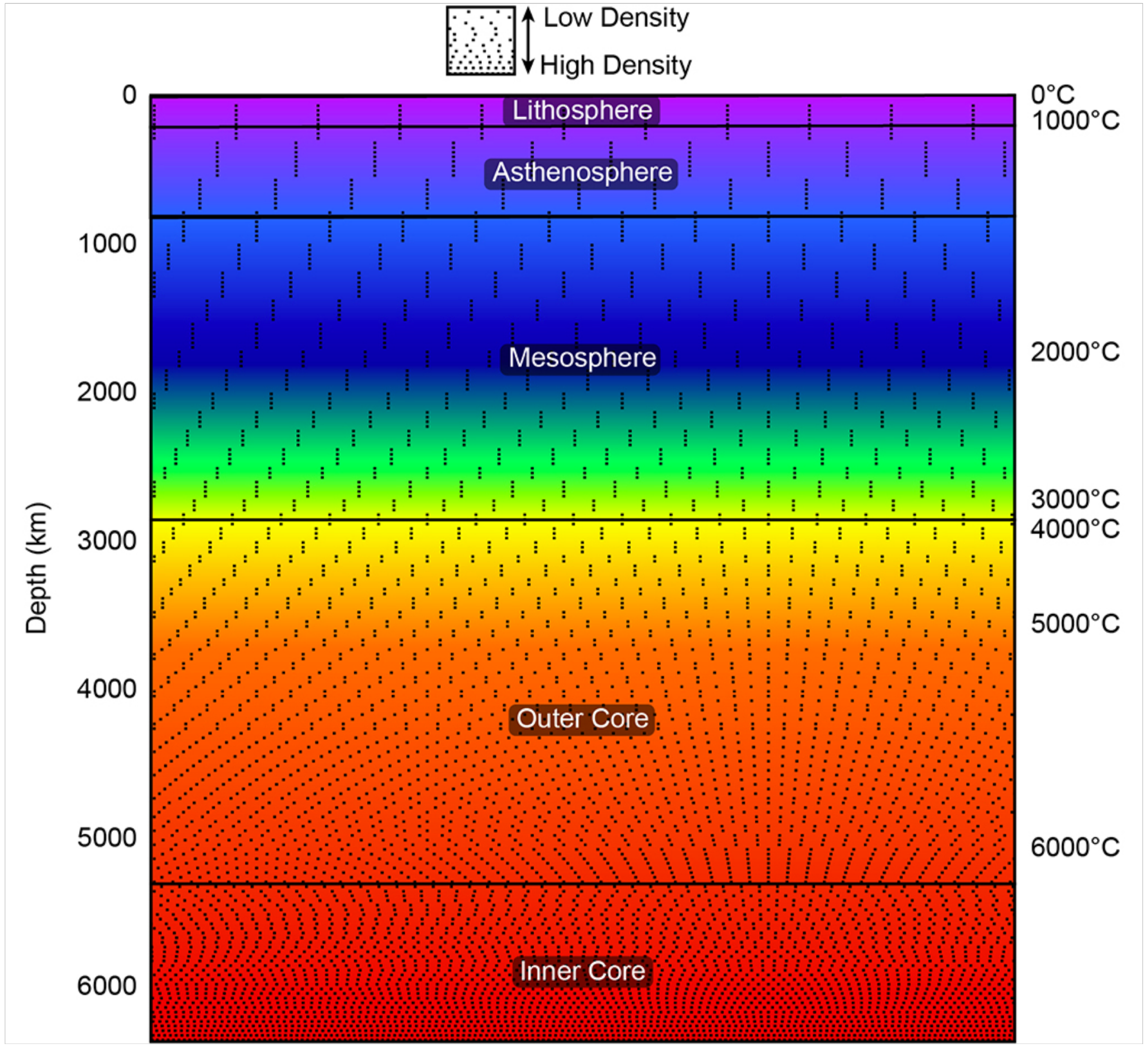
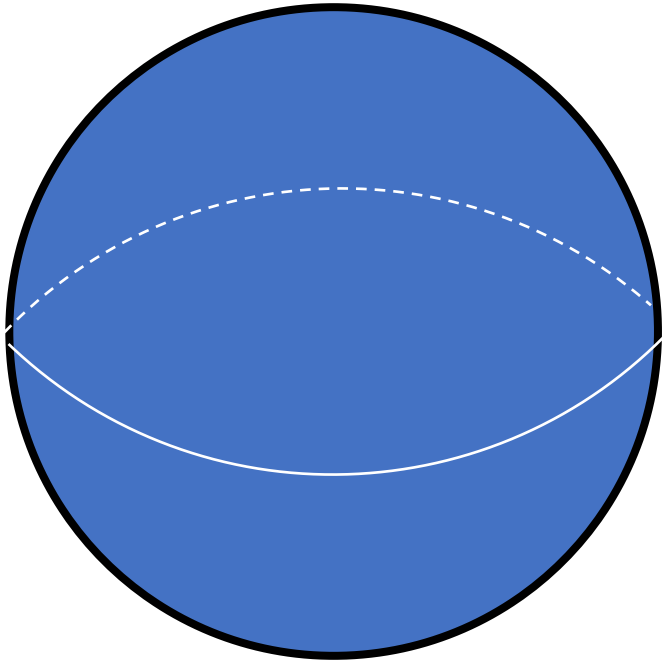
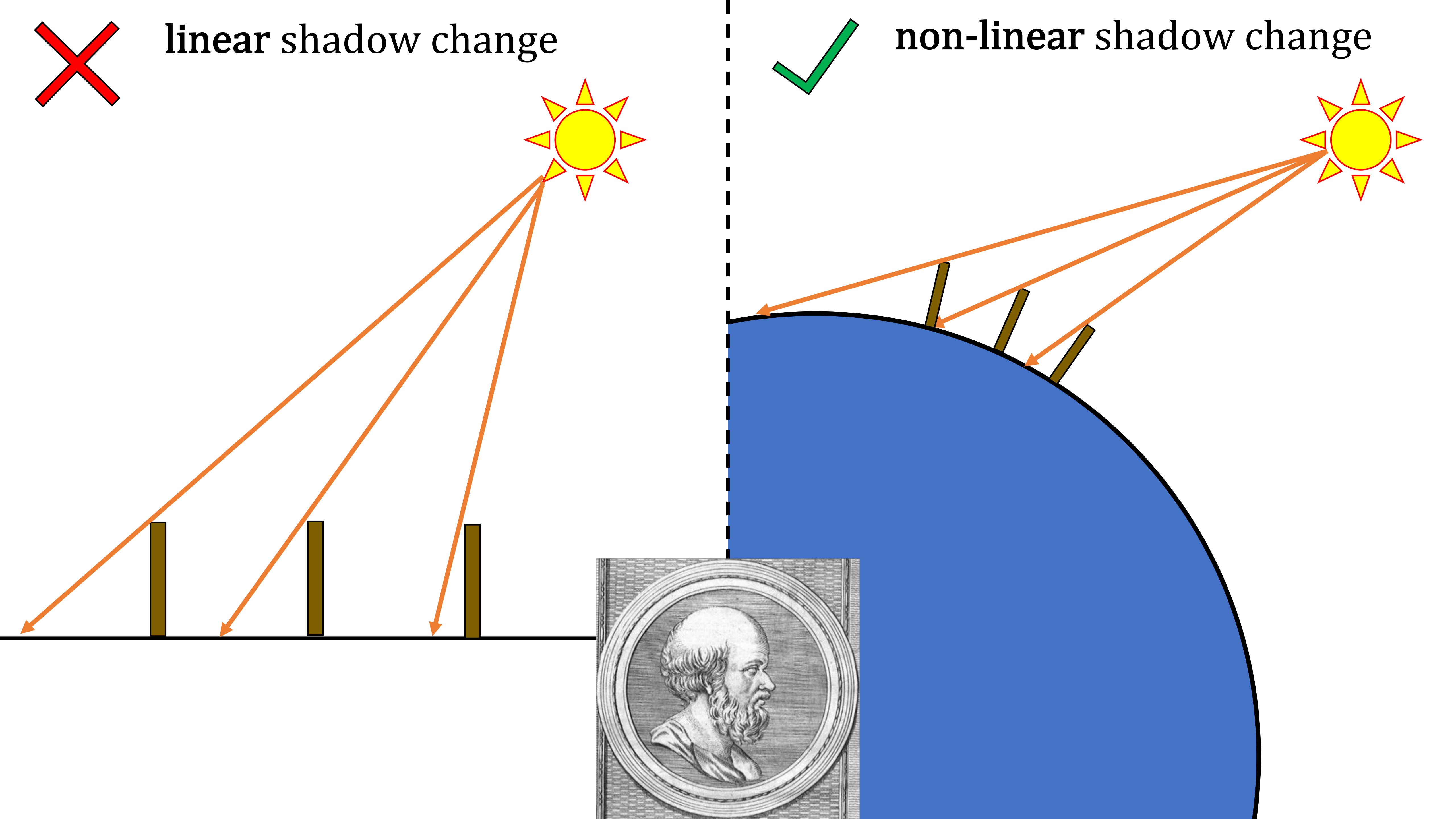
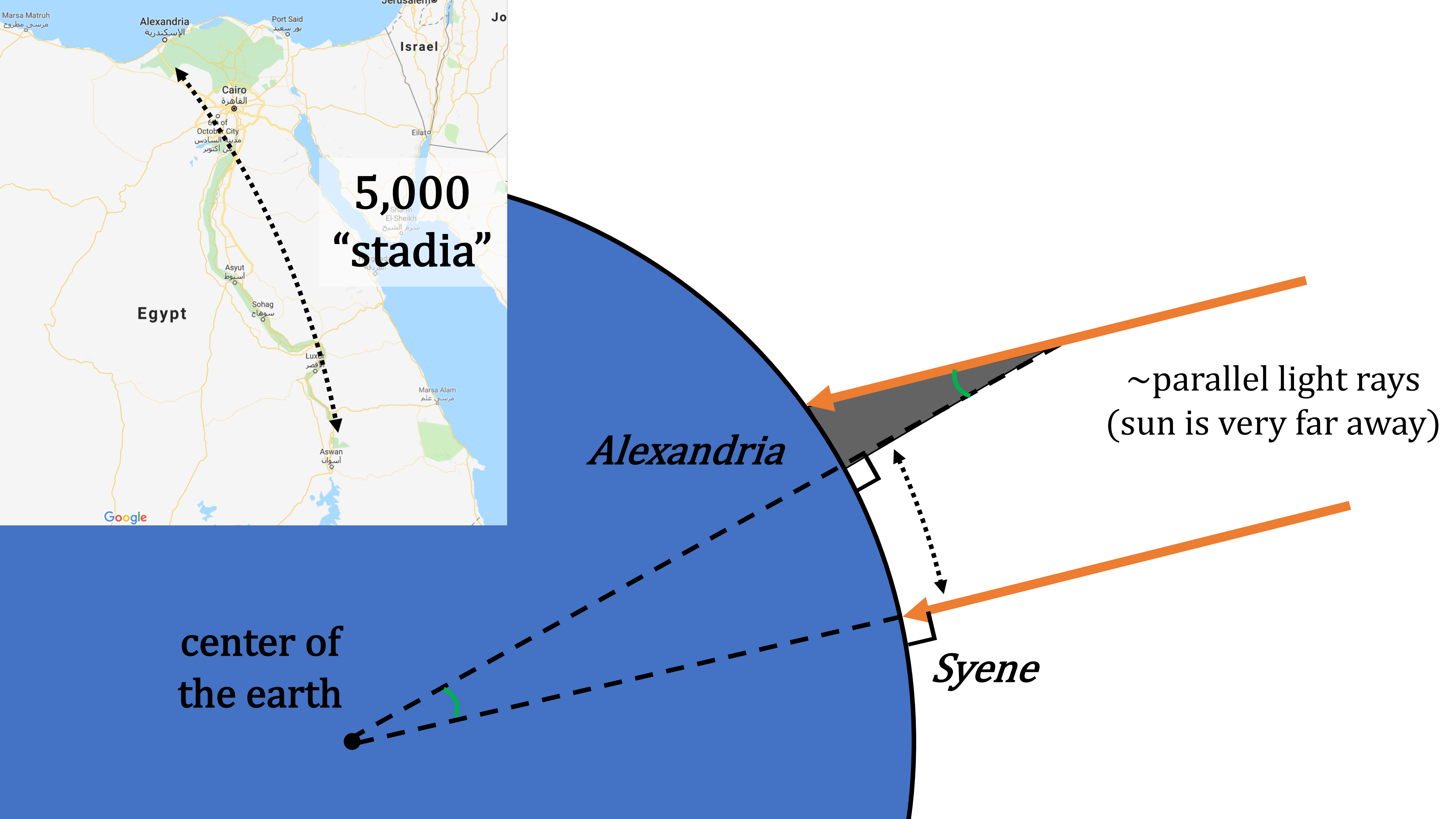
Estimating the Earth's Density





$$\rho = \frac{M}{V}$$

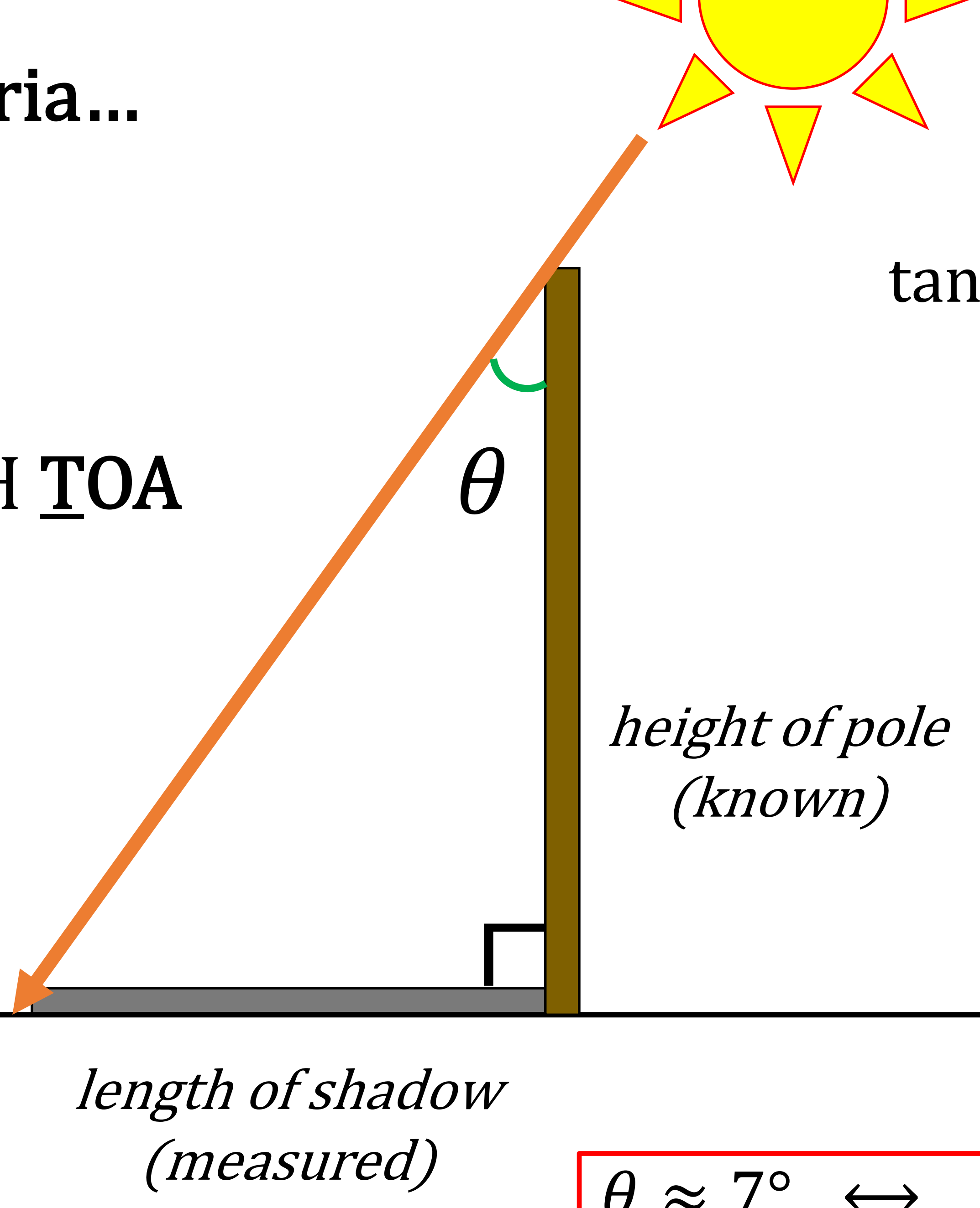

linear shadow change
non-linear shadow change



Marsa Matruh
Alexandria
Port Said
Israel
Cairo
Eilat
Asyut
Egypt
Sohag
Luxor
Marsa Alam
Aswan
Google
5,000
“stadia”
~parallel light rays
(sun is very far away)
Alexandria
Syene
center of
the earth

in Alexandria…

SOH CAH **TOA**

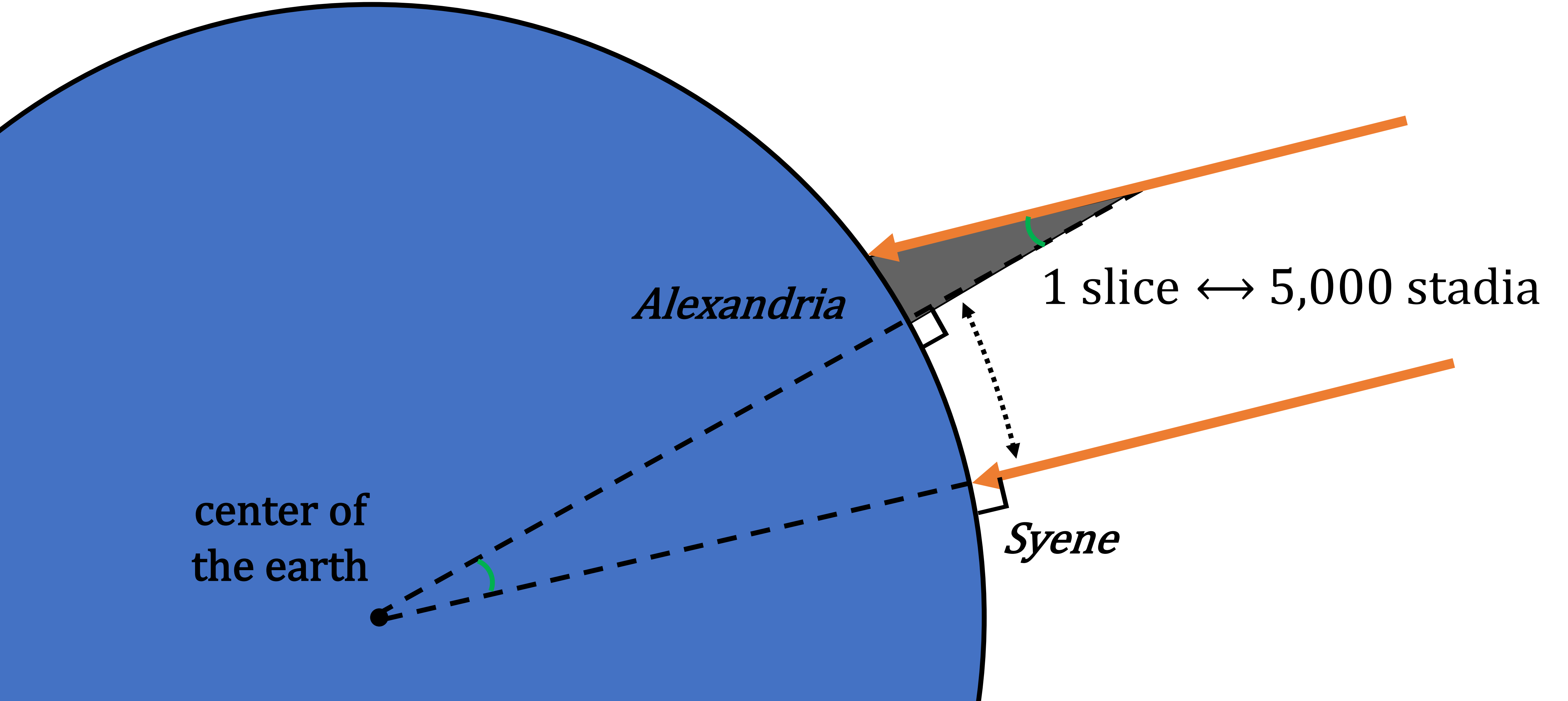


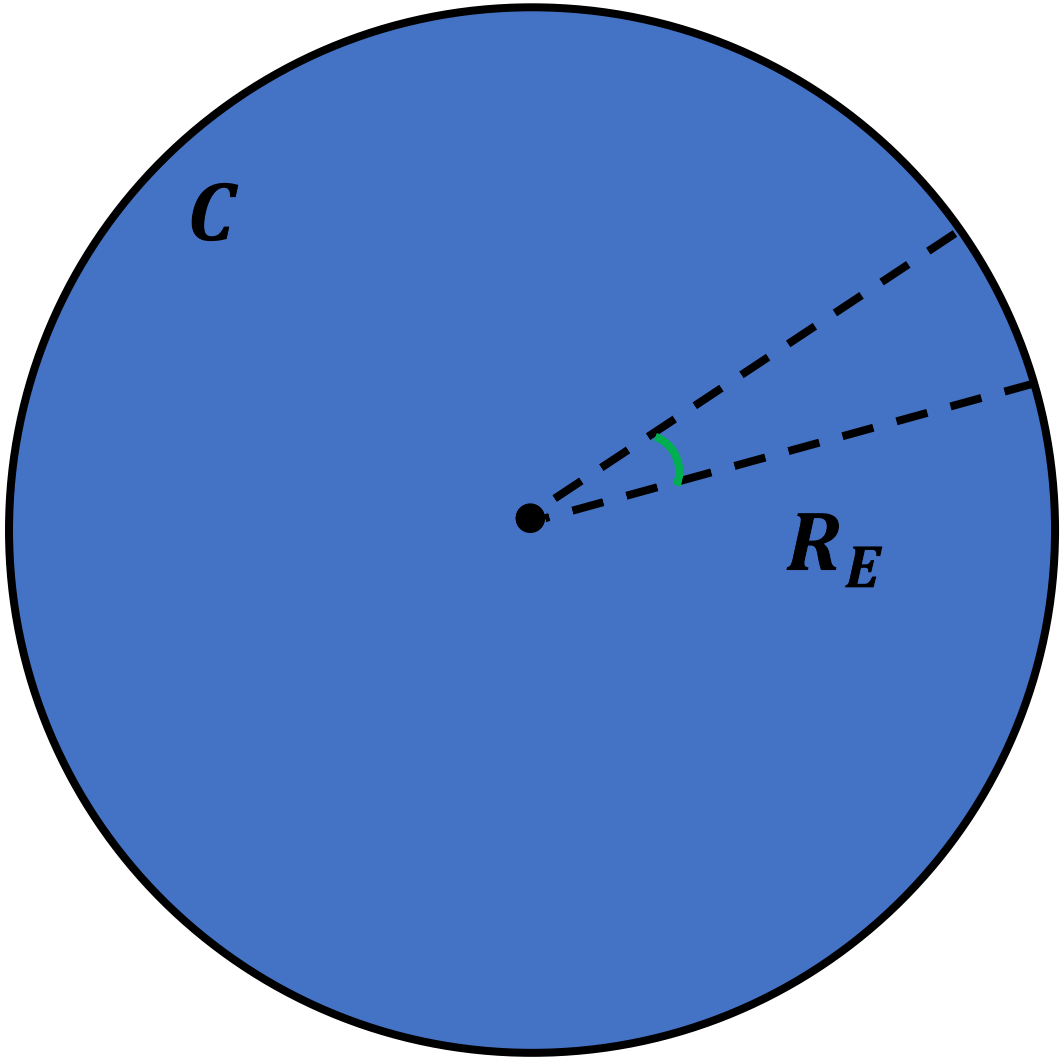
$$\tan\theta = \frac{\text{length of shadow}}{\text{height of pole}}$$

$$\theta = \tan^{-1}\left(\frac{\text{length of shadow}}{\text{height of pole}}\right)$$

$$\theta \approx 7^\circ \quad \longleftrightarrow \quad 1.9\% \text{ of earth's circumference}$$

$$\frac{\theta = 360^\circ \;\leftrightarrow\; 100\% \text{ of earth's circumference}}{\theta \approx 7^\circ \;\leftrightarrow\; 1.9\% \text{ of earth's circumference}} \Rightarrow \sim 51.4 \text{ slices}$$





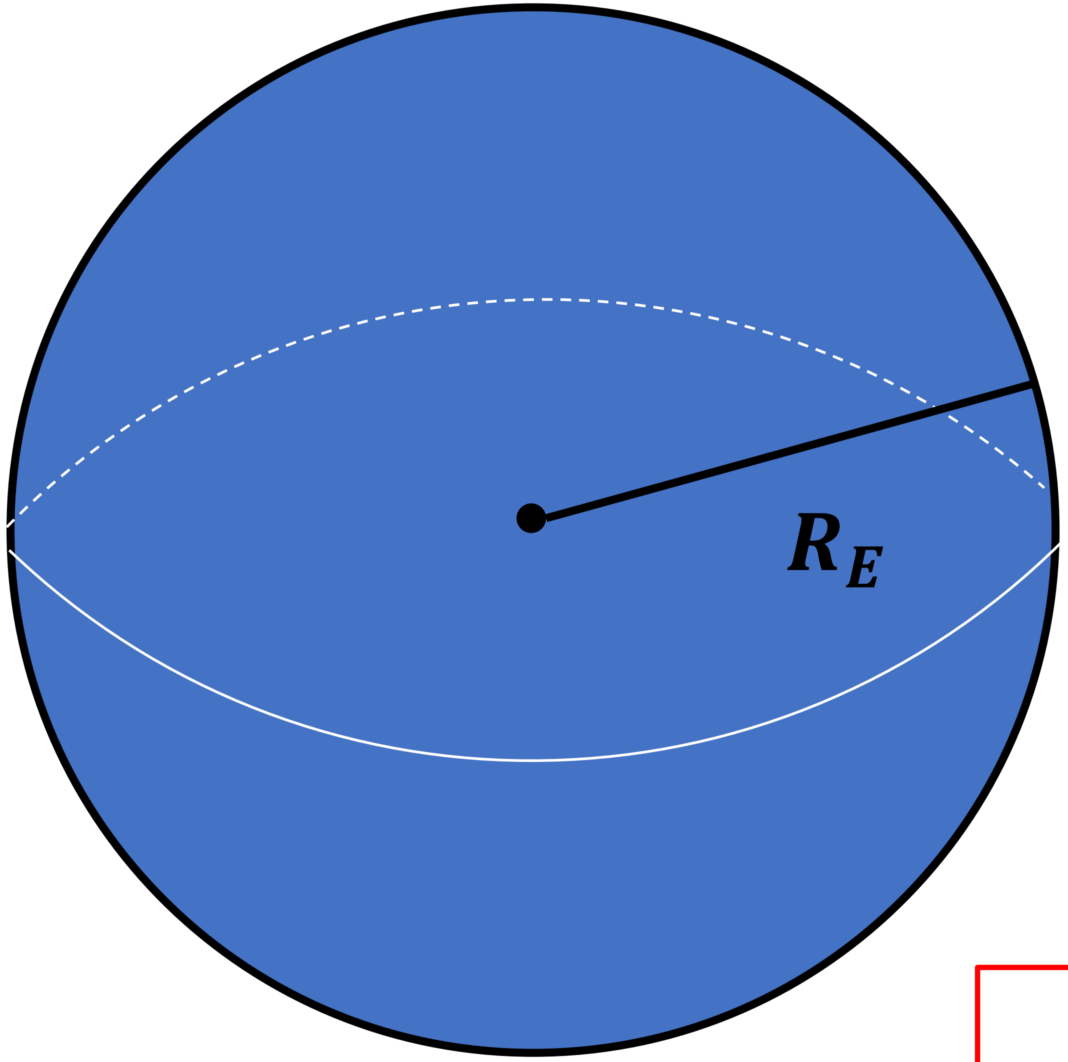
1 slice ⟷ 5,000 stadia

~51.4 total slices

$$\text{circumference} = 51.4 \times 5{,}000 \text{ stadia}$$
$$\approx 257{,}000 \text{ stadia}$$
$$\approx 44{,}100 \text{ km}$$

$$C = 2\,\pi\,R_E$$

$$R_E = \frac{C}{2\,\pi} = \frac{4.41 \times 10^7 \text{ m}}{2\,\pi} \approx 7.02 \times 10^6 \text{ m}$$

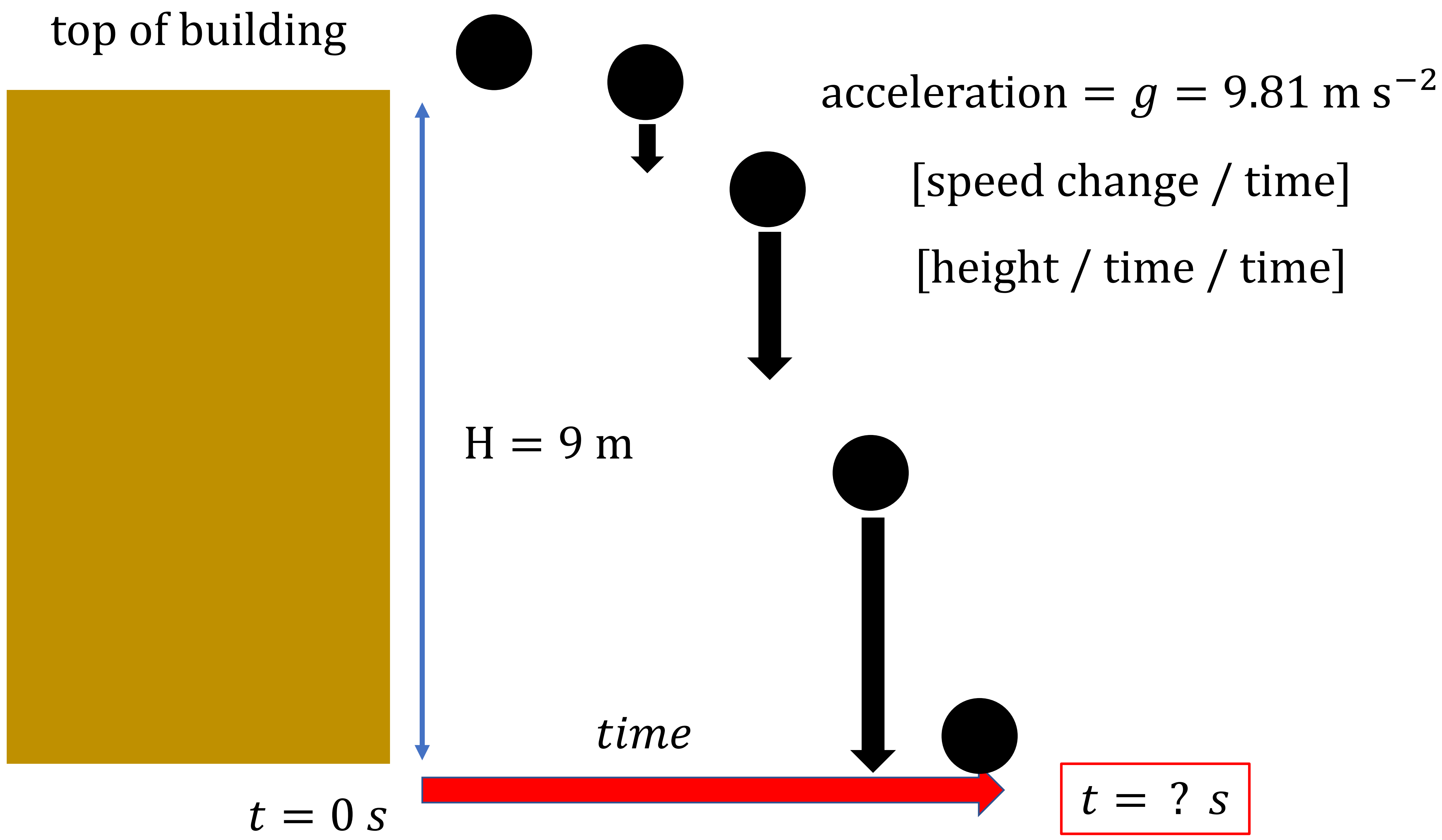


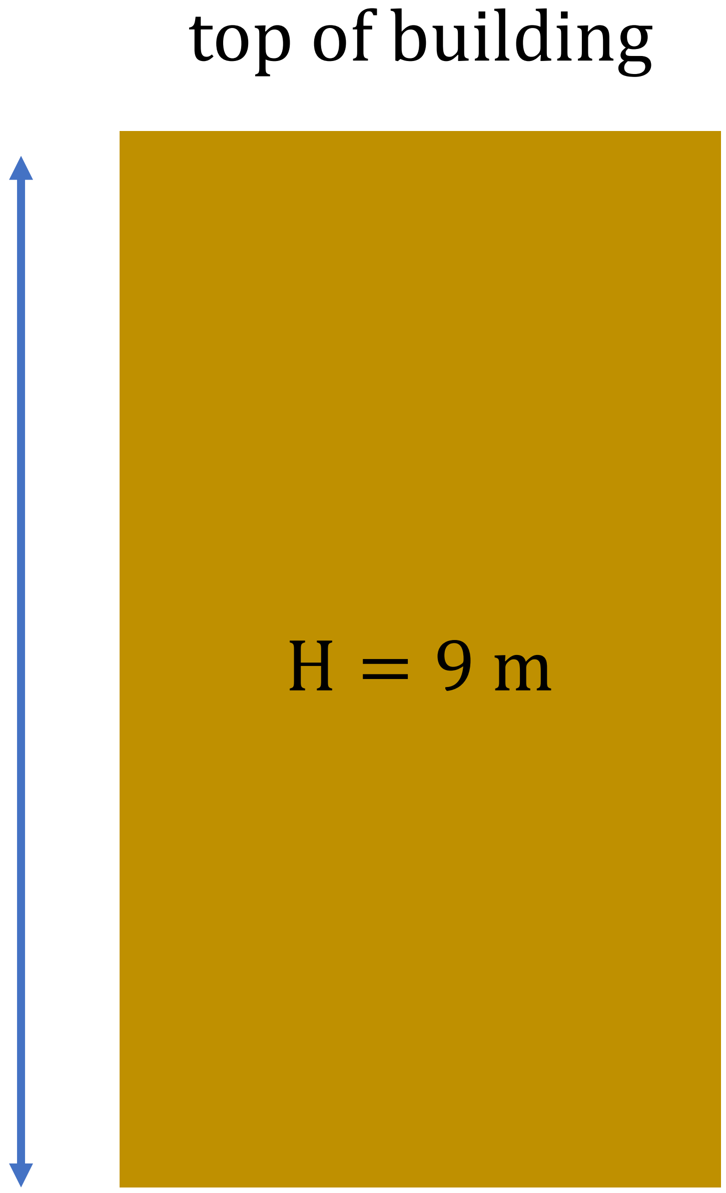
$$V_{sphere} = \frac{4}{3}\pi R^3$$

$$V_E = \frac{4}{3}\pi\,(7.02 \times 10^6\ \mathrm{m})^3$$

$$\approx 1.45 \times 10^{21}\ \mathrm{m}^3$$

$$\rho_E = \frac{M_E}{V_E} = \frac{M_E}{1.45 \times 10^{21}\ \mathrm{m}^3}$$


top of building
acceleration = g = 9.81 m s^{-2}
[speed change / time]
[height / time / time]
H = 9 m
$time$
$t = 0\ s$
$t = ?\ s$



$$H = \frac{1}{2} g t^2 \qquad 2H = g t^2$$

$$g = \frac{2H}{t^2} = \frac{18 \text{ m}}{t^2}$$

$$g_{ave} = \frac{G M_E}{{R_E}^2} \qquad {R_E}^2 g_{ave} = G M_E$$

$$M_E = \frac{{R_E}^2 g_{ave}}{G} \approx 7.25 \times 10^{24} \text{ kg}$$

#1: use (a) post height and (b) shadow length to find the **angle**

$$\theta = \tan^{-1}\left(\frac{\text{length of shadow}}{\text{height of pole}}\right) \quad \rightarrow \quad \text{keep calculator in DEGREE mode}$$

#2: $\text{circumference} = 857\text{ km} \times [360° / (\theta_{\text{Boulder}} - \theta_{\text{Las Cruces}})]$

#3: $R_E = \dfrac{\text{circumference}}{2\,\pi}$

#4: $V_E = \dfrac{4}{3}\,\pi\,{R_E}^3$

#5: $g = \dfrac{2\,(9\text{ m})}{t^2}$

#6: $M_E = \dfrac{{R_E}^2\;g_{ave}}{G}$

#7: $\rho_E = \dfrac{M_E}{V_E}$

Approximate Values

$$R_E = \frac{\text{circumference}}{2\,\pi} \approx 6.37 \times 10^6 \text{ m}$$

$$M_E = \frac{{R_E}^2\, g_{ave}}{G} \approx 5.97 \times 10^{24} \text{ kg}$$

$$\rho_E = \frac{M_E}{V_E} \approx 5{,}515.3 \text{ kg m}^{-3} = 5.52 \text{ g cm}^{-3}$$