Homework 2

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Problem 1

(a)

The standard equation for the viscosity of a medium with path length l_p is given by

$$\nu = C_s l_p \tag{1}$$

where ν is the viscosity
C_s is the sound speed

Therefore,
The viscosity is

$$\nu = 1 \times 10^5 \times 10$$

The viscosity is

$$\boxed{\nu = 1 \times 10^6 \mathrm{cm}^2 - \mathrm{s}^{-2}}$$

This value of viscosity is too low, so a good method of calculating viscosity is by using the molecular viscosity equation. This method uses a few assumptions, this is rather interesting because with little or no observational data we can roughly calculate the time taken for collapse.

The equation for the viscosity is given by

$$\nu = C_s H_z \alpha_v \tag{2}$$

where H_z is the scale height
α_v is the fudge factor

The scale height is given by

$$H_z = 4.8 \times 10^5 \sqrt{T} (\frac{R}{1.49 \times 10^{14}})^{\frac{3}{2}} (\frac{M_\odot}{M_c})^{\frac{1}{2}} \tag{3}$$

where T is the temperature at a distance R from the central mass M_c

$$C_s = \sqrt{\gamma R T}$$

$$\sqrt{T} = \frac{C_s}{\sqrt{R\gamma}}$$

we shall assume a $\gamma = 1.4$ and a central mass $M_C = M_\odot$ Substituting in (3)

$$H_z = 4.8 \times 10^5 \frac{C_s}{\sqrt{\gamma R}} (\frac{R}{1.48 \times 10^{13}})^{\frac{3}{2}}$$

$$H_z = 2.44 \times 10^{11} \mathrm{cm}^2 \mathrm{s}$$

substituting this in (2) with $\alpha_\nu = 1$

$$\nu = 10^5 \times 2.44 \times 10^{11}$$

$$\boxed{\nu = 2.44 \times 10^{16} \mathrm{cm}^2 - \mathrm{s}}$$

(b)

The time scale is given by

$$\tau_d = \frac{R^2}{\nu} \tag{4}$$

where τ_d is the diffusion time scale at a distance R with a viscosity ν
From the previous case we have $\nu = 1 \times 10^6$ Substituting in eq(4) we have

$$\tau_d = \frac{(10^{14})^2}{1 \times 10^6}$$

$$\tau_d = 10^{22}\text{s}$$

$$\boxed{\tau_d = 3.17 \times 10^{14}\text{years}}$$

This is obviously not true but if we employ the molecular reasoning argument from the previous case $R = 10^{14}\text{cm}$ and $\nu = 2.44 \times 10^{16}\text{cm}^2 - \text{s}$. Substituting these values in (4)

$$\tau_d = \frac{10^{28}}{2.06 \times 10^{16}}$$

$$\tau_d = 4.08 \times 10^{11}\text{s}$$

$$\boxed{\tau_d = 1.2877 \times 10^4 \text{ years}}$$

This represents a more sensible cogent answer.

Problem 2

If we continue to assume that the viscosity operates macroscopically in order toachieve a a time scale of 10^6years we will get an $\alpha_\nu = 10^{-16}$ which is an incredibly small value to be reasonable.
However if we assume the molecular scale, In order to achieve a time scale of 10^6 years we need a fudge factor $\alpha_\nu = 0.01$

Problem 3

(a)

The diffusion time scale is given by the eq(4). The viscosity ν is give by eq(2). $c_s = 10^5 r^{\frac{-1}{2}}$,$H_z = 0.1r$, substituting in eq(2) we get

$$\nu = \alpha_\nu 10^5 r^{\frac{-1}{2}} 0.1r$$

$$\nu = \alpha_\nu 10^4 r^{\frac{1}{2}}$$

Substituting $R = r$ in the eq(4)

$$\tau_d = \frac{r^2}{\alpha_\nu} 10^4 r^{\frac{1}{2}}$$

$$\boxed{\tau_d = 10^{-4}\alpha_\nu r^{\frac{3}{2}}}$$

(b)

For $\alpha_\nu = 0.01$ and $r = 1.49 \times 10^{13}$cm

$$\tau_d = 10^{-4} \times 10^{-2} \times (1.49 \times 10^{13})^{3/2}$$

$$\tau_d = 5.75 \times 10^{13}\text{s}$$

$$\boxed{\tau_d = 1.831 \times 10^{6}\text{years}}$$

For $\alpha_\nu = 0.01$ and $r = 7.5 \times 10^{13}$cm

$$\tau_d = 10^{-4} \times 10^{-2} \times (7.5 \times 10^{13})^{3/2}$$

$$\tau_d = 6.4303 \times 10^{14}\text{s}$$

$$\boxed{\tau_d = 2.047 \times 10^{7}\text{years}}$$