Lab 8

Parallax Measurements and Determining Distances

8.1 Introduction

How do we know how far away stars and galaxies are from us? Determining the distances to these distant objects is one of the most difficult tasks that astronomers face. Since we cannot simply pull out a very long ruler to make a few measurements, we have to use other methods.

Inside the solar system, astronomers can bounce a radar signal off of a planet, asteroid or comet to directly measure its distance. How does this work? A radar signal is an electromagnetic wave (a beam of light), so it always travels at the same speed, the speed of light. Since we know how fast the signal travels, we just measure how long it takes to go out and to return to determine the object's distance.

Some stars, however, are located hundreds, thousands or even tens of thousands of "light-years" away from Earth. A light-year is the distance that light travels in a single year (about 9.5 trillion kilometers). To bounce a radar signal off of a star that is 100 light-years away, we would have to wait 200 years to get a signal back (remember the signal has to go out, bounce off the target, and come back). Obviously, radar is not a feasible method for determining how far away the stars are.

In fact, there is one, and only one, direct method to measure the distance to a star: the "parallax" method. Parallax is the angle by which something *appears* to move across the sky when an observer looking at that object changes position. By observing the size of this angle and knowing how far the observer has moved, one can determine the distance to the object. Today you will experiment with parallax, and develop an appreciation for the small angles that astronomers must measure to determine the distances to stars.

To get the basic idea, perform the following simple experiment. Hold your thumb out in front of you at arm's length and look at it with your right eye closed and your left eye open. Now close your left eye and open your right one. See how your thumb appeared to move to the left? Keep staring at your thumb, and change eyes several times. You should see your thumb appear to move back and forth, relative to the background. Of course, your thumb is not moving. Your *vantage point* is moving, and so your thumb *appears* to move. That's the parallax effect!

How does this work for stars? Instead of switching from eye to eye, we shift the position of our entire planet! We observe a star once, and then wait six months to observe it again. In six months, the Earth will have revolved half-way around the Sun. This shift of two A.U. (twice the distance between the Earth and the Sun) is equivalent to the distance between your two eyes. Just as your thumb will appear to shift position relative to background objects when viewed from one eye and then the other, over six months a nearby star will appear to shift position in the sky relative to very distant stars.

8.1.1 Goals

The primary goals of this laboratory exercise are to understand the theory and practice of using parallax to find the distances to nearby stars, and to use it to measure the distance to objects for yourself.

8.1.2 Materials

All online lab exercise components can be reached from the GEAS project lab URL, listed below.

http://astronomy.nmsu.edu/geas/labs/labs.html

You will need the following items to perform your parallax experiment:

- a protractor (provided on page 27)
- a 30-foot long tape measure, or a shorter tape measure (or a yardstick) and at least 15 feet of non-stretchy string (not yarn)
- a thin object and a tall object with vertical sides, such as a drinking straw and a full soft drink can, or a chopstick and a soup can
- a piece of cardboard, 30 inches by 6 inches
- a pair of scissors, and a roll of tape
- a needle, and 18 inches of brightly colored thread
- 2 paper clips
- 3 coins (use quarters, or even heavier objects, if windy)

- a pencil or pen
- a calculator
- 2 large cardboard boxes, chairs, or stools (to create 2 flat surfaces a couple of feet off the ground; recommended but not required)
- a clamp, to secure your measuring device to a flat surface (helpful, but not required)

You will also need a computer with an internet connection, to analyze the data you collect from your parallax experiment.

8.1.3 Primary Tasks

This lab is built on two activities: 1) a parallax measurement experiment, to be performed in a safe, dry, well-lit space with a view of the horizon (or at least out to a distance of 200 feet), and 2) an application of the parallax technique to stars. Students will complete these two activities, answer a set of final (Post-Lab) questions, and write a summary of the laboratory exercise.

8.1.4 Grading Scheme

There are 100 points available for completing the exercise and submitting the lab report perfectly. They are allotted as shown below, with set numbers of points being awarded for individual questions and tasks within each section. Note that Section 8.6 (§8.6) contains 5 extra credit points.

Table 8.1: Breakdown of Points

Activity	Parallax Experiment	Stellar Parallax	Questions	Summary
Section	§8.2	§8.3	§8.4	§8.5
Page	3	17	22	24
Points	46	14	15	25

8.1.5 Timeline

Week 1: Read §8.1–§8.3, complete activities in §8.2 and §8.3, and begin final (Post-Lab) questions in §8.4. Identify any issues that are not clear to you, so that you can receive feedback and assistance from your instructors before Week 2.

Week 2: Finish final (Post-Lab) questions in §8.4, write lab summary, and submit completed lab report.

8.2 The Parallax Experiment

In this experiment, you will develop a better understanding of parallax by measuring the apparent shift in position of a nearby object relative to the background as you view it from two locations. You will explore the effects of changing the "object distance," the distance between an object and an "observer" (you), and of changing the "vantage point separation" (the distance between your two viewing locations).

8.2.1 Setting up your experiment

Our first step is to create a measuring device for observing angles between various object and landmarks, as shown in Figure 8.5. The more carefully you build your device the more accurate your results will be, so take your time and work carefully. You can put the device together ahead of time, and then conduct your experiment later if this is more convenient than completing the entire experiment all at once.

Find the first protractor provided in your lab manual at the end of this chapter (on page 27) and carefully cut it out. Make sure to follow the dotted lines, so that your edges are straight and form right angles (90° corners). Trim down a piece of cardboard (such as the side of a large box) to form a rectangle 30 inches wide and 6 inches tall. Cut the edges as perfectly straight as you can make them, again forming right angles.

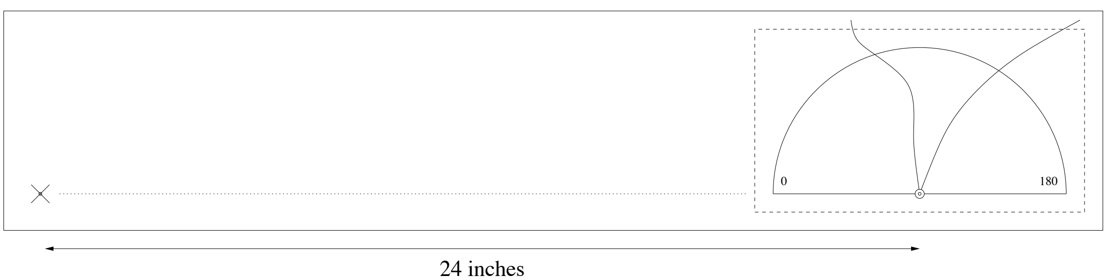

Figure 8.1: The measuring device has a base formed from a piece of cardboard, 30 inches wide and 6 inches high. The protractor is affixed to the right side, and an "X" is marked exactly 24 inches to the left of its origin. This two-foot distance represents the two astronomical units between the Earth's position around the Sun during January and July. Straightened paper clips are attached perpendicular to the surface at the protractor origin and the "X" mark, to mark sight-lines. A piece of thread is doubled and secured at the protractor origin, to create two threads that can be rotated around the protractor to mark various angles between 0 and 180 degrees.

Place the cardboard frame in front of you on a table, extending 15 inches on either side of you. Now place the protractor on the right end of the cardboard. Tape it securely to the cardboard, making sure that the dotted line forming the edge of the protractor paper is parallel to the long edge of the cardboard. Take your tape measure or yardstick, and measure a distance of two feet (24 inches) to the left of the origin of the protractor. (The origin is the dot in the center of the small circle, at the point where the dashed lines pointing to the numbers 10, 20, and so on up to 170, all meet.) Make sure to align your tape measure precisely with the line on the protractor extending from 180 to 0, and don't let it tilt. Mark the point 24 inches to the left with a small "X".

Next take a threaded needle, and thrust it through the origin of the protractor. Pull just enough thread through the hole that you can cut the needle free, and tape the two ends of thread securely to the back of the cardboard. You should have two pieces of thread coming up through the origin, each long enough to reach past the arc of numbers and trail off of the edge of the protractor paper.

Now take your two paper clips. Unfold one bend in each one, making the resultant long stick as straight as possible. This is very important, so spend a bit of time getting the bend out and straightening any kinks. The remaining portion will have two bends; force the first into a perfect right angle and open up the last one as well. Your paper clip should now have the form of a small flagpole, with a broad "V" shaped base. If you hold the base on a table, the flagpole should point upward. Tape over the tips of the flagpoles with small pieces of tape, so that they do not scratch anything or anyone.

Place the first paper clip on top of the origin of the protractor, and tape it into place. Don't secure it until you have checked that the flagpole is centered directly over the center of the origin when viewed from all angles around it (you will probably have to touch up your paper clip angles a bit as you do this). This is another critical step, so take your time and do it well.

Place the second paper clip on top of the "X" mark 24 inches to the left of the protractor origin (at the left end of the cardboard). Again take your time and make sure that it is located directly above the mark, and points directly upward.

Our next step is to construct a support for the thin object which you will observe. If you are using a drinking straw and a soft drink can, tape the straw securely to the side of the can, pointing straight up, so that the top of the straw lies at least 6 inches above the table. If you are using a thin chopstick and a soup can, tape the chopstick securely to the side of the can, pointing straight up, so that the top of the chopstick lies at least 6 inches above the table.

You will be lining up the position of the top of your thin object (the tip of the straw, or the thinner end of the chopstick) with your paper clip markers by eye, so it needs to be held straight, without drooping or falling.

8.2.2 Taking parallax measurements

Complete the following, answering the 10 questions and completing Tables 8.2–8.5. Filling in the three tables correctly is worth 22 points, and the associated questions are worth 20 additional points (2 points per question).

You will need to find a clear, flat fairly-open space (ideally 30 feet long), with a view to the horizon (or to at least 200 feet away), as shown in Figure 8.2. An empty parking lot, a large room with an expansive view of the horizon (look for a ten-foot wide window on the first few floors of a library or a community center, with a view of mountains or a radio tower), or

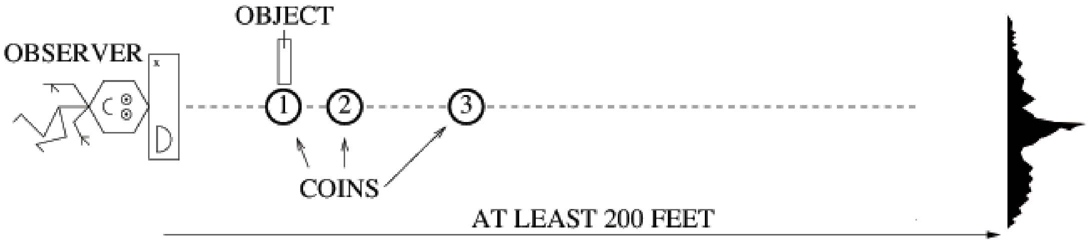

Figure 8.2: Basic layout of the parallax experiment. Lay the coins in a fairly straight line, between 4 and 30 feet away from the measuring device. They are marking the position at which you will place objects for distance estimation, so if you are using existing objects such as street sign or lamp posts, chimneys, or trees, you will not need them.

a backyard with a low fence might all be possible places to work. Be sure that your location is safe, well-lit, and not too windy.

You will need a stable support for your measuring device, such as a wall, a table, a stool, or cardboard box. The support should raise it high enough off the ground that you can comfortably sight along both paper clip posts toward features on the horizon, and should keep it level. It is important that your measuring device does not shift position as you work, even slightly. If you have a clamp, use it to secure your measuring device in place; otherwise, tape may help. You are going to use the device to estimate the distance to several objects between you and the horizon (or a landmark at least 200 feet in front of you).

Once you have your measuring apparatus in place, select an initial location at which to place an object, some five feet or so in front of it. If there are existing objects with clearly-defined vertical lines in front of you, such as a post supporting a street lamp, a chimney, or a radio antenna on a car, feel free to use them. Otherwise, place your foreground object (the can with an attached straw or chopstick) on top of a stool or cardboard box, and place your first coin on the ground next to it at the same distance from the measuring device as the object. Be sure to match the position of the coin with the position of the straw or chopstick, not with the supporting stool or box.

Before committing to a particular position, check that you can sight along both paper clip posts (first one, and then the other) and the object, and have a clear view to the horizon or to a distant landmark. If you need to shift your object (or your measuring device) slightly to the left or right, that is fine. (You can sight along the measuring device and direct your lab partner to shift the object for you if you are working in a team, to save time.)

You are now going to estimate the distance to the object, directly and via the parallax method. First, measure directly by running the tape measure from the object to the center of the line between your two paper clip posts on your measuring device. (Your partner can help by holding one end of the tape measure for you.) This is the equivalent of measuring the distance between a nearby star and the Sun, if we had a really long, long tape measure. Record this distance, in units of inches, in Table 8.2 under "Position 1." (If you prefer to use the metric system, then give yourself a pat on the back and replace inches with centimeters throughout this exercise. Be sure to indicate the change of units clearly in Tables 8.2, 8.3, and 8.5, however.) We will use the parallax method to compute this distance independently without leaving our measuring device, and then we will compare our two distance measurements.

Select two more existing objects anywhere between 7 and 30 feet in front of your device, or place two more coins to mark the position at which you will place your foreground object in turn. Record the distance between each object and the center of the measuring device, and record the values in the remaining two columns of Table 8.2. If your tape measure is not long enough to extend all the way from the measuring apparatus to the third object position, use a long piece of string to measure the distance piece by piece.

1. Estimate the distance to the horizon, or to a distant landmark in line with your measuring device and your object at Position 1, by eye. (This can be a very rough estimate.)

________________________. (2 points)

Table 8.2: Direct Measurements of Distance[a]

Measure distances in inches, to the nearest half-inch.		
Position 1	Position 2	Position 3

(4 points)

[a]Distance from observer and apparatus to three foreground objects.

While it is easy to directly measure these distances for sign posts 20 feet away from us, it is impossible to measure them directly for stars, as they lie many light-years away from Earth. We will use these measurements as "controls," and compare our parallax-derived distances to them at the end of the experiment.

You will now measure two angles for each of the three object positions, repeating each set of measurements three times and recording the data in Table 8.3. By conducting three trials you will be able to see how much your measurements vary from trial to trial, and get a sense of the measurement errors.

Begin by sitting down behind the left end of your measuring device. Sight down the line formed by the paper clip post and the object at Position 1. Identify a distant landmark which lines up with these two objects, such as a radio tower, the edge of a building, or a feature on a mountain range. Hold the image of that feature in your head!

Now shift over to the right end of our measuring device. Ask your partner to press down firmly on the middle of the cardboard measuring device, so that you do not accidentally jiggle it. Sight down this second paper clip post, and find the same distant landmark. Because you will have shifted your position by a mere two feet, it will appear at the same position on your horizon. Gently pick up one of the threads attached to the protractor, and line it up with the paper clip post and the distant landmark, taking care not to disturb the measurement device. When you have it in place on top of the protractor, tape it in place gently (or have your partner do so for you). Use a piece of tape with most of the glue removed (stick it to a spare piece of paper or to your clothes a few times to strip the glue), so that you can place and remove the thread repeatedly without tearing the paper protractor.

Now find the foreground object, and sight down the paper clip post and the object. While the object and the distant landmark lined up together when viewed from the left, by shifting your vantage point you will observe that the foreground object appears to shift to the left across your field of view. Pick up the second thread, and use it to mark the angle on the protractor for the apparent position of the foreground object.

The two pieces of thread indicate the angles at which the landmark and the foreground object appeared to you on the protractor. Record the angle of the landmark as "Angle 1" in the first row of Table 8.3, and the angle of the object as "Angle 2" in the next column. (You will fill in the final four columns later.) There are small lines on the protractor for every degree between 0 and 180; record the angle at which the threads lie to the nearest tenth of a degree. The difference between these angles tells you the angular shift of the foreground object, or the amount by which it appeared to move across the sky, when you shifted your vantage point by two feet. (From the left vantage point, we selected our distant landmark so that Angle 1 and Angle 2 were identical, so we don't need to record them and calculate their difference.)

Now move on to Positions 2 and 3. Either shift your foreground object to the position of the second (and then the third) coin, or shift your attention to a slightly more distant existing object in your field of view. Again, line up the left paper clip post, the foreground object, and a distant landmark in a single line. You can use the same landmark if your additional objects also line up with it, or change to another landmark. Then observe through your right vantage point, and record the angular position of the landmark and the foreground object, in the first two columns of the second and third rows of Table 8.3.

At this point you should have completed the first two columns of the first three rows of Table 8.3. We will now repeat these measurements two more times, in order to measure how repeatable our results are. The variation in measured angles will give us an estimate of the measurement errors of our technique. As you are measuring angles to the nearest tenth of a degree, there will be variations from trial to trial, and for the most distant objects, the variation could easily be more than a degree. Try not to look at the results of the first trial while conducting the second and third trials, so that you do not bias your data. You want your measurements to be independent of each other. Conduct the second and third trials in turn, and fill in the rest of the first two columns in the table.

DO NOT perform all three trials of any object position at the same time – that would defeat the purpose of taking three independent measurements.

Table 8.3: Parallax Measurements

Record angles to the nearest tenth-degree, distances to the nearest half-inch.						
	Angle 1 (°)[a]	Angle 2 (°)[b]	2α (°)[c]	α (°)	r/d[d]	d (in)[e]
Trial 1						
Posn. 1						
Posn. 2						
Posn. 3						
Trial 2						
Posn. 1						
Posn. 2						
Posn. 3						
Trial 3						
Posn. 1						
Posn. 2						
Posn. 3						

(16 points)

[a]Landmark angle; [b]foreground object angle; [c] 2α = Angle 1 − Angle 2; [d] r/d values from Table 8.4 (page 15); [e] $d = \frac{r}{r/d}$, where $2r$ is the vantage point separation (24 inches).

2. Estimate the *uncertainty* in your measurement of the object's apparent shift. For example, do you think your recorded measurements could be off by ten degrees? One degree? One tenth of a degree? Compare the measurements made at each position from trial to trial, to help you estimate the reliability of your measurements. (2 points)

8.2.3 Dependence of parallax on vantage point separation

Now that we understand how the apparent shift of an object changes as its distance from the observer changes, let's explore what happens when the distance between the vantage points changes.

3. What would happen if the vantage points were farther apart? We separated our vantage points by two feet, to simulate the two astronomical units by which the Earth shifts position over a six-month period. What if we had used a separation of ten feet instead? How would you expect the angular shift of the object (the difference between Angle 1 and Angle 2) to change? (Note that there is no wrong answer to this question. The point is to take a guess, and then to verify or to disprove it.) (2 points)

4. Repeat the experiment with the object at Position 3, but this time estimate the apparent shift from positions separated by four feet (shift the whole measuring device an additional two feet to the right after selecting your landmark on the left). If you are working inside and have a limited view of the horizon, however, don't shift the right vantage point so far that your foreground object moves out of the window and is lost from sight. By how many degrees did the object move using the more widely separated vantage points? (2 points)

5. For an object at a fixed distance, how does the apparent shift change as you observe from more widely separated vantage points? (2 points)

At this point, you should have completed your measurements with the parallax apparatus. You can pack it up, and complete this exercise at a location of your choice.

8.2.4 Measuring distances using parallax

We have seen that an object's apparent shift relative to background objects depends both on the distance between the object and the observer and on the separation between the observer's two vantage points. We can now turn this around: if we can measure the apparent shift and the separation of the two vantage points, we should be able to calculate the distance to an object. This is very handy, as it provides a way of measuring distance without actually having to go all the way to an object. Since we cannot travel to the stars, this is an excellent way to measure their distances from us.

We will now see how parallax can be used to determine the distances to the objects in your experiment based only on your measurements of their apparent changes in angular position ("apparent shifts") and the measurement of the separation of your two vantage points (your "baseline").

Angular shift

6. The apparent shift of the object is caused by looking at the object from two different vantage points. Qualitatively, what do you see changing from viewpoint to viewpoint? As a foreground object moves farther away from you, does its apparent shift increase or decrease? (2 points)

To check your answer, consider the apparent angular shift for the two objects shown in Figure 8.3.

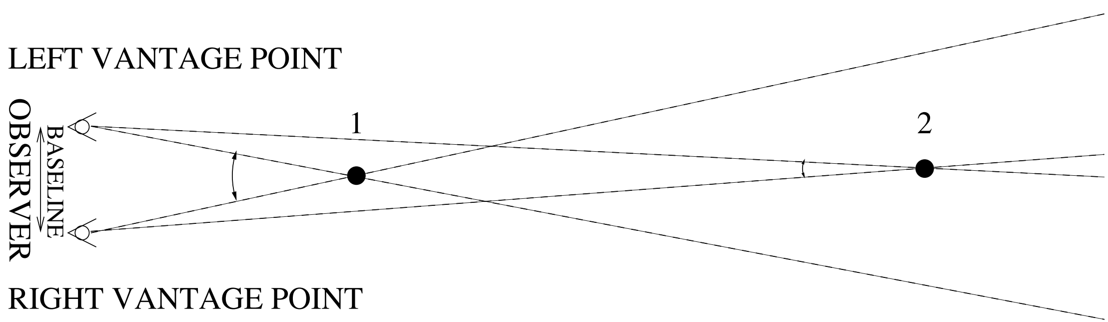

Figure 8.3: Layout of the parallax experiment, comparing the angular shift for foreground objects at two positions.

Distance between the vantage points (the "baseline")

We used a baseline of two feet, or 24 inches, in our experiment. Each foot represents the distance between the Earth and the Sun, as over the course of six months the Earth shifts its position around the Sun by two such astronomical units.

Using parallax measurements to determine the distance to an object

To determine the distance to an object for which you have a parallax measurement, you can construct an imaginary triangle between the two different vantage points and the object, as shown in Figure 8.4.

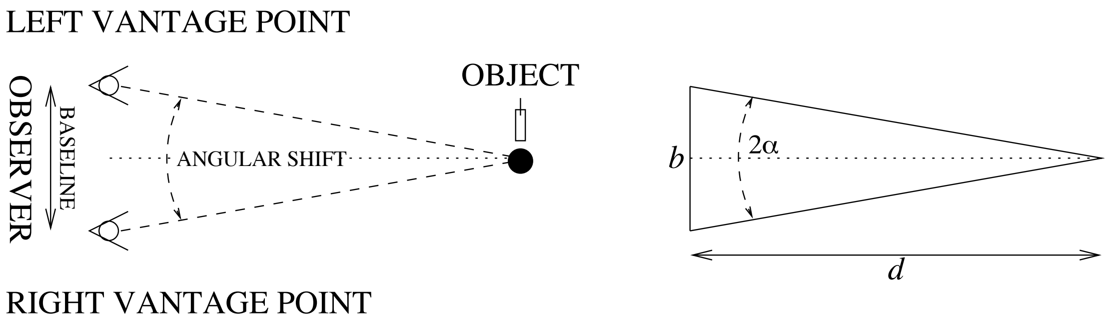

Figure 8.4: The parallax experiment, with real objects (left) and expressed mathematically (right). Can you find the same triangle on both sides?

The angular shifts that you have just calculated correspond to the angle 2α on the diagram, and the distance between your vantage points (your "baseline") corresponds to the distance b. The distance to the object, which you wish to determine, is d.

Let's recast this slightly now, to better match the way in which we observe stars from Earth. The baseline is equivalent to twice the orbital radius of the Earth, so we'll define "b" to be "2r" now, as shown in Figure 8.5. We can also separate 2α into two equal angles, each with a value of α. This figure emphasizes that from the left vantage point, the foreground object and a landmark on the horizon line up with each other, but from the right, they are separated by an angle of 2α.

The 2α angle appears in three places on the diagram, and shows us that a right triangle (a triangle with a right angle) can be formed with height r, width d, and angle α. This triangle holds the key to determining "d."

Can you see that the difference between Angle 1 and Angle 2, the two angles that you measured in your experiment, is equal to 2α? Take a good look at Figure 8.5 to ensure that this makes sense to you. Once it does, fill in the third column of Table 8.3 with a value for 2α (Angle 1 – Angle 2). Having done so, divide your values by a factor of two and fill in values for α in the fourth column.

People have been studying right triangles for thousands of years, and so we know a fair bit about how they work. One of their most useful properties is the fact that if you know *the ratio of the height to the width* of a right triangle, you can determine the values for the interior angles – you know exactly how large α is. Figure 8.6 illustrates this fact. It shows three different right triangles. Though each triangle has a different height and a different width, because the ratio of the height to the width is the same in each case, the value of α is also the same for all three triangles.

If you have studied trigonometry, you may recognize that this ratio, r/d, is the tangent of α. However, you do not need to be familiar with tangents to understand the clear connection

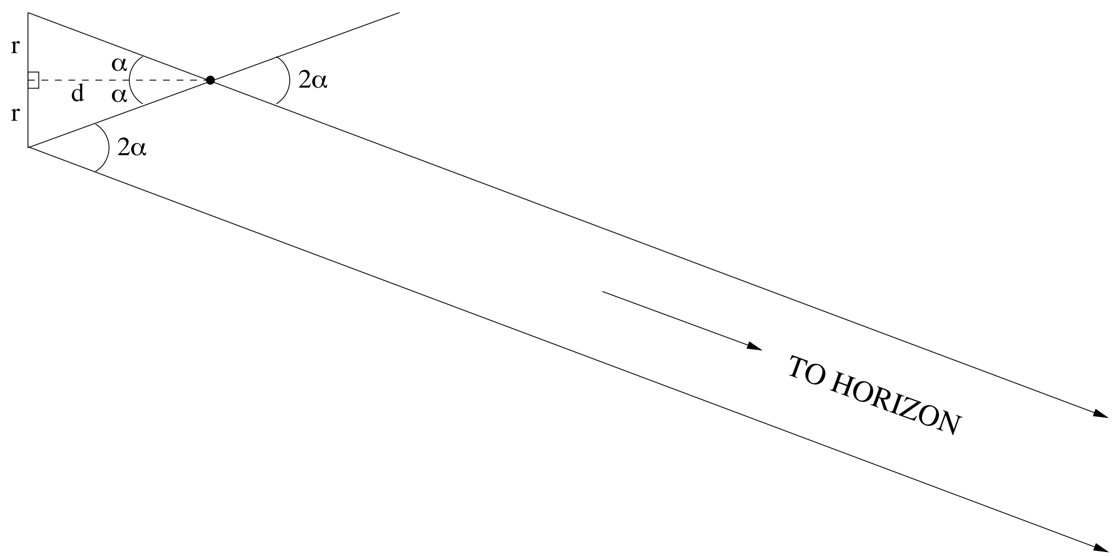

Figure 8.5: Our two vantage points are separated by a distance of $2r$, equivalent to twice the Earth's orbital radius. A landmark on the horizon will appear at the same angle from both vantage points, because it is so far away, but a nearby object a distance d away will appear to shift position, by an angle of 2α.

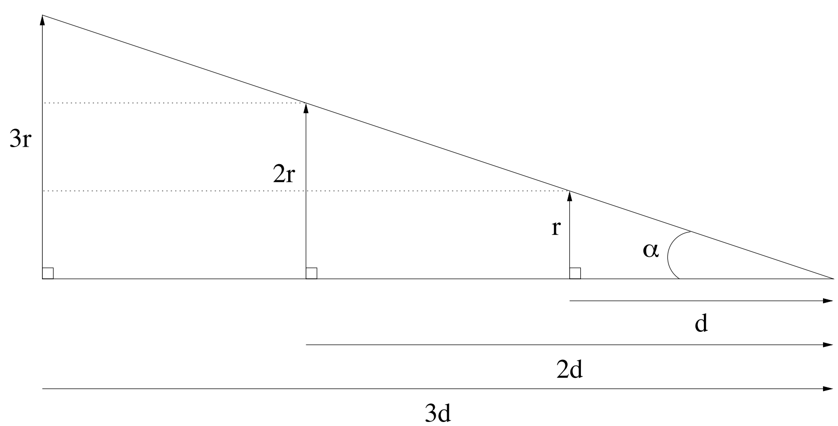

Figure 8.6: The angle α in a right triangle is determined uniquely by the ratio of its height, r, to its width d. We see here that if we double the height and width, or triple them both, α remains unchanged.

between r/d and α. We have established that there is a unique relationship between the value of r/d and the value of α for a right triangle. Figure 8.7 emphasizes this fact, showing how α changes value as we vary the value of d for right triangles of height r.

In studying our foreground objects, what do we know? We know that we can define a useful right triangle that has a height r equal to one foot, a width d equal to the distance to the

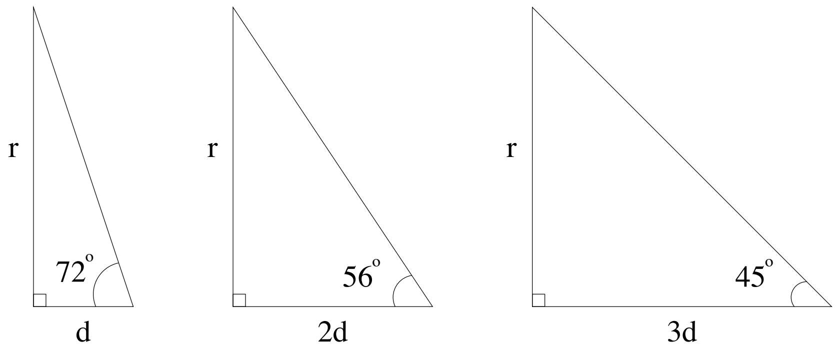

Figure 8.7: If we know the height r and the width d of a right triangle, we know the value of its interior angles. We see here that as we double and then triple the width of such a triangle, the labeled angle decreases in response.

object, and that we have determined values for the angle α to match. From the preceding discussion, we can see that with r and α known, we can determine d!

Table 8.4 provides the last piece of the puzzle. It tabulates values of r/d for values of α ranging from zero to 20 degrees. Read this table by combining each set of two columns in order. The first, third, fifth, seventh, and ninth columns list values for α. Take your first measurement of α, listed in the fourth column of the first row of Table 8.3, and find it in Table 8.4, reading off the appropriate value of r/d from the column *to the right*. For example, if $\alpha = 0.1°$ then $r/d = 0.0017$, and if $\alpha = 20.0°$ then $r/d = 0.3640$. If you have a value for α which lies between two values listed in the table, just average the r/d values for the nearest two α values listed. For example, if your value for α is 19.95° then $r/d = (0.3620 + 0.3640)/2 = 0.3630$.

If you are comfortable using a calculator to calculate tangent values you may of course do so instead. Just remember that the values for α in Table 8.3 are listed in units of degrees, and make sure that your calculator is expecting to work with angles in degrees, not radians. As a check, calculate $tan(10)$. You should get a value of 0.1763.

Look up the values of r/d now for the nine values of α you added to Table 8.3, and place them in the fifth column of that table.

Our final step is very straight-forward. We know the values for r and for r/d, and we need to solve for d.

$$d = \frac{r}{r} \times d = r \times \frac{d}{r} = \frac{r}{r/d}.$$

Calculate values for d for your nine rows of Table 8.3, by dividing your value for r, in feet, by your dimensionless (with no units) values for r/d. Then multiply your answer by twelve to convert from feet to inches, and fill these values for d into the final column of Table 8.3.

Table 8.4: Height-to-Width Ratios (r/d) for Right Triangles

α (°)	r/d	α (°)	r/d	α (°)	r/d	α (°)	r/d	α (°)	r/d
0.1	0.0017	4.1	0.0717	8.1	0.1423	12.1	0.2144	16.1	0.2886
0.2	0.0035	4.2	0.0734	8.2	0.1441	12.2	0.2162	16.2	0.2905
0.3	0.0052	4.3	0.0752	8.3	0.1459	12.3	0.2180	16.3	0.2924
0.4	0.0070	4.4	0.0769	8.4	0.1477	12.4	0.2199	16.4	0.2943
0.5	0.0087	4.5	0.0787	8.5	0.1495	12.5	0.2217	16.5	0.2962
0.6	0.0105	4.6	0.0805	8.6	0.1512	12.6	0.2235	16.6	0.2981
0.7	0.0122	4.7	0.0822	8.7	0.1530	12.7	0.2254	16.7	0.3000
0.8	0.0140	4.8	0.0840	8.8	0.1548	12.8	0.2272	16.8	0.3019
0.9	0.0157	4.9	0.0857	8.9	0.1566	12.9	0.2290	16.9	0.3038
1.0	0.0175	5.0	0.0875	9.0	0.1584	13.0	0.2309	17.0	0.3057
1.1	0.0192	5.1	0.0892	9.1	0.1602	13.1	0.2327	17.1	0.3076
1.2	0.0209	5.2	0.0910	9.2	0.1620	13.2	0.2345	17.2	0.3096
1.3	0.0227	5.3	0.0928	9.3	0.1638	13.3	0.2364	17.3	0.3115
1.4	0.0244	5.4	0.0945	9.4	0.1655	13.4	0.2382	17.4	0.3134
1.5	0.0262	5.5	0.0963	9.5	0.1673	13.5	0.2401	17.5	0.3153
1.6	0.0279	5.6	0.0981	9.6	0.1691	13.6	0.2419	17.6	0.3172
1.7	0.0297	5.7	0.0998	9.7	0.1709	13.7	0.2438	17.7	0.3191
1.8	0.0314	5.8	0.1016	9.8	0.1727	13.8	0.2456	17.8	0.3211
1.9	0.0332	5.9	0.1033	9.9	0.1745	13.9	0.2475	17.9	0.3230
2.0	0.0349	6.0	0.1051	10.0	0.1763	14.0	0.2493	18.0	0.3249
2.1	0.0367	6.1	0.1069	10.1	0.1781	14.1	0.2512	18.1	0.3269
2.2	0.0384	6.2	0.1086	10.2	0.1799	14.2	0.2530	18.2	0.3288
2.3	0.0402	6.3	0.1104	10.3	0.1817	14.3	0.2549	18.3	0.3307
2.4	0.0419	6.4	0.1122	10.4	0.1835	14.4	0.2568	18.4	0.3327
2.5	0.0437	6.5	0.1139	10.5	0.1853	14.5	0.2586	18.5	0.3346
2.6	0.0454	6.6	0.1157	10.6	0.1871	14.6	0.2605	18.6	0.3365
2.7	0.0472	6.7	0.1175	10.7	0.1890	14.7	0.2623	18.7	0.3385
2.8	0.0489	6.8	0.1192	10.8	0.1908	14.8	0.2642	18.8	0.3404
2.9	0.0507	6.9	0.1210	10.9	0.1926	14.9	0.2661	18.9	0.3424
3.0	0.0524	7.0	0.1228	11.0	0.1944	15.0	0.2679	19.0	0.3443
3.1	0.0542	7.1	0.1246	11.1	0.1962	15.1	0.2698	19.1	0.3463
3.2	0.0559	7.2	0.1263	11.2	0.1980	15.2	0.2717	19.2	0.3482
3.3	0.0577	7.3	0.1281	11.3	0.1998	15.3	0.2736	19.3	0.3502
3.4	0.0594	7.4	0.1299	11.4	0.2016	15.4	0.2754	19.4	0.3522
3.5	0.0612	7.5	0.1317	11.5	0.2035	15.5	0.2773	19.5	0.3541
3.6	0.0629	7.6	0.1334	11.6	0.2053	15.6	0.2792	19.6	0.3561
3.7	0.0647	7.7	0.1352	11.7	0.2071	15.7	0.2811	19.7	0.3581
3.8	0.0664	7.8	0.1370	11.8	0.2089	15.8	0.2830	19.8	0.3600
3.9	0.0682	7.9	0.1388	11.9	0.2107	15.9	0.2849	19.9	0.3620
4.0	0.0699	8.0	0.1405	12.0	0.2126	16.0	0.2867	20.0	0.3640

7. Based on your estimate of the uncertainty in the angular measurements of 2α, estimate the uncertainty in your measurements of the object distances. (Note that there is no wrong answer to this question. The point is to take a guess, and then to verify or to disprove it.) (2 points)

8. Now look at the spread in the three values for each position in the last column of Table 8.3. Is this spread consistent with your estimated uncertainty? (2 points)

How good are your parallax-derived distances?

At this point, you are ready to average your distance measurements together, and compute their standard deviations. Access the plotting tool listed for this lab exercise from the GEAS project lab exercise web page (see the URL on page 2 in §8.1.2). You can use the plotting tool to create histograms of your distance measurements if you have a user account, entering the three values measured in Trials 1 through 3 for each quantity in turn. Once you enter your three values the web page will will display their mean value and associated error (you do not need to log in to see this information). Record the averaged values shown for each plot (the "mean value", or μ), and the associated errors (σ). In order to see how good your distance measurements are, calculate these averages and errors for each of the distance estimates d recorded three times in the final column of Table 8.3.

9. Now compare the distances that you calculated for each position using the parallax method to the distances that you measured directly at the beginning of the experiment (in Table 8.2). How well did the parallax technique work? Are the differences between the direct measurements and your parallax-derived measurements within your errors (within 2σ)? (2 points)

Table 8.5: Comparison of Average Distances

	Direct Distance (from Table 8.2)	Parallax Distance (from Table 8.3)
Measured and parallax-derived distances from observer to object, in inches. For parallax distances record the average values (μ) of the three trials, and the errors (σ) calculated with the plotting tool in this form: $nn.n \pm n.n$.		
Position 1		
Position 2		
Position 3		

(6 points)

10. If the differences are larger than 5σ, can you think of a reason why your measurements might have some additional error in them? We might call this a "systematic" error, if it is connected to a big approximation in our observational setup. (2 points)

8.3 Calculating Astronomical Distances With Parallax

Complete the following section, answering each of the four questions in turn. (Each question is worth either 3 or 5 points.)

8.3.1 Distances on Earth and within the Solar System

1. We have just demonstrated how parallax works on a small scale, so now let us move to a larger playing field. Use the information in Table 8.4 to determine the angular shift (2α, in degrees) for Organ Summit, the highest peak in the Organ Mountains, if you observed it with a baseline $2r$ of not 2 feet, but 300 feet, from NMSU. Organ Summit is located 12 miles from Las Cruces. (*If you are working from another location, select a mountain, sky scraper, or other landmark at a similar distance to use in place of the Organ Summit.*) There are 5,280 feet in a mile. (3 points)

You should have gotten a small angle!

The smallest angle that the best human eyes can resolve is about 0.02 degrees. Obviously, our eyes (with an internal baseline of only 3 or 4 inches) provide an inadequate baseline for measuring large distances. How could we create a bigger baseline? Surveyors use a "transit," a small telescope mounted on a protractor, to carefully measure angles to distant objects. By positioning the transit at two different spots separated by exactly 300 feet (and carefully measuring this baseline), they will observe a much larger angular shift. Recall that when you increased the distance between your two vantage points, the angular shift increased. This means that if an observer has a larger baseline, he or she can measure the distances to objects which lie farther away. With a surveying transit's 300-foot baseline, it is thus fairly easy to measure the distances to faraway trees, mountains, buildings or other large objects here on Earth.

2. What about an object farther out in the solar system? Consider our near neighbor, planet Mars. At its closest approach, Mars comes to within 0.4 A.U. of the Earth. (Remember that an A.U. is the average distance between the Earth and the Sun, or 1.5×10^8 kilometers.) At such a large distance we will need an even larger baseline than a transit could provide, so let us assume we have two telescopes in neighboring states, and calculate the ratio r/d for Mars for a baseline of 1000 kilometers. Can you even find this value in Table 8.4? (3 points)

You should get a value for r/d which lies well beyond the bounds of Table 8.4, less than 5×10^{-5}. For the correct value, the equivalent value of 2α is 0.00019 degrees, or 7 arcseconds (where there are 60 arcminutes per degree, and 60 arcseconds per arcminute).

8.3.2 Distances to stars, and the "parsec"

The angular shifts for even our closest neighboring planets are clearly quite small, even with a fairly large baseline. Stars, of course, are *much* farther away. The nearest star is 1.9×10^{13} miles, or 1.2×10^{18} inches, away! At such a tremendous distance, the apparent angular shift is extremely small. When observed through the two vantage points of your two eyes, the angular shift of the nearest star corresponds to the apparent diameter of a human hair seen at the distance of the Sun! This is a truly tiny angle and totally unmeasurable by eye.

Like geological surveyors, we can improve our situation by using two more widely separated vantage points. In order to separate our two observations as far as possible from each other, we will take advantage of the Earth's motion around the Sun. The Earth's orbit forms a large circle around the Sun, and so by observing a star from first one position and then waiting six months for the Earth to revolve around to the other side of the Sun, we will achieve a separation of two A.U. (twice the average distance between the Earth and the Sun). This is the distance between our two vantage points, labeled b in Figure 8.8.

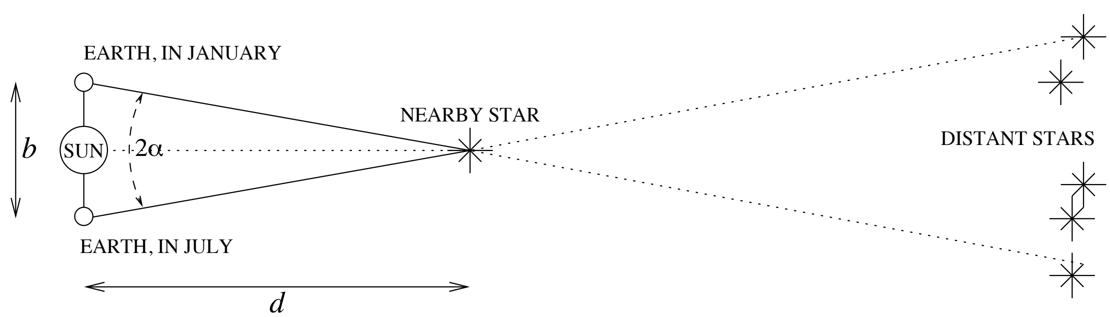

Figure 8.8: The parallax experiment, as done from Earth over a period of six months.

An A.U. (astronomical unit) is equal to 1.5×10^8 kilometers, so b is equal to twice that, or 300 million kilometers. Even though this sounds like a large distance, we find that the apparent angular shift (2α in Figure 8.5) of even the nearest star is only about 0.00043 degrees, or the width of a quarter coin observed from two miles away. This is unobservable by eye, which is why we cannot directly observe parallax by looking at stars with the naked eye. However, such angles are relatively easy to measure using modern telescopes and instruments. (Note that the ancient Greeks used the lack of observable parallax angles to argue that the Sun rotated around the Earth rather than the reverse, having greatly underestimated the distances to the nearest stars.)

Let us now discuss the idea of angles that are smaller than a degree. Just as a clock ticks out hours, minutes, and seconds, angles on the sky are measured in degrees, arcminutes, and arcseconds. A single degree can be broken into 60 arcminutes, and each arcminute contains 60 arcseconds. An angular shift of 0.02 degrees is thus equal to 1.2 arcminutes, or to 72 arcseconds. Since the angular shift of even the nearest star (Alpha Centauri) is only 0.00042 degrees (1.5 arcseconds), we can see that arcseconds will be a most convenient unit to use when describing them. Astronomers append a double quotation mark ($''$) at the end of the angle to denote arcseconds, writing $\alpha = 0.75''$ for the nearest star. When astronomers talk about the "parallax" or "parallax angle" of a star, they mean α.

The small angle approximation

If we look closely at the values for α and r/d in the first two columns of Table 8.4, we will notice an interesting, useful fact. For small values of α, $\alpha = 57.3 \times r/d$, and thus $r/d = \frac{\alpha}{57.3}$. Please don't take our word for it – check for yourself.

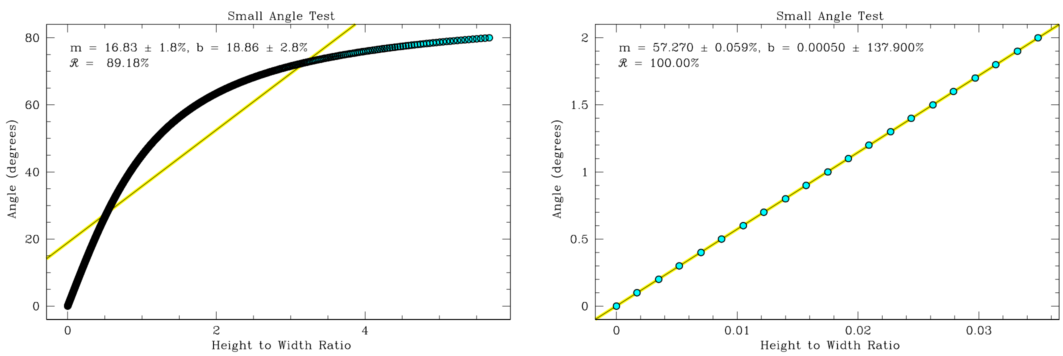

Figure 8.9: This figure plots the angle α on the y-axis and the height to width ratio r/d on the x-axis. On the left, we see the relationship for angles between zero and 80°, while on the right we restrict our range to run from zero to two degrees. In each case, the yellow line represents an attempt to fit a straight line to the points. The slope of the yellow line (m) is listed on each plot. For which values of α is there a linear relationship between α and r/d?

Based on the data shown in Figure 8.9, is a straight line a good fit to either set of data (for $\alpha \leq 80°$, or just up to 2°)? What is the slope, in the linear region? Are you now comfortable using this approximation to shift between α and r/d? (2 points)

This relationship is known as the *small angle approximation*. Why does it work? For small angles like those listed in the first column, the value of an angle, in radians, is equal to its tangent, or equal to r/d. It takes 360 degrees to form a full circle, or 2π radians (a convenient alternate unit for angles). We can thus divide our values for α in degrees by $360/2\pi$, or 57.3, to convert to radians, and then r/d is equal to α. Our expression for d thus simplifies to

$$d = \frac{57.3 \times r}{\alpha}.$$

For stellar parallaxes, r is one astronomical unit, so if α is measured in degrees,

$$d = \frac{57.3 \times 1\,\text{A.U.}}{\alpha},$$

and if α is measured in arcseconds,

$$d = \frac{57.3 \times 1\,\text{A.U.}}{\alpha/3,600} = \frac{206,265}{\alpha}\,\text{A.U.}$$

Astronomers have defined a new unit, the "parsec," a unit of distance equal to 206,265 astronomical units. The parallax angle expression thus simplifies neatly to

$$d\,(\text{pc}) = \frac{1}{\alpha\,('')},$$

where d is measured in units of parsecs, and α is measured in units of arcseconds.

The word parsec comes from the phrase "**par**allax **sec**ond." By definition, an object at a distance of 1 parsec has a parallax of $1''$. How far away is a star with parallax angle of $\alpha = 1''$, in units of light-years? It is is 3.26 light-years from our solar system. (To convert parsecs into light years, you simply multiply by 3.26 light-years per parsec.)

An object at 10 parsecs (32.6 light-years) has a parallax angle of $0.1''$, and an object at 100 parsecs has a parallax angle of $0.01''$. Remember that the farther away an object is from us, the smaller its parallax angle will be. The nearest star has a parallax of $\alpha = 0.78''$, and is thus at a distance of $1/\alpha = 1/0.75 = 1.3$ parsecs.

You may use the words parsec, kiloparsec, megaparsec and even gigaparsec in astronomy. These names are just shorthand methods of talking about large distances. A kiloparsec is 1,000 parsecs, or 3,260 light-years. A megaparsec is one million parsecs, and a gigaparsec is a whopping one billion parsecs! The parsec may seem like a strange unit at first, but it is ideal for describing the distances between stars within our galaxy.

4. Let's work through a couple of examples. (6 points)

(a) If a star has a parallax angle of $\alpha = 0.25''$, what is its distance (in parsecs)?

(b) If a star is 5 parsecs away from Earth, what is its parallax angle (in arcseconds)?

(c) If a star lies 5 parsecs from Earth, how many light-years away is it?

8.4 Final (Post-Lab) Questions

1. How does the parallax angle of an object change as it moves away from us? As we can only measure angles to a certain accuracy, is it easier to measure the distance to a nearby star or to a more distant star? Why? (3 points)

2. Relate the experiment you did in the first part of this lab to the way that parallax is used to measure the distances to nearby stars. Describe the process an astronomer goes through to determine the distance to a star using the parallax method. What did your two vantage points represent in the experiment? (5 points)

3. Imagine that you observe a star field twice, with a six-month gap between your observations, and that you see the two sets of stars shown in Figure 8.10:

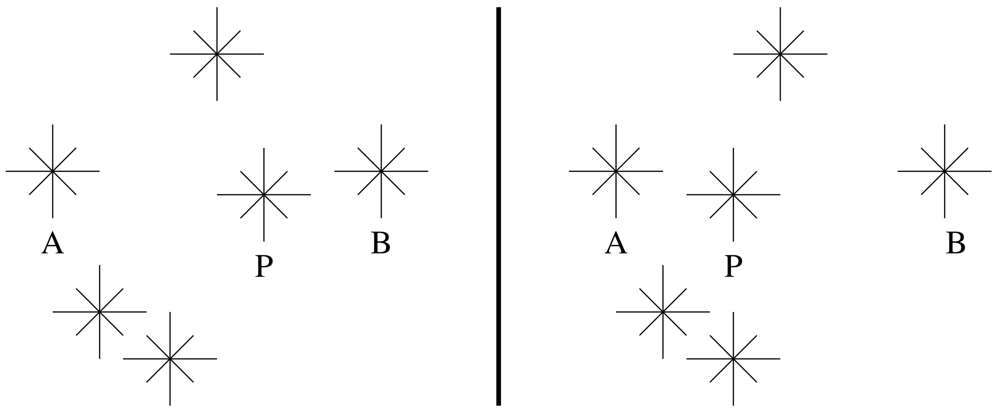

Figure 8.10: A star field, viewed from Earth in January (left) and again in July (right). Which star do you think lies closest to Earth?

The nearby star marked P appears to move between the two images, because of parallax. Consider the two images to be equivalent to the measurements that we made in our experiment, where each image represents the view of an object relative to a distant object as seen through one of your eyes. None of the stars but P change position; they correspond to the distant objects in our experiment.

If the angular distance between Stars A and B is 0.5 arcseconds then how far away would you estimate that Star P lies from Earth?

First, estimate how far Star P has moved between the two images relative to the constant distance between Stars A and B. This tells you the apparent angular shift of P (2α). You can then use the parallax equation ($d = 1/\alpha$) to estimate the distance to Star P. (3 points)

4. Astronomers like Tycho Brahe made careful naked eye observations of stars in the late 1600s, hoping to find evidence of semi-annual parallax shifts for those which were nearby and so weigh in on the growing debate over whether or not the Earth was in motion around the Sun. If the nearest stars (located 1.3 or more parsecs from Earth) were 100 times closer to us, or if the resolving power of the human eye (0.02 degrees) was improved by a factor of 100, could he have observed such shifts? Explain your answer. (4 points)

8.5 Summary

Summarize the important concepts discussed in this lab. Include a brief description of the basic principles of parallax and how astronomers use parallax to determine the distances to nearby stars. (25 points)

Be sure to think about and answer the following questions:

- Does the parallax method work for all of the stars we can see in our Galaxy? Why, or why not?

- Why is it so important for astronomers to determine the distances to the stars which they study?

Use complete sentences, and be sure to proofread your summary. It should be 300 to 500 words long.

8.6 Extra Credit

Use the web to learn about the planned Space Interferometry Mission (SIM). What are its goals, and how will it work? How accurately will it be able to measure parallax angles? How much better will SIM be than the best ground-based parallax measurement programs? Be sure that you understand the units of milliarcseconds ("mas") and microarcseconds, and can use them in your discussion. (5 points)

Be sure to cite your references, whether they are texts or URLs.

Protractor

Cut along outer dotted line.