I. Bremsstrahlung (following: Rybicki & Lightman Ch 5).

= free - free radiation

Process: ionized plasma, encounters between electrons & ions causes deflections of electrons
→ electrons are being accelerated & will radiate.

Under most conditions: velocity distributions of ions & electrons is Maxwellian (TE distrib.) and we speak of thermal bremsstrahlung.

The word bremsstrahlung means "braking radiation". This refers to the slowing down of the electrons when approaching from a given direction to the protons/ions.

Approach will be classical, rather than quantum mech., this is valid as long as $\sim kT \gg h\nu$
$\sim m v_e^2$

Note: collisions between the same particles produce no net dipole radiation, because

dipole moment $= \Sigma e_i \vec{r}_i \propto \Sigma m_i \vec{r}_i$
the latter is the center of mass, which is either at rest or in constant motion, but experiences no net acceleration.

Another way to state this is that each particle moves exactly opposite to its partner so their individual contributions to the time-varying electric fields at large distances cancels. (Shu)

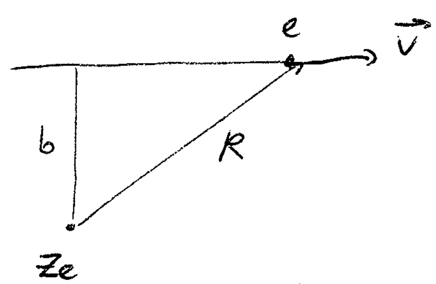
EMISSION from SINGLE-SPEED ELECTRONS

Hence we consider <u>electron-ion collisions</u>, or rather <u>encounters</u>. In these encounters, the <u>electrons</u> radiate primarily, since $|\vec{a}| \propto \frac{1}{m}$, for equal charges.

<u>Approximations:</u>

1. ion mass >> electron mass → electron moves in <u>fixed</u> Coulomb field of ion.
2. speed of electrons is large → in a single encounter the electron motion barely deviates from its original direction.

= $\boxed{\text{small angle scattering regime}}$ (not essential approx.)



b = impact parameter
$\vec{v}$ = electron velocity

$\vec{d} = -e\vec{R}$ = dipole moment
$\ddot{\vec{d}} = -e\dot{\vec{v}}$

Take FT, note that $FT(\ddot{\vec{d}}) = -\omega^2 \hat{\vec{d}}(\omega)$ RL 3.3

$$\rightarrow \quad -\omega^2 \hat{\vec{d}}(\omega) = -\frac{e}{2\pi}\int_{-\infty}^{\infty} \dot{\vec{v}}\, e^{i\omega t}\, dt \qquad (*)$$

Derive expressions for $\hat{\vec{d}}(\omega)$ in limits for large and small ω:

collision time $\tau \equiv \frac{b}{v}$

if $\omega\tau \gg 1$ then $e^{i\omega t}$ oscillates rapidly, and $(*) \rightarrow 0$
if $\omega\tau \ll 1$ $\quad e^{i\omega t} \sim 1$

$$\rightarrow \quad \hat{d}(\omega) \sim \begin{cases} \frac{e}{2\pi\omega^2}\Delta\vec{v} & \omega\tau \ll 1 \\ 0 & \omega\tau \gg 1 \end{cases}$$

$\Delta\vec{v}$ = change in velocity during encounter

We had derived before the spectrum of dipole radiation:

$$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3} |\hat{d}(\omega)|^2$$

so now we find $$\frac{dW}{d\omega} = \begin{cases} \frac{2e^2}{3\pi c^3} |\Delta\vec{v}|^2 & \omega\tau \ll 1 \\ 0 & \omega\tau \gg 1 \end{cases} \quad \text{and } \tau \equiv \frac{b}{v}$$

No dependence on ω! (Impulse has flat spectrum).

Clearly, the task is to estimate $\Delta\vec{v}$.

We use our approximation that the path of the electron is almost linear;
$\Delta\vec{v}$ is mainly in $\perp$ direction;

$$\Delta v = \frac{Ze^2}{m} \int_{-\infty}^{\infty} \frac{b\,dt}{(b^2+v^2t^2)^{3/2}} = \frac{2Ze^2}{mbv}$$

(use Coulomb Force:
$$F = m_e \frac{\Delta v}{\Delta t} = \frac{Ze^2}{R^2})$$
$$R^2 = b^2 + v^2t^2$$
and $\perp$ comp. of $F \propto \frac{b}{R^3}$)

So, for small-angle-scattering:

$$\frac{dW(b)}{d\omega} = \begin{cases} \frac{8Z^2e^6}{3\pi c^3 m^2 v^2 b^2} & b \ll \frac{v}{\omega} \\ 0 & b \gg \frac{v}{\omega} \end{cases} \quad (**)$$

(no interaction for large b).

The next step is to determine the total spectrum for a medium with
n_i = ion density
n_e = electron density
v = (fixed) speed of the electrons.
but "all" impact parameters

$n_e v$ = incident flux of electrons on 1 ion.
area of impact: $2\pi b\,db$

$$(***) \rightarrow \frac{dW}{d\omega\, dV\, dt} = n_e n_i\, 2\pi v \int_{b_{min}}^{\infty} \frac{dW(b)}{d\omega}\, b\, db$$

Note here already the essential feature of Bremsstrahlung, the proportionality with $\underline{n_e n_i}$.

RL discuss that the approximations we have used ($\omega\tau \ll 1$, $\omega\tau \gg 1$) are not a limitation here;

subst. (**) into (***)

$$\rightarrow \frac{dW}{d\omega\, dV\, dt} = \frac{16 e^6}{3 c^3 m^2 v} n_e n_i Z^2 \int_{b_{min}}^{b_{max}} \frac{db}{b}$$

$$= [\qquad\qquad\qquad] \ln\left(\frac{b_{max}}{b_{min}}\right)$$

b_{max} is value of b beyond which $b \ll \frac{v}{\omega}$ no longer true. So $b_{max} \lesssim \frac{v}{\omega}$.

Take $b_{max} \equiv \frac{v}{\omega}$ (only log. dependence)

$\boxed{\text{Next: estimate value of } b_{min}}$ (the minimum impact parameter)

1 for which value of b_{min} is small-angle scattering still valid? Occurs for $\Delta v \sim v$
we take $b_{min}^{(1)} = \frac{4Ze^2}{\pi m v^2}$ (or: $\frac{2Ze^2}{mv^2}$) (pg 147,)

2 quantum limit (scales can not be too small for classical approach to work)

$$\left.\begin{aligned} \Delta x\, \Delta p &\gtrsim \hbar \\ \Delta x &\sim b_{min} \\ \Delta p &\sim mv \end{aligned}\right\} \quad b_{min}^{(2)} = \frac{\hbar}{mv}$$

If $b_{min}^{(1)} \gg b_{min}^{(2)}$ then classical approach is valid.

$$\frac{2Ze^2}{mv^2} \gg \frac{\hbar}{mv} \rightarrow \frac{2Ze^2}{\hbar} \gg v \quad \text{or} \quad v^2 \ll \frac{4Z^2e^4}{\hbar^2}$$

? factor 2

$$\rightarrow \frac{1}{2}mv^2 \ll \left(\frac{Z^2e^4 m}{2\hbar^2}\right) = Z^2 \underbrace{\left(\frac{e^4 m}{2\hbar^2}\right)}_{R = \text{ Rydberg energy}} \quad (\text{not important})$$

(*) If $\frac{1}{2}mv^2 \gg Z^2 R$, quantum effects are important. In that case, reasonable results are still obtained by taking $b_{min} = b_{min}^{(2)}$

[We deal with the b_{min}, b_{max} issue, and the potential importance of quantum effects, by introducing the $\boxed{\text{Gaunt factor}}$ $g_{ff}(v, \omega)$ which includes these effects:

$$\frac{dW}{d\omega\, dV\, dt} = \frac{16\pi e^6}{3\sqrt{3}\, c^3 m^2 v} n_e n_i Z^2 g_{ff}(v, \omega) \qquad \binom{**}{**}$$

$$g_{ff} = \frac{\sqrt{3}}{\pi} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

$\boxed{\text{Next step}}$: We are not dealing with particles at one speed v, but there is a $\boxed{\text{velocity distribution}}$

Maxwellian distrib. $\rightarrow$ thermal Bremsstrahlung

$$dP \propto e^{-E/kT} d^3\vec{v} = \exp\left(-\frac{mv^2}{2kT}\right) d^3\vec{v}$$

Here, $d^3\vec{v} = 4\pi v^2 dv$ for isotropic distribution,

so $dP \propto v^2 \exp\left(-\frac{mv^2}{2kT}\right) dv$

We have to integrate $\binom{**}{**}$ over this distribution function. <u>What are integration limits?</u>

<u>Not</u> $0 \rightarrow \infty$ for v, because:

$h\nu \leq \frac{1}{2} m v^2$ by necessity.

$$\rightarrow \quad \frac{dW(T,\omega)}{dV\,dt\,d\omega} = \frac{\int_{v_{min}}^{\infty} \frac{dW(v,\omega)}{d\omega\,dV\,dt} v^2 \exp\left(-\frac{mv^2}{2kT}\right) dv}{\int_0^{\infty} v^2 \exp\left(-\frac{mv^2}{2kT}\right) dv}$$

where $v_{min} \equiv \left(\frac{2h\nu}{m}\right)^{1/2}$

$d\omega = 2\pi\, d\nu$

$$\rightarrow \quad \frac{dW}{dV\,dt\,d\nu} = \frac{2^5 \pi e^6}{3mc^3}\left(\frac{2\pi}{3km}\right)^{1/2} \frac{1}{\sqrt{T}} Z^2 n_e n_i\, e^{-h\nu/kT}\, \bar{g}_{ff}$$

$$\equiv \varepsilon_\nu^{ff}$$

$$\boxed{\varepsilon_\nu^{ff} = 6.8\times 10^{-38}\, Z^2 n_e n_i \frac{1}{\sqrt{T}} e^{-h\nu/kT}\, \bar{g}_{ff}}$$

So up to cutoff, ε_ν is independent of ν

$\rightarrow$ <u>flat spectrum</u> (<u>optically thin emission!</u>)

$\bar{g}_{ff}$ = velocity averaged Gaunt factor.

$\frac{1}{\sqrt{T}}$ dependence, because of $\frac{1}{v}$ dep. in $\binom{**}{**}$

RL discuss the values the Gaunt factor takes in various cases (see their Fig 5.2).

Note cutoff in spectrum $e^{-h\nu/kT}$! What is cause?

Total power: integrate over frequency
see RL for expressions.

Absorption coefficient

We have calculated $\boxed{\varepsilon_\nu^{ff} = \text{emissivity}}$
From that we obtain $\boxed{j_\nu^{ff} = \frac{\varepsilon_\nu}{4\pi} = \text{emission coefficient.}}$

Since we are dealing with thermal emission, we can use Kirchoff's law to obtain the abs. coeff. (simple!)

$$j_\nu = \alpha_\nu B_\nu(T) \rightarrow$$

$$\alpha_\nu^{ff} = 3.7 \times 10^8 \frac{1}{\sqrt{T}} Z^2 n_e n_i \nu^{-3} (1 - e^{-h\nu/kT}) \bar{g}_{ff}$$

two regimes:

a $h\nu \gg kT$, $\alpha_\nu^{ff} \propto \nu^{-3}$

b $h\nu \ll kT$, $\alpha_\nu^{ff} = 0.018\, T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}_{ff}$

limit a is not important for thermal bremsstrahlung, because we cannot have $h\nu \gg kT$!
The most common case is $\alpha_\nu^{ff} \propto \nu^{-2}$!

What does this imply? (for spectrum)

$$\tau_\nu = \int \alpha_\nu \, ds;$$ assume plasma of uniform temp.

we see that $\tau_\nu \propto \int n_e n_i \, ds \propto \underbrace{\int n_e^2 \, ds}_{\text{Emission measure}}$

(units of pc cm^{-6})

You may sometimes find an expression for

$$\tau_\nu \propto \underbrace{T^{-1.35}\,\nu^{-2.1}}\int n_e^2\,ds$$

slightly different than before, because approximate g_{ff} has been calculated.

The dependence of α_ν on ν^{-2} implies that the emission becomes optically thick at low freq.
We have:

① optically thin case:

$$I_\nu = \tau_\nu S_\nu = \tau_\nu B_\nu(T) \propto \nu^{-2}\nu^2 = \nu^0$$

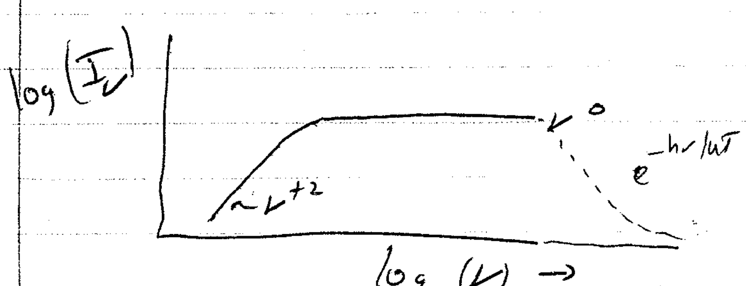
Rayleigh Jeans part
(OK for $T \sim 10^4$ K, radio regime)

$\rightarrow I_\nu \propto \nu^0$ flat spectrum

② optically thick case:

$$I_\nu = S_\nu(1-e^{-\tau_\nu})$$
$$= B_\nu(1-e^{-\tau_\nu}) \rightarrow B_\nu(T) \text{ for } \tau_\nu \to \infty$$

so $\boxed{I_\nu \propto \nu^{+2}}$ black body!



Applications: { HII regions, $T \sim 10^4$ K → radio free-free
Hot Ionized Medium, $T \sim 10^6$ K → X-ray free-free

FyI.

PROBLEMS

5.1—Consider a sphere of ionized hydrogen plasma that is undergoing spherical gravitational collapse. The sphere is held at constant isothermal temperature T_0, uniform density and constant mass M_0 during the collapse, and has decreasing radius $R(t)$. The sphere cools by emission of bremsstrahlung radiation in its interior. At $t=t_0$ the sphere is optically thin.

a. What is the total luminosity of the sphere as a function of M_0, $R(t)$ and T_0 while the sphere is optically thin?

b. What is the luminosity of the sphere as a function of time after it becomes optically thick?

c. Give an implicit relation, in terms of $R(t)$, for the time t_1 when the sphere becomes optically thick.

d. Draw a qualitative curve of the luminosity as a function of time.

5.1

a. The optically thin luminosity is equal to the volume $V=(4/3)\pi R^3(t)$ times the power radiated per unit volume, Eq. (5.15b):

$$\mathcal{L}_{\text{thin}} = 1.7\times 10^{-27} n_e n_p T_0^{1/2} V,$$

where we have taken $\bar{g}_B = 1.2$. Now, $n_e = n_p = M_0/m_p V$, where $m_p =$ hydrogen mass. Thus

$$\mathcal{L}_{\text{thin}} = 1.6\times 10^{20} M_0^2 T_0^{1/2} R^{-3}(t). \quad \longrightarrow \propto R(t)^{-3}$$

b. The optically thick luminosity is equal to the surface area $4\pi R^2(t)$ times the blackbody flux, Eq. (1.43):

$$\mathcal{L}_{\text{thick}} = 7.1\times 10^{-4} T_0^4 R^2(t). \quad \propto R^{+2}(t)$$

c. The transition between thick and thin cases occurs roughly when $\mathcal{L}_{\text{thin}} \approx \mathcal{L}_{\text{thick}}$. Setting the above expressions equal for $t=t_0$ we obtain

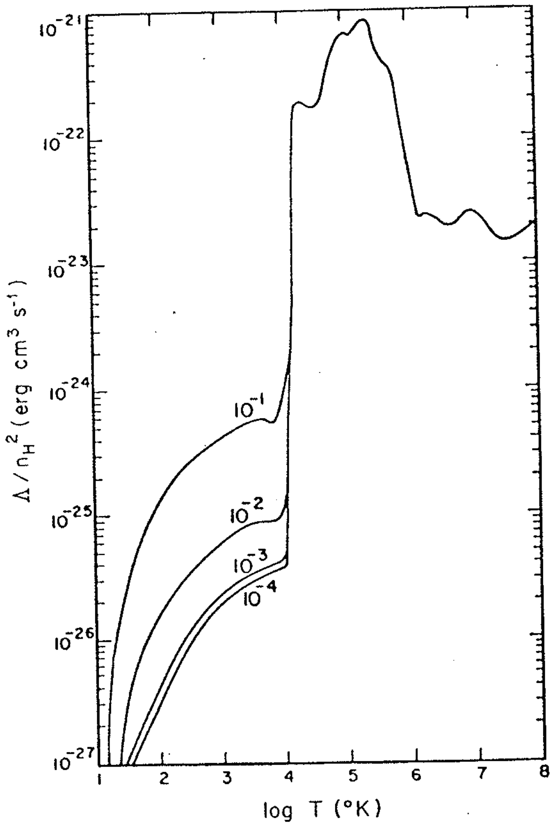
$$R(t_0) \approx 4.7\times 10^4 M_0^{2/5} T_0^{-7/10}.$$

[An alternate solution follows by setting $\alpha_R^{ff} R(t_0) \approx 1$, using Eq. (5.20). This yields a result of the same form, but with coefficient 2.0×10^4.]

d. See Fig. S.7. (Radius shrinks, so L grows to max as R^{-3} then drops as R^{+2}).

Do pb. 5.2 as well (yourself).

Source: Rybicki & Lightman

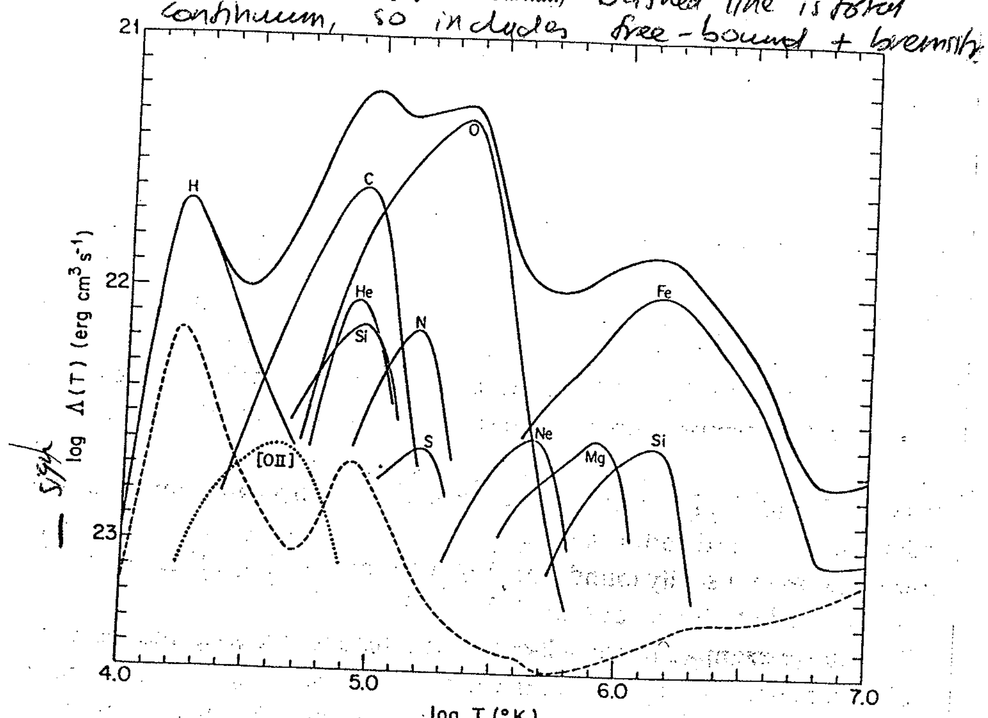


Spitzer
ch 6

Bremsstrahlung at different temp.
Plus excited lines from highly ionized species.

Figure 6.2 Cooling function for interstellar gas [6]. Values of $\Lambda(T)/n_H^2$ are shown as functions of the temperature T. For $T < 10{,}000°K$, the different curves represent different values of $x \equiv n_e/n_H$, while for $T > 10{,}000°K$, collisional ionization is assumed for all elements. Depletion and the possible presence of dust grains [7] or of H_2 are ignored.

Fig. 10.2. Radiative cooling function $\Lambda(T)$, for coronal model. Contributions to cooling from individual elements are indicated. (From Gaetz & Salpeter (1983); courtesy of *Astrophysical Journal*.) Dashed line is total continuum, so includes free-bound + bremsstr.



Ch4 Special relativity (from Rybicki & Lightman)

Principal of relativity: all laws of physics hold in inertial frames of reference.

inertial frame = frames at rest or moving at constant velocity.

Note: coordinate frame in a freely falling object is also a local inertial frame, even though it is not moving at constant velocity. (a non-rotating)

This is an extension introduced by Einstein in general relativity.

Einstein's postulate: speed of light, c, is the same in all reference frames whether they move or are stationary.

(consequence of relativity principle;
if it were not the same, an observer could determine whether he was at rest or moving;
cf Michelson-Morley experiment).

Also: speed of light is maximum possible speed

Special relativity: physical processes in inertial frames of reference, without gravity

general relativity: includes gravity and arbitrarily accelerated motions

Terminology

K': inertial frame in which process occurs in a system that is at rest with respect to K'

K: inertial frame of observer

If velocity difference between K and K' is large, the observer will see the processes occurring in K' distorted

both in space and time.
Special relativity provides the formulae to describe the events properly in both systems.

We will use this result in discussing radiation from fast moving charges.

concepts

(1) There is no absolute standard for a state of rest.
Motion or being at rest is only defined relative to some other system

(It is interesting to note though, that we can define the motion of the earth w.r.t. microwave background, and one may wonder if that defines a particular frame "at rest" in universe)

(b) We talk about events which are defined by both a place and a time.
So defined by four coordinates

event: specified by world points (x, y, z, t)
particle: specified by world line (connecting world points)

(c) "distance" between events:

$$s_{ab}^2 = -\left[(x_a - x_b)^2 + (y_a - y_b)^2 + (z_a - z_b)^2 - c^2(t_a - t_b)^2\right]$$

so in principle we could define $\tau = ict$ as an "imaginary time"

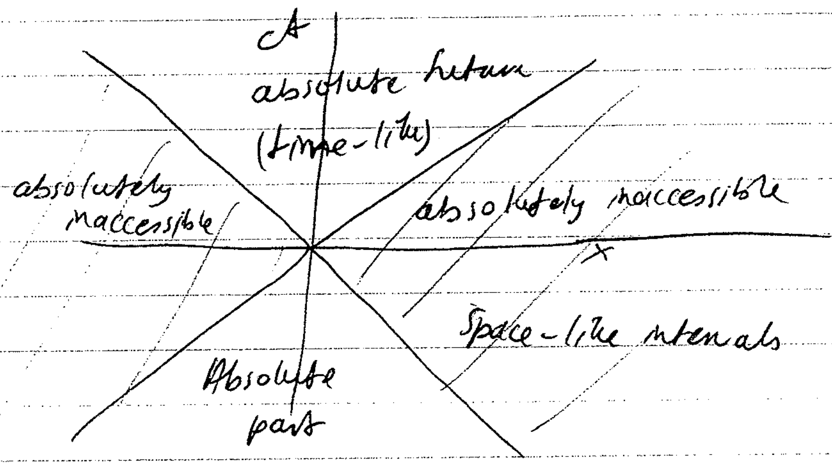
The above metric for s_{ab} is basically Euclidean; it is called the Minkowski space
Usually written in differential form:

$$ds^2 = -(dx^2 + dy^2 + dz^2 - c^2 dt^2)$$

(d) if $s_{ab}^2 > 0$: time-like interval
$s_{ab}^2 < 0$: space-like interval

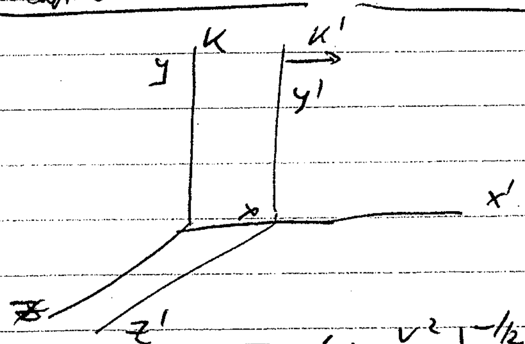
$s_{ab}^2 = 0$ for $v_x^2 + v_y^2 + v_z^2 \equiv v^2 = c^2$
which defines the light cone

World diagram (take $y = z = 0$):



(e) proper time = time read on a clock moving with the reference frame of an observer (invariant!)
proper length = length of an object measured in reference frame of observer.

Relative motion & Lorentz transformations



K' moves with $+v$ relative to K.

At $t = t' = 0$ two coordin. systems coincide;

$$\gamma \equiv \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

Then:
$$\begin{cases} x' = \gamma(x - vt) \\ y' = y \\ z' = z \\ t' = \gamma\left(t - \frac{v}{c^2}x\right) \end{cases}$$

and inverse:
$$x = \gamma(x' + vt'), \quad y = y', \quad z = z', \quad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

Note that the inverse transformations simply follow from replacing v by $-v$; check this yourself!

Consequences of these transformations

1. Length contraction (= Lorentz contraction)

rigid rod of length $L_0 = x_2' - x_1'$ is at rest in K'! What is length in K?

$L = x_2 - x_1$; x_1, x_2 are the positions of the ends of the rod at same time t in frame K;

$L_0 = x_2' - x_1' = \gamma(x_2 - x_1) = \gamma L$

$$\text{so} \quad L = \frac{L_0}{\gamma} = \sqrt{1-\left(\frac{v}{c}\right)^2}\, L_0 \quad \text{so} \quad \underline{\underline{L < L_0}}.$$

Vice versa, if the stick were at rest in K, then an observer in K' would see a shortening of the rod.

Apparent contradiction is due to the fact that the definition of simultaneous events is different in the two systems; temporal simultaneity is not Lorentz invariant.

2. Time Dilation

Clock at rest at origin of K' measures a time interval $T_0 = t_2' - t_1'$. What would be measured in K?

$x' = 0$ of clock

$\rightarrow \quad T = t_2 - t_1 = \gamma(t_2' - t_1') = \gamma T_0$

so T is larger by factor $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ than T_0.

clock seems to run slower in the (apparently) moving frame.

Again, the effect is present for both, i.e. an observer in frame K' thinks that clocks run slower in frame K. Apparent contradiction is again due to ambiguity in defining simultaneous events in two systems.

Discuss briefly twin paradox!

Observational evidence: lifetime of unstable cosmic ray particles entering atmosphere at $v \lesssim c$.

3. Transformation of velocities

Point with velocity $\vec{u}'$ in K'; what is its velocity $\vec{u}$ in K?

Write Lorentz transformations as:

$$dx = \gamma\,(dx' + v\,dt') \qquad dz = dz' \quad dy = dy'$$

$$dt = \gamma\,(dt' + \frac{v}{c^2}\,dx')$$

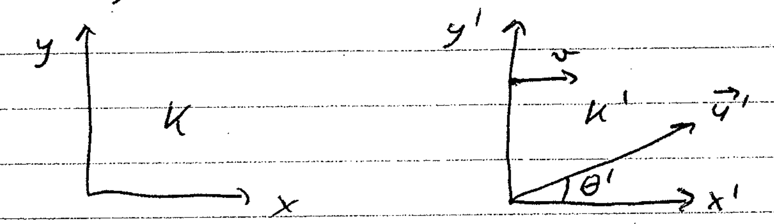
then $$u_x = \frac{dx}{dt} = \frac{\gamma\,(dx' + v\,dt')}{\gamma\,(dt' + v\,dx'/c^2)} = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}}$$

$$u_y = \frac{u_y'}{\gamma\left(1 + \frac{v u_x'}{c^2}\right)}$$

so $u_y \neq u_y'$! $u_z \neq u_z'$ due to change in measuring of time in two systems.

$$u_z = \frac{u_z'}{\gamma\left(1 + \frac{v u_x'}{c^2}\right)}$$

In a diagram:



Generalization to arbitrary velocity $\vec{v}$ of system K': (cf above). Can be stated in terms of components of $\vec{u}$ perpendicular to and parallel to $\vec{v}$:

$$u_{\parallel} = \frac{u_{\parallel}' + v}{\left(1 + \frac{v u_{\parallel}'}{c^2}\right)} \qquad u_{\perp} = \frac{u_{\perp}'}{\gamma\left(1 + \frac{v u_{\parallel}'}{c^2}\right)}$$

What about the directions of the velocities in both frames?

<u>aberration formula</u>: $\tan\theta = \frac{u_\perp}{u_\parallel} = \frac{u'\sin\theta'}{\gamma(u'\cos\theta' + v)}$ (verify!)

where θ' = angle between $\vec{u}'$ and $\vec{v}$ (see sketch),
$u' = |\vec{u}'|$, $u' = u'\sin\theta'$ and $u_\parallel' = u'\cos\theta'$

(note: aberration also occurs "classically" (e.g. cycling in rain)

Note: azimuthal angle ϕ between $\vec{v}$ and $\vec{u}'$ remains unchanged.

Special case: $u' = c$ speed of light $\rightarrow$ <u>aberration of light</u>

$\boxed{\tan\theta = \frac{\sin\theta'}{\gamma(\cos\theta' + v/c)}}$, $\boxed{\cos\theta = \frac{\cos\theta' + v/c}{1 + (v/c)\cos\theta'}}$ (see transf. of u_x!).

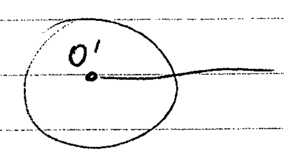
Consider case where $\theta' = \frac{\pi}{2}$, so photon emitted $\perp$ to v in K'

Then: $\tan\theta = \frac{c}{\gamma v}$

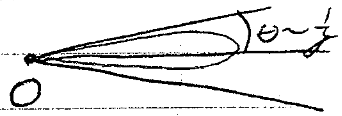
$\sin\theta = \frac{1}{\gamma}$

for $v \sim c$, $\gamma \gg 1$, so θ becomes small: $\theta \sim \frac{1}{\gamma}$

So if photons are emitted isotropically in K':



then observer in K sees: emission in a narrow cone



→ <u>relativistic beaming effect</u>

<u>Addition of velocities</u>

Again, consider $u_z' = u_y' = 0$, $u_x' = u'$ then $u_y = u_z = 0$ and

$$u_x = u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

This is in effect the sum of two velocities v and u', as seen in frame K.

Special cases:

① If $u' = v = c$, then $u = c$

② If $u' < c$ and $v = c$, or if $u' = c$ and $v < c$,
$u = c$
→ constancy of speed of light in all inertial frames.

③ if $u' < c$ and $v < c$ then always $u < c$
(prove yourself)

So never a collision with effective velocity $> c$!

Consequence: you can never accelerate a particle to the speed of light; each additional velocity Δv resulting from the acceleration will still never add up to the speed of light.

Doppler effect

Periodic phenomenon in frame K' will appear to have a longer period by factor γ in frame K of local observer. In addition, if we measure the arrival times of pulses, that propagate with velocity c (or also for other propagation velocities), there is an additional effect, due to the delay times of (light) propagation (i.e. the source has moved farther away between two pulses).

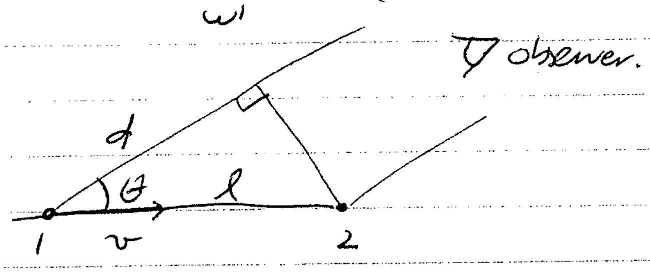
joint effect = Doppler effect (classically, only the 2nd effect was considered, not the change in time).

In frame K: source moves from $1 \to 2$ in the time it emits one period of radiation, at velocity v.

ω' = freq. of radiation in rest frame of emitting source

→ time to move from point 1 to point 2 in frame K:
(observer's frame)

$$\Delta t = \frac{2\pi\gamma}{\omega'} \quad \text{(time dilation effect)}$$



from diagram: $l = v\,\Delta t$
$d = v\,\Delta t \cos\theta$

Δt_A = difference in arrival time from pulses emitted at 1 and 2

= Δt − (time it takes pulse of light to travel distance d)

$$= \Delta t - \frac{d}{c} = \Delta t\left(1 - \frac{v}{c}\cos\theta\right)$$

→ observed frequency $\boxed{\omega = \frac{2\pi}{\Delta t_A} = \frac{\omega'}{\gamma\left(1-\frac{v}{c}\cos\theta\right)}}$

= relativistic Doppler formula.

Factor γ^{-1} due to relativistic theory.
$\left(1 - \frac{v}{c}\cos\theta\right)$ is "classical". ($v\cos\theta = v_{radial}$!)

We see that even for $\cos\theta = 0$, there is still a Doppler effect (a redshift). This is called the <u>transverse</u> Doppler effect

We can also write: $\omega' = \omega\gamma\left(1 - \frac{v}{c}\cos\theta\right)$

and the inverse: $\omega = \omega'\gamma\left(1 + \frac{v}{c}\cos\theta'\right)$

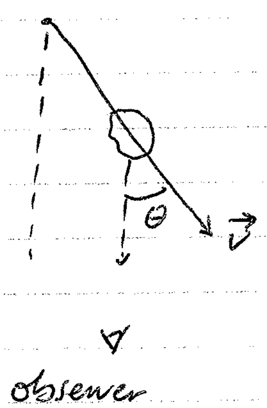
<u>verify this!</u>

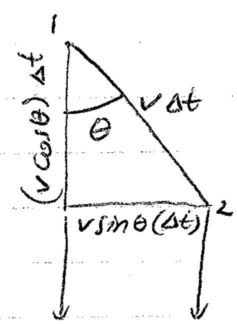
$\boxed{\text{Example of relativistic effects:}}$

$\boxed{\text{Apparent superluminal motion in quasars}}$

First consider the general situation (RL pb 4.7)

Object emits a radiating blob of material at speed v at an angle θ to the line-of-sight of a distant observer





a What is apparent transverse velocity inferred by observer?
(= angular velocity on sky times distance to object)
See second sketch:
- blob moves from $1 \to 2$ in time Δt
- point 2 is closer to observer than point 1
- apparent time difference between light received by observer:

$$(\Delta t)_{app} = \Delta t \left(1 - \frac{v}{c}\cos\theta\right) \quad \text{(cf Doppler effect)}$$

- Apparently velocity on sky is:

$$v_{app} = \frac{v\,\Delta t \sin\theta}{(\Delta t)_{app}} = \frac{v \sin\theta}{1 - \frac{v}{c}\cos\theta}$$

b Show that v_{app} can exceed c; find the angle for which v_{app} is maximum, and show that $v_{max} = \gamma v$

To find max, differentiate:

$$\left(\frac{dv_{app}}{d\theta}\right)_{\theta=\theta_c} = 0$$

$$\frac{v\cos\theta}{1-\frac{v}{c}\cos\theta} + \frac{-v\sin\theta}{(1-\frac{v}{c}\cos\theta)^2}\cdot\left(-\frac{v}{c}\sin\theta\right) = 0$$

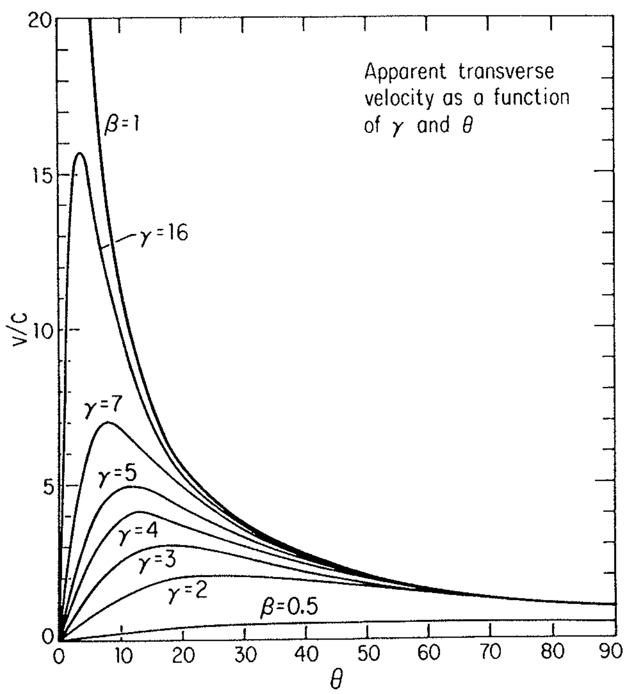
$$\leadsto \quad \cos\theta\Big|_{\theta=\theta_c} = \frac{v}{c} = \beta$$

and $\sin\theta_c = \sqrt{1-\beta^2} = \gamma^{-1}$

so $v_{max} = \dfrac{v\sqrt{1-\beta^2}}{1-\beta^2} = \gamma v \gg c$ for $\gamma \gg 1$

In addition to this apparently superluminal motion, the emission from a blob moving towards an observer will also be greatly boosted in flux compared to that of an blob moving away from us.

The figure below shows the apparent $\frac{v}{c}$ for various values of γ and θ



Source: Gal & Extragal. Radio astron. 2nd ed.

Fig. 13.9. Apparent transverse velocity as a function of the Lorentz factor, γ, and the inclination to the line of sight, θ.

The above superluminal motion has actually been detected in (and in Galactic binaries.) quasars, which are active galactic nuclei emitting jets of relativistic particles from the nuclear region. The diagram shows images of the nearby quasar 3C273 taken at different epochs. When interpreting the redshift $^{(z)}$ of the quasar as a measure of its distance through the Hubble law ($\frac{v}{c} \approx z$; $d = \frac{v}{H_0}$), the measured angular separation of the blobs on the sky would imply superluminal motion, in a classical interpretation.

The above model is not the only way to explain these effects.

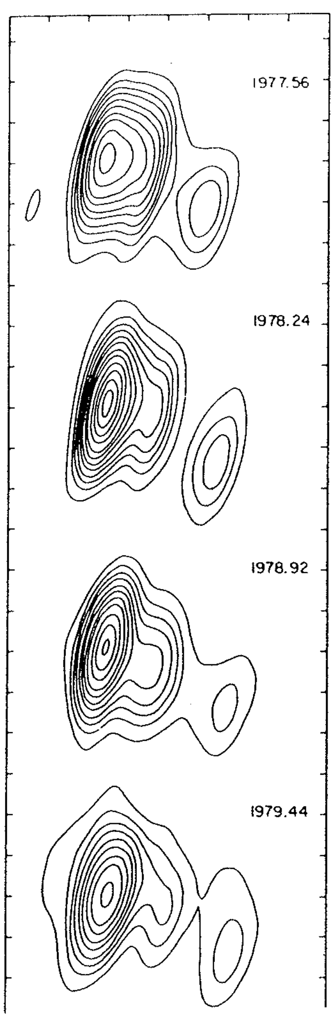
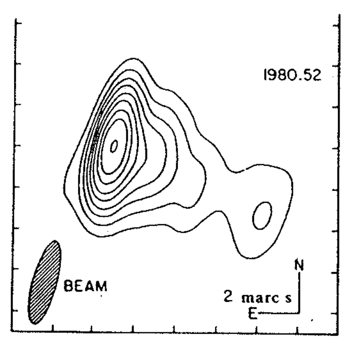


Fig. 13.7. Changes in the structure of the quasar 3C273 observed between 1977 and 1981 (Pearson et al. 1981. Reprinted by permission from Nature, Vol. 290, No. 5805, pp. 365. Copyright(c) 1981, Macmillan Magazines Ltd.



"Omitted":

- 4-vectors ("four vectors") (space, time e.g. velocity, wave vector, four-current, four potential, etc.).
- transformation of $\vec{E}, \vec{B}$ fields under LT.
- Mechanics & invariances, including tensor notation.
- Angular distrib of received & emitted power.

Some key aspects:

- relativistically moving charge creates a pulse of radiation as it passes by. (even though it is not accelerating!).
- relativistic beaming effects (of dipole pattern emitted by a single electron) if radiating particles are traveling at high speed.
- time dilation changes gyro resonance freq. (see next pages).

(- See section 10.8 WRH for a discussion of special relativistic effects, and 10.9 for discussion of effects on the radiation pattern.)

(WRH = Wilson, Rohlfs & Hüttemeister)