1.1 Introduction

Astronomy is a physical science. Just like biology, chemistry, geology, and physics, astronomers collect data, analyze that data, attempt to understand the object/subject they are looking at, and submit their results for publication. Along the way astronomers use all of the mathematical techniques and physics necessary to understand the objects they examine. Thus, just like any other science, a large number of mathematical tools and concepts are needed to perform astronomical research. In today’s introductory lab, you will review and learn some of the most basic concepts necessary to enable you to successfully complete the various laboratory exercises you will encounter later this semester. When needed, the weekly laboratory exercise you are performing will refer back to the examples in this introduction—so keep the worked examples you will do today with you at all times during the semester to use as a reference when you run into these exercises later this semester (in fact, on some occasions your TA might have you redo one of the sections of this lab for review purposes).

1.2 The Metric System

Like all other scientists, astronomers use the metric system. The metric system is based on powers of 10, and has a set of measurement units analogous to the English system we use in everyday life here in the US. In the metric system the main unit of length (or distance) is the meter, the unit of mass is the kilogram, and the unit
of liquid volume is the liter. A meter is approximately 40 inches, or about 4” longer than the yard. Thus, 100 meters is about 111 yards. A liter is slightly larger than a quart (1.0 liter = 1.101 qt). On the Earth’s surface, a kilogram = 2.2 pounds. In the Astronomy 110 labs you will mostly encounter units of length/distance (variations on the meter).

As you have almost certainly learned, the metric system uses prefixes to change scale. For example, one thousand meters is one “kilometer”. One thousandth of a meter is a “millimeter”. The prefixes that you will encounter in this class are listed in Table 1.

<table>
<thead>
<tr>
<th>Prefix Name</th>
<th>Prefix Symbol</th>
<th>Prefix Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Giga</td>
<td>G</td>
<td>1,000,000,000 (one billion)</td>
</tr>
<tr>
<td>Mega</td>
<td>M</td>
<td>1,000,000 (one million)</td>
</tr>
<tr>
<td>kilo</td>
<td>k</td>
<td>1,000 (one thousand)</td>
</tr>
<tr>
<td>centi</td>
<td>c</td>
<td>0.01 (one hundredth)</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>0.001 (one thousandth)</td>
</tr>
<tr>
<td>micro</td>
<td>µ</td>
<td>0.0000001 (one millionth)</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
<td>0.0000000001 (one billionth)</td>
</tr>
</tbody>
</table>

In the metric system 3,600 meters is equal to 3.6 kilometers; while 0.8 meter is equal to 80 centimeters, which in turn equals 800 millimeters, etc. In the lab exercises this semester we will encounter a large range in sizes and distances. For example, you will measure the sizes of some objects/things in class in millimeters, talk about the wavelength of spectral lines in nanometers, and measure the sizes of features on the Sun that are larger than 100,000 kilometers.

### 1.2.1 Beyond the Metric System

When we talk about the sizes or distances to those objects beyond the surface of the Earth, we begin to encounter very large numbers. For example, the average distance from the Earth to the Moon is 384,000,000 meters or 384,000 kilometers (km). The distances found in astronomy are usually so large that we have to switch to a unit of measurement that is much larger than the meter, or even the kilometer. In and around the solar system, astronomers use “Astronomical Units”. An Astronomical Unit is the mean distance between the Earth and the Sun. One Astronomical Unit (AU) = 149,600,000 km. For example, Jupiter is about 5 AU from the Sun, while Pluto’s average distance from the Sun is 39 AU. With this change in units, it is easy to talk about the distance to other planets. It is more convenient to say that Saturn is 9.54 AU away than it is to say that Saturn is 1,427,184,000 km from Earth.
When we talk about how far away the stars are in our own Milky Way galaxy, we have to switch to an even larger unit of distance to keep the numbers manageable. One such unit is the “light year”. A light year (ly) is the distance light travels in one year. The speed of light is enormous: 300,000 kilometers per second (km/s) or 186,000 miles per second. Since one year contains 31,536,000 seconds, one ly = 9,460,000,000,000 km! The nearest star, Alpha Centauri, is 4.2 ly away. The Milky Way galaxy is more than 150,000 light years across. The nearest galaxy with a size similar to that of the Milky Way, the Andromeda Galaxy (see the sky chart for November online at http://astronomy.nmsu.edu/tharriso/skycharts.html for a picture and description of the Andromeda galaxy), is 2.2 million light years away!

In the Parallax lab we will introduce the somewhat odd unit of “parsecs”. For now, we will simply state that one parsec ("pc") = 3.26 ly. Thus, Alpha Centauri is 1.28 pc away. During the semester you will frequently hear the term parsec, kiloparsec (1 thousand pc), Megaparsec (1 million pc), and even the term Gigaparsec (1 billion pc). Astronomers have borrowed the prefixes from the metric system to construct their own shorthand way of describing extremely large distances. The Andromeda Galaxy is at a distance of 700,000 pc = 0.7 Megaparsecs.

1.2.2 Changing Units and Scale Conversion

Changing units (like those in the previous paragraph) and/or scale conversion is something you must master during this semester. The concept is fairly straightforward, so let’s just work some examples.

1. Convert 34 meters into centimeters

Answer: Since one meter = 100 centimeters, 34 meters = 3,400 centimeters.

2. Convert 34 kilometers into meters:

3. If one meter equals 40 inches, how many meters are there in 400 inches?

4. How many centimeters are there in 400 inches?

5. How many parsecs are there in 1.4 Mpc?

6. How many AU are there in 299,200,000 km?
One technique that you will use this semester involves measuring a photograph or image with a ruler, and converting the measured number into a real unit of size (or distance). One example of this technique is reading a road map. In the next figure is a map of the state of New Mexico. Down at the bottom left hand corner is a scale in Miles and Kilometers.

Map Exercises (using a ruler determine):

1) How many kilometers is it from Las Cruces to Albuquerque?
1.3 Squares, Square Roots, and Exponents

In several of the labs this semester you will encounter squares, cubes, and square roots. Let us briefly review what is meant by such terms as squares, cubes, square roots and exponents. The square of a number is simply that number times itself: $3 \times 3 = 3^2 = 9$. The exponent is the little number “2” above the three. $5^2 = 5 \times 5 = 25$. The exponent tells you how many times to multiply that number by itself: $8^4 = 8 \times 8 \times 8 \times 8 = 4096$. The square of a number simply means the exponent is 2 (three squared = $3^2$), and the cube of a number means the exponent is three (four cubed = $4^3$). Here are some examples:

1) $7^2 = 7 \times 7 = 49$
2) $7^5 = 7 \times 7 \times 7 \times 7 \times 7 = 16,807$
3) The cube of 9 = $9^3 = 9 \times 9 \times 9 = 729$
4) The exponent of $12^{16}$ is 16
5) $2.56^3 = 2.56 \times 2.56 \times 2.56 = 16.777$

**Your turn:**

7) $6^3 = $

8) $4^4 = $

9) $3.1^2 = $
The concept of a square root is easy to understand, but is much harder to calculate (we usually have to use a calculator). The square root of a number is that number whose square is the number: the square root of 4 = 2 because $2 \times 2 = 4$. The square root of 9 is 3 ($9 = 3 \times 3$). The mathematical operation of a square root is usually represented by the symbol “√”, as in $\sqrt{9} = 3$. But mathematicians also represent square roots using a fractional exponent of one half: $9^{1/2} = 3$. Likewise, the cube root of a number is represented as $27^{1/3} = 3$ ($3 \times 3 \times 3 = 27$). The fourth root is written as $16^{1/4} (= 2)$, and so on. We will encounter square roots in the algebra section shortly. Here are some examples/problems:

1) $\sqrt{100} = 10$

2) $10.5^3 = 10.5 \times 10.5 \times 10.5 = 1157.625$

3) Verify that the square root of 17 ($\sqrt{17} = 17^{1/2}$) = 4.123

1.4 Scientific Notation

The range in numbers encountered in Astronomy is enormous: from the size of subatomic particles, to the size of the entire universe. You are certainly comfortable with numbers like ten, one hundred, three thousand, ten million, a billion, or even a trillion. But what about a number like one million trillion? Or, four thousand one hundred and fifty six million billion? Such numbers are too cumbersome to handle with words. Scientists use something called “Scientific Notation” as a short hand method to represent very large and very small numbers. The system of scientific notation is based on the number 10. For example, the number $100 = 10 \times 10 = 10^2$. In scientific notation the number 100 is written as $1.0 \times 10^2$. Here are some additional examples:

Ten $= 10 = 1 \times 10 = 1.0 \times 10^1$
One hundred $= 100 = 10 \times 10 = 10^2 = 1.0 \times 10^2$
One thousand $= 1,000 = 10 \times 10 \times 10 = 10^3 = 1.0 \times 10^3$
One million $= 1,000,000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^6 = 1.0 \times 10^6$

Ok, so writing powers of ten is easy, but how do we write 6,563 in scientific notation? $6,563 = 6563.0 = 6.563 \times 10^3$. To figure out the exponent on the power of ten, we simply count-up the numbers to the left of the decimal point, but do not include the left-most number. Here are some other examples:
1,216 = 1216.0 = 1.216 \times 10^3
8,735,000 = 8735000.0 = 8.735000 \times 10^6
1,345,999,123,456 = 1345999123456.0 = 1.345999123456 \times 10^{12}

Your turn! Work the following examples:

121 = 121.0 = 
735,000 =
999,563,982 =

Now comes the sometimes confusing issue: writing very small numbers. First, lets look at powers of 10, but this time in fractional form. The number 0.1 = 1/10. In scientific notation we would write this as $1 \times 10^{-1}$. The negative number in the exponent is the way we write the fraction 1/10. How about 0.001? We can rewrite 0.001 as $1/10 \times 1/10 \times 1/10 = 0.001 = 1 \times 10^{-3}$. Do you see where the exponent comes from? Starting at the decimal point, we simply count over to the right of the first digit that isn’t zero to determine the exponent. Here are some examples:

0.121 = 1.21 \times 10^{-1}
0.000735 = 7.35 \times 10^{-4}
0.0000099902 = 9.9902 \times 10^{-6}

Your turn:

0.0121 =
0.0000735 =
0.0000000999 =
−0.121 =
There is one issue we haven’t dealt with, and that is when to write numbers in scientific notation. It is kind of silly to write the number 23.7 as $2.37 \times 10^1$, or 0.5 as $5.0 \times 10^{-1}$. You use scientific notation when it is a more compact way to write a number to insure that its value is quickly and easily communicated to someone else. For example, if you tell someone the answer for some measurement is 0.0033 meter, the person receiving that information has to count over the zeros to figure out what that means. It is better to say that the measurement was $3.3 \times 10^{-3}$ meter. But telling someone the answer is 215 kg, is much easier than saying $2.15 \times 10^2$ kg. It is common practice that numbers bigger than 10,000 or smaller than 0.01 are best written in scientific notation.

How do we multiply and divide two numbers in Scientific Notation? It is a three step process: 1) multiply (divide) the numbers out front, 2) add (subtract) the exponents, and 3) reconstruct the number in Scientific Notation. It is easier to just show some examples:

$$(2 \times 10^4) \times (3 \times 10^5) = (2 \times 3) \times 10^{(4+5)} = 6 \times 10^9$$

$$(2.00 \times 10^4) \times (3.15 \times 10^7) = (2.00 \times 3.15) \times 10^{(4+7)} = 6.30 \times 10^{11}$$

$$(2 \times 10^4) \times (6 \times 10^5) = (2 \times 6) \times 10^{(4+5)} = 12 \times 10^9 = 1.2 \times 10^{10}$$

$$(6 \times 10^4) \div (3 \times 10^8) = (6 \div 3) \times 10^{(4-8)} = 2 \times 10^{-4}$$

$$(3.0 \times 10^4) \div (6.0 \times 10^8) = (3.0 \div 6.0) \times 10^{(4-8)} = 0.5 \times 10^{-4} = 5.0 \times 10^{-5}$$

Your turn:

$$(6 \times 10^3) \times (3 \times 10^2) =$$

$$(8.0 \times 10^{18}) \div (4.0 \times 10^{14}) =$$

Note how we rewrite the exponent to handle cases where the number out front is greater than 10, or less than 1.

1.5 Algebra

Because this is a freshman laboratory, we do not use high-level mathematics. But we do sometimes encounter a little basic algebra and we need to briefly review the main
concepts. Algebra deals with equations and “unknowns”. Unknowns, or “variables”, are usually represented as a letter in an equation: \( y = 3x + 7 \). In this equation both “\( x \)” and “\( y \)” are variables. You do not know what the value of \( y \) is until you assign a value to \( x \). For example, if \( x = 2 \), then \( y = 13 \) (\( y = 3 \times 2 + 7 = 13 \)). Here are some additional examples:

\[
y = 5x + 3, \text{ if } x=1, \text{ what is } y? \quad \text{Answer: } y = 5 \times 1 + 3 = 5 + 3 = 8
\]

\[
q = 3t + 9, \text{ if } t=5, \text{ what is } q? \quad \text{Answer: } q = 3 \times 5 + 9 = 15 + 9 = 24
\]

\[
y = 5x^2 + 3, \text{ if } x=2, \text{ what is } y? \quad \text{Answer: } y = 5 \times (2^2) + 3 = 5 \times 4 + 3 = 20 + 3 = 23
\]

What is \( y \) if \( x = 6 \) in this equation: \( y = 3x + 13 = \)

These problems were probably easy for you, but what happens when you have this equation: \( y = 7x + 14 \), and you are asked to figure out what \( x \) is if \( y = 21 \)? Let’s do this step by step, first we re-write the equation:

\[
y = 7x + 14
\]

We now substitute the value of \( y \) (\( y = 21 \)) into the equation:

\[
21 = 7x + 14
\]

Now, if we could get rid of that 14 we could solve this equation! Subtract 14 from both sides of the equation:

\[
21 - 14 = 7x + 14 - 14 \quad \text{(this gets rid of that pesky 14!)}
\]

\[
7 = 7x \quad \text{(divide both sides by 7)}
\]

\[
x = 1
\]

Ok, your turn: If you have the equation \( y = 4x + 16 \), and \( y = 8 \), what is \( x \)?

We frequently encounter more complicated equations, such as \( y = 3x^2 + 2x - 345 \), or \( p^2 = a^3 \). There are ways to solve such equations, but that is beyond the scope of our introduction. However, you do need to be able to solve equations like this: \( y^2 = 3x + 3 \) (if you are told what “\( x \)” is!). Let’s do this for \( x = 11 \):
Copy down the equation again:

\[ y^2 = 3x + 3 \]

Substitute \( x = 11 \):

\[ y^2 = 3 \times 11 + 3 = 33 + 3 = 36 \]

Take the square root of both sides:

\[ (y^2)^{1/2} = (36)^{1/2} \]

\[ y = 6 \]

Did that make sense? To get rid of the square of a variable you have to take the square root: \((y^2)^{1/2} = y\). So to solve for \(y^2\), we took the square root of both sides of the equation.

### 1.6 Graphing and/or Plotting

The last subject we want to discuss is graphing data, and the equation of a line. You probably learned in high school about making graphs. Astronomers frequently use graphs to plot data. You have probably seen all sorts of graphs, such as the plot of the performance of the stock market shown in the next figure (1.2). A plot like this shows the history of the stock market versus time. The “x” (horizontal) axis represents time, and the “y” (vertical) axis represents the value of the stock market. Each place on the curve that shows the performance of the stock market is represented by two numbers, the date (x axis), and the value of the index (y axis). For example, on May 10 of 2004, the Dow Jones index stood at 10,000.

Plots like this require two data points to represent each point on the curve or in the plot. For comparing the stock market you need to plot the value of the stocks versus the date. We call data of this type an “ordered pair”. Each data point requires a value for \(x\) (the date) and \(y\) (the value of the Dow Jones index). In the next table is the data for how the temperature changes with altitude near the Earth’s surface. As you climb in altitude the temperature goes down (this is why high mountains can have snow on them year round, even though they are located in warm areas). The data in this table is plotted in the next figure.
Figure 1.2: The change in the Dow Jones stock index over one year (from April 2003 to July 2004).
Looking at the plot of temperature versus altitude, we see that a straight line can be drawn through the data points. We can figure out the equation of this straight line and then predict the temperature at any altitude. In high school you learned that the equation of a line was $y = mx + b$, where “$m$” is the “slope” of the line, and “$b$” is the “$y$ intercept”. The $y$ intercept is simply where the line crosses the $y$-axis. In the plot, the $y$ intercept is at 59.0, so $b = 59$. So, we can rewrite the equation for this line as $y = mx + 59.0$. How can we figure out $m$? Simple, pick any other data point and solve the equation—let’s choose the data at 10,000 feet. The temperature ($y$) is 23.3 at 10,000 feet ($= x$): $23.3 = 10000m + 59$. Subtracting 59 from both sides shows $23.3 - 59 = 10000x + 59 - 59$, or $-35.7 = 10000m$. To find $m$ we simply divide both sides by 10,000: $m = -35.7/10000 = -0.00357$. In scientific notation, the equation for the temperature vs. altitude is $y = -3.57 \times 10^{-3}x + 59.0$. Why is the slope negative? What is happening here? As you go up in altitude, the temperature goes down. Increasing the altitude ($x$) decreases the temperature ($y$). Thus, the slope has to be negative.

Using the equation for temperature versus altitude just derived, what is the temperature at 20,000 feet?

Ok, your turn. On the blank sheet of graph paper in Figure 1.4 plot the equation $y = 2x + 2$ for $x = 1, 2, 3$, and $x = -1, -2, -3$. What is the $y$ intercept of this line? What is its slope?

While straight lines and perfect data show up in science from time to time, it is actually quite rare for real data to fit perfectly ontop of a line. One reason for this is that all measurements have error. So, even though there might be a perfect relationship between $x$ and $y$, the noise of the measurements introduces small deviations from the line. In other cases, the data are approximated by a line. This is sometimes called a best-fit relationship for the data. An example of a plot with real data is shown in Figure 1.5. In this case, the data suggest that there is a general trend between
Figure 1.3: The change in temperature as you climb in altitude with the data from the preceding table. At sea level (0 ft altitude) the surface temperature is 59°F. As you go higher in altitude, the temperature goes down.

the absolute magnitude ($M_V$) and the Orbital Period in certain types of binary stars. But some other factor plays a role in determining the final relationship, so some stars do not fit very well, and hence their absolute magnitudes cannot be estimated very well from their orbital periods (the vertical bars associated with each data point are error bars, and represent the measurement error).
Figure 1.4: Graph paper for plotting the equation $y = 2x + 2$. 


Figure 1.5: The relationship between absolute visual magnitude ($M_V$) and Orbital Period for cataclysmic variable binary stars.