1 Estimating the Earth's Density

Names: ____

Determining Earth's Radius

- Take a post outside, into the sunlight, and measure the length of the post with the tape measure.
- Place one end of the post on the ground, and hold the post as vertical as possible.
- Using the tape measure provided, measure to the nearest 1/2 centimeter the length of the shadow cast by the post; this shadow length should be measured three times, by three separate individuals; record these shadow lengths in Table 1.
- You will be provided with the length of a post and its shadow measured simultaneously today in Boulder, Colorado.
- Proceed through the calculations described after Table 1, and write your answers in the appropriate locations in Table 1. **(10 points)**

Table 1: **Angle Data**

Location	Post Height (cm)	Shadow Length (cm)	Angle (Degrees)
Las Cruces Shadow #1			
Las Cruces Shadow #2			
Las Cruces Shadow #3			
Average Las Cruces Angle:			
Boulder, Colorado			

Angle Determination:

TANGENT of the ANGLE = far-side length/ near-side length

Since you know the length of the post (the near-side length, which you have measured) and the length of the shadow (the far-side length, which you have also measured, three separate times), you can determine the shadow angle from your measurements, using the ATAN, or TAN^{-1} capability on your calculator (these functions will give you an angle if you provide the ratio of the height to length):

$$\text{ANGLE} = \text{ATAN (shadow length / post length)}$$
or
$$\text{ANGLE} = \text{TAN}^{-1}\text{(shadow length / post length)}$$

Calculate the shadow angle for each of your three shadow-length measurements, and also for the Boulder, Colorado shadow-length measurement. Write these angle values in the appropriate locations in Table 1, then calculate the average of the three Las Cruces shadow angles, and write the value on the "Average Las Cruces Angle" line.

The angles you have determined are: 1) an estimate of the angle (latitude) difference between Las Cruces and the latitude at which the Sun appears to be directly overhead (which is currently $\sim$ 12 degrees south of the equator since we are experiencing early northern autumn), *and* 2) the angle (latitude) difference between Boulder, Colorado and the latitude at which the Sun appears to be directly overhead. The difference (Boulder angle minus Las Cruces angle) between these two angles is the angular (latitude) separation between Las Cruces and Boulder, Colorado. We will now use this information and our knowledge of the actual distance (in kilometers) between Las Cruces' latitude and Boulder's latitude. This distance is:

857 kilometers north-south distance between Las Cruces and Boulder, Colorado

In the same way that Eratosthenes used his measurements (just like those you have made today), we can now determine an estimate of the Earth's circumference. Using your calculated Boulder Shadow Angle and your Average Las Cruces Shadow Angle values, calculate the corresponding EARTH CIRCUMFERENCE value, and write it below:

Average Earth Circumference (kilometers) =

857 kilometers $\times$ (360°)/(Boulder angle —Avg LC Angle) =

857 $\times$ [360°/(_________ − _________)] = __________________ km (2 points)

The CIRCUMFERENCE value you have just calculated is related to the RADIUS via the equation:

$$\text{EARTH CIRCUMFERENCE} = 2 \times \pi \times \text{EARTH RADIUS}$$

which can be converted to RADIUS using:

$$\text{EARTH RADIUS} = R_E = \text{EARTH CIRCUMFERENCE} / (2 \times \pi)$$

For your calculated CIRCUMFERENCE, calculate that value of the Radius (in units of kilometers) in the appropriate location below:

AVERAGE EARTH RADIUS VALUE = R_E = ______________________ kilometers (3 points)

Convert this radius (R_E) from kilometers to meters, and enter that value in Table 3. *(Note we will use the radius in meters the rest of this lab.)*

Determining the Earth's Mass

Tasks:

- Using a stopwatch, measure the amount of time required for a dropped object (from the top of the Astronomy Building) to fall 9.0 meters (28.66 feet). Different members of your group should take turns making the fall-time measurements; write these fall time values for two "drops" in the appropriate location in Table 2. **(10 points for a completed table)**
- **Use the equation: Acceleration = [2.0 x Fall Distance] / [(Time to fall)^2]**

  and your measured Time to Fall values and the measured distance (9.0 meters) of Fall to determine the gravitational acceleration due to the Earth; write these acceleration values (in units of meters per second per second) in the proper locations in Table 2.

Table 2: **Time of Fall Data**

	Time to Fall	Fall Distance	Acceleration
Object Drop #1		9 meters	
Object Drop #2		9 meters	
Average =			

- Now, knowing the magnitude of the average acceleration that Earth's gravity imposes upon a dropped object, we will now use the "Gravity" equation to get M_E:

  Gravitational acceleration = $G \times M_E/R_E^2$ (where R_E must be in meters!)

By rearranging the Gravity equation to solve for M_E, we can now make an estimate of the Earth's mass:

$\mathbf{M_E = Average\ Acceleration \times (R_E)^2 / G =}$ ______________________________ **(5 points)**

Write the value of M_E (in kilograms) in Table 3 below.

Determining the Earth's Density

- Calculate the volume (V_E) of the Earth given your determination of its radius *in meters!*:

$$\mathbf{V_E = (4/3) \times \pi \times R_E^3}$$

  and write this value in the appropriate location in Table 3 below.

- *Divide your value of M_E (that you entered in Table 3) by your estimate of V_E that you just calculated (also written in Table 3):* the result will be your estimate of the Average Earth Density in units of kilograms per cubic meter. Write this value in the appropriate location in Table 3.

- *Divide your AVERAGE ESTIMATE OF EARTH'S DENSITY value that you just calculated by the number 1000.0*; the result will be your estimated Earth density value in units of grams per cubic centimeter (the unit in which most densities are tabulated). Write this value in the appropriate location in Table 3.

Table 3: **Data for the Earth**

Estimate of Earth's Radius:	______________________________ **m**	(4 points)
Estimate of Earth's Mass:	______________________________ **kg**	(4 points)
Estimate of Earth's Volume:	______________________________ $\mathbf{m^3}$	(4 points)
Estimate of Earth's Density:	__________________________ $\mathbf{kg/m^3}$	(4 points)
Converted Density of the Earth:	_________________________ $\mathbf{gm/cm^3}$	(4 points)

In-Lab Questions:

1. Is your calculated value of the (Converted) Earth's density GREATER THAN, or LESS THAN, or EQUAL TO the actual value (see the Introduction) of the Earth's density? If your calculated density value is not identical to the known Earth density value, calculate the "percent error" of your calculated density value compared to the actual density value **(2 points)**:

   PERCENT ERROR =

   $$\frac{100\% \times (\text{CALCULATED DENSITY} - \text{ ACTUAL DENSITY})}{\text{ACTUAL DENSITY}} = \underline{\hspace{3cm}}$$

2. You used the AVERAGE Las Cruces shadow angle in calculating your estimate of the Earth's density (which you wrote down in Table 3). If you had used the LARGEST of the three measured Las Cruces shadow angles shown in Table 1, would the Earth density value that you would calculate with the LARGEST Las Cruces shadow angle be larger than or smaller than the Earth density value you wrote in Table 3? Think before writing your answer! Explain your answer. **(5 points)**

3. If the Las Cruces to Boulder, Colorado distance was actually 200 km in length, but your measured fall times did not change from what you measured, would you have calculated a larger or smaller Earth density value? Explain the reasoning for your answer. **(3 points)**

4. If we had conducted this experiment on the Moon rather than here on the Earth, would your measured values (fall time, angles and angle difference between two locations separated north-south by 857 kilometers) be the same as here on Earth, or different? Clearly explain your reasoning. [It might help if you draw a circle representing Earth and then draw a circle with $1/4^{\text{th}}$ of the radius of the Earth's circle to represent the Moon.] **(5 points)**