Lab 4

Parallax Measurements and Determining Distances

4.1 Introduction

How do we know how far away stars and galaxies are from us? Determining the distances to these distant objects is one of the most difficult tasks that astronomers face. Since we cannot simply pull out a very long ruler to make a few measurements, we have to use other methods.

Inside the solar system, astronomers can bounce a radar signal off of a planet, asteroid or comet to directly measure its distance. How does this work? A radar signal is an electromagnetic wave (a beam of light), so it always travels at the same speed, the speed of light. Since we know how fast the signal travels, we just measure how long it takes to go out and to return to determine the object's distance.

Some stars, however, are located hundreds, thousands or even tens of thousands of "light-years" away from Earth. A light-year is the distance that light travels in a single year (about 9.5 trillion kilometers). To bounce a radar signal off of a star that is 100 light-years away, we would have to wait 200 years to get a signal back (remember the signal has to go out, bounce off the target, and come back). Obviously, radar is not a feasible method for determining how far away the stars are.

In fact, there is one, and only one, direct method to measure the distance to a star: the "parallax" method. Parallax is the angle by which something *appears* to move across the sky when an observer looking at that object changes position. By observing the size of this angle and knowing how far the observer has moved, one can determine the distance to the object. Today you will experiment with parallax, and develop an appreciation for the small angles that astronomers must measure to determine the distances to stars.

To get the basic idea, perform the following simple experiment. Hold your thumb out in front of you at arm's length and look at it with your left eye closed and your right eye open. Now close your right eye and open your left one. See how your thumb appeared to move to the right? Keep staring at your thumb, and change eyes several times. You should see your thumb appear to move back and forth, relative to the background. Of course, your thumb is not moving. Your *vantage point* is moving, and so your thumb *appears* to move. That's the parallax effect!

How does this work for stars? Instead of switching from eye to eye, we shift the position of our entire planet! We observe a star once, and then wait six months to observe it again. In six months, the Earth will have revolved half-way around the Sun. This shift of two A.U. (twice the distance between the Earth and the Sun) is equivalent to the distance between your two eyes. Just as your thumb will appear to shift position relative to background objects when viewed from one eye and then the other, over six months a nearby star will appear to shift position in the sky relative to very distant stars.

4.1.1 Goals

The primary goals of this laboratory exercise are to understand the theory and practice of using parallax to find the distances to nearby stars, and to use it to measure the distance to objects for yourself.

4.1.2 Materials

All online lab exercise components can be reached from the GEAS project lab URL, listed below.

http://astronomy.nmsu.edu/geas/labs/labs.html

You will need the following items to perform your parallax experiment:

- a wall-mountable parallax ruler, and a protractor (provided on pages 35–43)
- a pair of scissors, and a roll of strong tape
- a yardstick or a tape measure (35 feet long, if available)
- at least 35 feet of non-stretchy string (not yarn)
- a ruler
- 4 coins (use quarters, or even heavier objects, if windy)
- 2 pencils or pens
- a calculator
- a thin object and a tall object, such as: a toothpick and an 6-inch stack of textbooks, **OR** a thin straw and a filled soft drink can, **OR** a thin chopstick and a soup can

- a friendly assistant

Your assistant could be an adult or an older child, and needs no special knowledge of astronomy. S/he will assist you in marking points on your parallax ruler and measuring their separations, measuring the angle between two lines-of-sight, and measuring the spacing between your two eyes.

Wear clothes which can get slightly dusty, as you will be lying down on the ground while making some of your measurements. You may also want to bring an old towel, to protect your clothing and shield yourself from the temperature of the ground.

You will also need a computer with an internet connection, and a calculator, to analyze the data you collect from your parallax experiment.

4.1.3 Primary Tasks

This lab comprises two activities: 1) a parallax measurement experiment, to be performed in a safe, dry, well-lit space measuring at least 35 feet by eight feet, and 2) an application of the parallax technique to stars, which can be recorded either on paper or directly on a computer. Students will complete these two activities, answer a set of final (post-lab) questions, write a summary of the laboratory exercise, and create a complete lab exercise report via the online Google Documents system (see http://docs.google.com).

The activities within this lab are are a combination of field-work and computer-based ones, so you may either read most of this exercise on a computer screen, typing your answers to questions directly within the lab report template at Google Documents, or you may print out the lab exercise, make notes on the paper, and then transfer them into the template when you are done. We strongly recommend that you print out at least pages 10–27 for use in conducting the parallax experiment in §4.2.

4.1.4 Grading Scheme

There are 100 points available for completing the exercise and submitting the lab report perfectly. They are allotted as shown below, with set numbers of points being awarded for individual questions and tasks within each section. Note that Section 4.6 (§4.6) contains 4 extra credit points.

Table 4.1: Breakdown of Points

Activity	Parallax Experiment	Stellar Parallax	Questions	Summary
Section	§4.2	§4.3	§4.4	§4.5
Page	12	27	30	33
Points	39	7	19	35

4.1.5 Timeline

Week 1: Read §4.1–§4.3, complete activities in §4.2 and §4.3, and begin final (post-lab) questions in §4.4. Identify any issues that are not clear to you, so that you can receive feedback and assistance from your instructors before Week 2. Enter your preliminary results into your lab report template, and make sure that your instructors have been given access to it so that they can read and comment on it.

Week 2: Finish final (post-lab) questions in §4.4, write lab summary, and submit completed lab report.

4.2 The Parallax Experiment

In this experiment, you will develop a better understanding of parallax by measuring the apparent shift in position of an object along a parallax ruler (described below). You will explore the effects of changing the "object distance," the distance between an object and an "observer" (you), and of changing the "vantage point separation" (the distance between your two eyes).

4.2.1 Setting up your experiment

Our first step is to set up the parallax experiment. You will need to find a location with a wall wide enough to hold a six-foot-long ruler and a clear, flat open space (ideally 35 feet long) in front of it, as shown in Figure 4.1. A safe driveway, a fairly flat backyard, a long hallway, or a racquetball court or gymnasium might all be possible places to work. Be sure that your location is safe, well-lit, and not too windy.

Find the four pages of "ruler segment" provided in your lab manual at the end of this chapter (pages 35–41). To create a wall-mounted "parallax ruler," cut out the eight ruler segments, and tape them together. (You can also cut out the protractor on page 43 at this time.) The segments will form a six-foot ruler, with inch and foot divisions clearly marked. Now tape your parallax ruler to the wall, roughly 6 inches above the ground. Make sure that the sheets are securely attached to each other and to the wall, so that your ruler does not fall apart in the middle of the lab. It is also important that the ruler be placed as horizontally as possible, and be attached without sagging at any point.

The next step is to construct a support for the thin object which you will observe. If you are using a toothpick and a stack of textbooks, stack the books on top of each other neatly and tape the toothpick securely to the side of the top book. It should point up straight, like a flagpole, and extend well above the height of the book. If you are using a thin straw and a soft drink, tape the straw securely to the side of the can, pointing straight up, so that the top of the straw lies at least 6 inches above the ground. If you are using a thin chopstick and a soup can, tape the chopstick securely to the side of the can, pointing straight up, so that the top of the chopstick lies at least 6 inches above the ground.

The toothpick is the most ideal thin object of the three, because it is so narrow. If you use a thicker object, such as a straw or chopstick, be very careful to always train your eye along the same side (left or right) of the object when you observe it.

You will be lining up the position of the top of your thin object (the tip of the toothpick, one end of the straw, or the thinner end of the chopstick) with positions along the parallax ruler by eye, so it needs to be held straight, without drooping or falling.

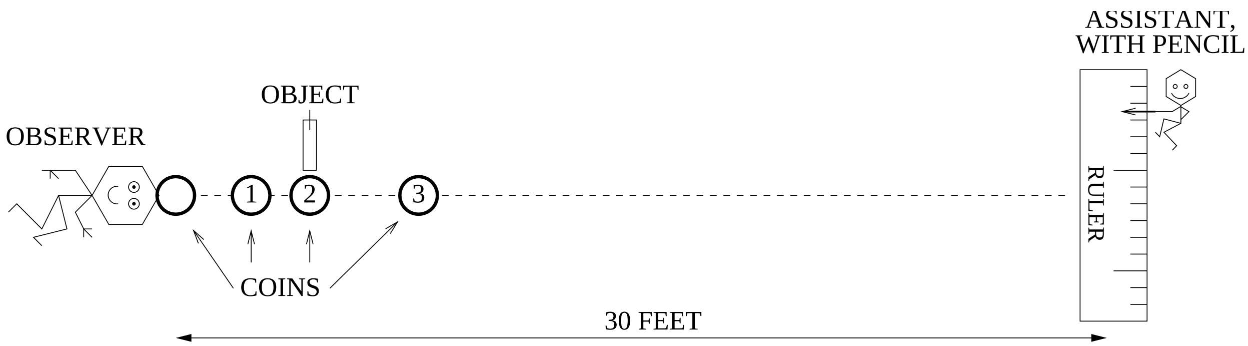


Figure 4.1: Basic layout of the parallax experiment. Lay the coins in a straight line along the string or tape measure, between the observer and the center of the ruler.

4.2.2 Taking parallax measurements

Complete the following, answering questions and filling in the blanks in Tables 4.2–4.5 as requested. (Filling in the four tables correctly is worth 18 points, and the associated questions are worth 19 additional points.)

You will need to have an assistant available for this part of the lab exercise. You will be the "observer," and your assistant will kneel next to the parallax ruler with a pencil and mark positions on it at your command.

Ask your assistant to hold one end of the long piece of string on the ground directly below the **center** of the parallax ruler. (If your tape measure is long enough, you may use it instead of string.) Take the other end of the string and walk 30 feet (12 really large paces) directly away from the center of the parallax ruler. We will refer to this position as the "observation spot." Pull the string taut and set it on the ground. Mark your observation spot on the ground with a coin next to the string, so that you can return to it later.

Measure how far your observation spot lies from the wall, and record your measurement here, to a fraction of an inch: ______________. (1 point)

If your tape measure is less than 30 feet long, you will need to do this in two (or more) stages. Measure to roughly 90% of the length of your tape measure away from the ruler, and carefully place a second coin at this location. Then measure the remaining distance to the observation spot, add the distances, and pick up the second coin.

You will now place three more coins along the string, between one and five feet away from the observation spot. Place the first coin just over a foot in front of the observation spot, the second coin four to six inches further away, and the third coin roughly five feet in front of the observation spot.

Before you continue, you should check that your detailed observations will be possible at the first location. Place the thin object on top of or next to the first coin, trying to position it exactly above either one edge or the center of the coin. (You may place the object just to the side of the string, if you are worried about the coin being moved, but make sure that the thin object lies the same distance from the observation spot as does the coin.) Now lie down flat behind the observation spot and look forward, so that your eyes are resting lightly on your folded hands above the observation spot and you can see both the thin object and the parallax ruler.

Look at the thin object through first one eye and then the other, and make sure that it lines up with the ruler, and does not "spill off" either end. If the thin object lines up with a position less than 0 feet, or more than 6 feet, then shift the first coin and the thin object forward an inch, and check again. Repeat as necessary. (If you need to shift the first coin forward more than an inch, then you may want to shift the second coin forward by a similar amount as well.)

You will now measure two numbers at each of the three object positions marked with coins, repeating each set of measurements three times.

First, use your tape measure to measure the distance along the ground between the coin at the observation point and each of the three coins at object positions (in inches). These are the true distances to the object positions. Record this information under the column labeled Trial 1 for Position 1, Position 2, and Position 3 in Table 4.2. Repeat this process two more times, entering the measurements into the next two columns. **The largest source of error in this measurement will be your ability to place your eyes repeatably directly above the coin at the observation point, so hold your head as carefully as possible when making measurements.**

While it is easy to measure these distances for toothpicks less than 30 feet away from us, it is impossible to measure them directly for stars, as they lie many light-years away from Earth. We will use these measurements as "controls," and compare our parallax-derived distances to them at the end of the experiment.

Second, place your thin object above or just next to the coin at Position 1. You measured the distance from the observation spot to the center, or one edge, of the coin, so be sure to align the object at precisely the same location.

Now measure the apparent shift of the object against the wall-mounted ruler as you observe it through first one eye and then the other. Lie down behind the observation spot again, so that your eyes are directly above the coin which marks your position. As you did previously (when observing your thumb), you will look at the thin object with first your right eye

Table 4.2: Direct Measurements of Distance

Make all measurements in fractions of inches.			
	Trial 1	Trial 2	Trial 3
Distances from observer to object, in inches.			
Position 1			
Position 2			
Position 3			

(3 points)

and then your left eye (you may want to close or cover the other eye as you make the measurements). Your eyes, the thin object, and the parallax ruler should all lie in a line roughly 6 inches above the ground.

You assistant will help you to measure the number of inches on the parallax ruler by which the object appears to move as you switch eyes. Begin by looking through one eye, and have your assistant hold up a pencil vertically in front of the ruler. Direct him or her to shift it to the left or right along the ruler, until it appears to you to lie directly behind the thin object. (You may want to call out "shift four inches to the left," or "shift just a quarter-inch to the right," to help your assistant to move quickly to the correct position. However, if you find that speaking while measuring causes your head to move too much, then use a set of hand signals to indicate "shift left," "shift right," "make smaller shifts (getting close)," and "stop.") When you are happy with the position of the pencil, have your assistant write a small "1" at exactly this position along the ruler. Now switch to your other eye, and have your assistant write a second "1" at the matching position along the ruler.

Your assistant can now measure the distance between the two "1"s on the ruler with the tape measure, and read it off to you in inches. (If your assistant is too young to do this with confidence, you can come up to the ruler to help him or her with this task.) Estimate the shift in position to within a fraction of an inch – your measurements might be 2.3 inches or 2 3/4 inches, rather than just 2 or 3 inches. Write down this number in the first column of the first line of Table 4.3.

Now repeat this measurement (the apparent shift against the wall ruler) with the thin object placed at Positions 2 and 3, marking the ruler with small "2" and "3" symbols. Remember to leave the coins undisturbed at their positions, so that you can go back to them later. Record these data in the second and third lines of the first column (labeled Trial 1) of Table 4.3. Then have your assistant carefully cross out the two "1"s, the two "2"s, and the two "3"s on the ruler, so that they will not be confused with later marks.

Once you have made your measurements once for all three objects, return the thin object to the first position and repeat your measurements two more times. Be sure that you go through one full set of measurements (closest, middle, and farthest distances) before you repeat the process. You will perform each measurement three times, recording the data in the columns for Trial 1 through Trial 3, to estimate how repeatable your measurements are.

Table 4.3: Parallax Measurements

Record all measurements in fractions of inches, or in fractions of degrees.			
	Trial 1	Trial 2	Trial 3
Eye-to-eye shifts (apparent linear shifts) along parallax ruler for objects, in inches.			
Position 1			
Position 2			
Position 3			
Measuring the entire parallax ruler. *Record the number of degrees per inch to three decimal places (n.nnn).*			
Angular width (degrees) Avg:			
Linear width (inches)		Degrees per inch	
Eye-to-eye shifts (apparent angular shifts) along parallax ruler for objects, in degrees.			
Position 1			
Position 2			
Position 3			
Baseline (inches) Avg:			

(9 points)

DO NOT perform all three measurements of any object at the same time – that would defeat the purpose of taking three independent measurements.

If your assistant is curious, you may switch places and let him/her take a set observations! However, do not enter them into Table 4.3 unless your assistant has exactly the same size head (the distance between the eyes, or "baseline," as labeled in Figure 4.2) as you do – can you see why?

Estimate the *uncertainty* in your measurement of the object's apparent shift. For example, do you think your recorded measurements could be off by a foot? An inch? A quarter of an inch? Compare the measurements made at each position from trial to trial, to help you estimate the reliability of your measurements. (2 points)

4.2.3 Dependence of parallax on vantage point separation

Now that we understand how the apparent shift of an object changes as its distance from the observer changes, let's explore what happens when the distance between the vantage points changes.

What would happen if the vantage points were farther apart? For example, imagine that you had a huge head and your eyes were two feet rather than several inches apart. How would you expect the apparent shift of the object relative to the parallax ruler to change? (Note that there is no wrong answer to this question. The point is to take a guess, and then to verify or to disprove it.) (1 point)

Repeat the experiment with the object at its farthest distance, but this time measure the apparent shift by using just one eye, and moving your whole head roughly one foot to each side to get more widely separated vantage points.

By how many inches did the object move using the more widely separated vantage points? ______________. (1 point)

For an object at a fixed distance, how does the apparent shift change as you observe from more widely separated vantage points? (1 point)

4.2.4 Measuring distances using parallax

We have seen that an object's apparent shift relative to background objects (such as the parallax ruler) depends both on the distance between the object and the observer and on the separation between the observer's two vantage points. We can now turn this around: if we can measure the apparent shift and the separation of the two vantage points, we should be able to calculate the distance to an object. This is very handy, as it provides a way of measuring distance without actually having to go all the way to an object. Since we cannot travel to the stars, this is an excellent way to measure their distances from us.

We will now see how parallax can be used to determine the distances to the objects in your experiment based only on your measurements of their apparent changes in position ("apparent shifts") and the measurement of the separation of your two vantage points (your two eyes), your "baseline."

Angular shift

As we now know, the apparent shift of the object across the parallax ruler is caused by looking at the object from two different vantage points, in this case, from the different positions of your two eyes. Qualitatively, what do you see changing from eye to eye? As an object gets farther away from you, is its apparent shift smaller or larger? (1 point)

Up until this point, we have measured the apparent shift in inches along our parallax ruler.

Since this is truly a length measurement, we can refer to this as an apparent "linear" shift. Are inches, however, really the most appropriate units? The linear shifts that we have measured actually depend on the distance of the parallax ruler from us. If we had placed our parallax ruler farther away, the actual number of inches that the object would have appeared to move along the ruler would have been larger. Consider what would happen to the measured "eye-to-eye shift" in Figure 4.2 if the ruler were moved farther to the right (farther away from the observer).

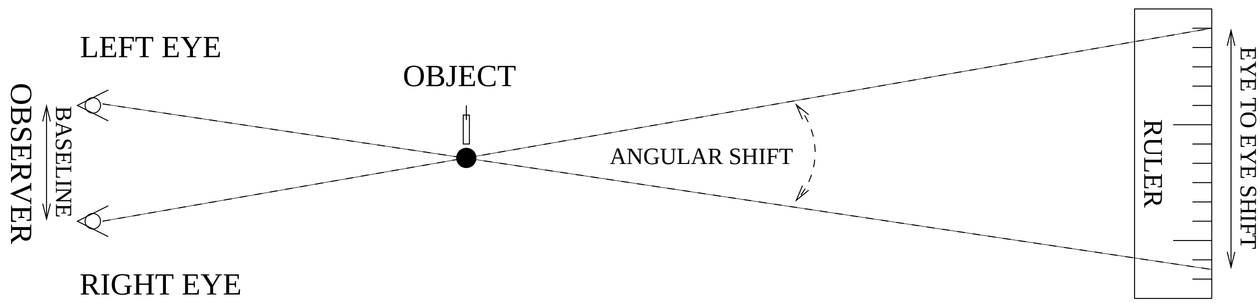


Figure 4.2: Layout of the parallax experiment, defining the baseline, angular shift, and eye-to-eye shift.

A much more useful measurement is the apparent "angular" shift of the object, the *angle* by which the object shifts relative to the background. As can be seen in Figure 4.2, the angular shift does not depend on the location of the background.

The apparent angular shift is measured in *degrees*, where 360 degrees make up a full circle. How do we measure the apparent angular shift for our object at its three positions?

First, let's get a qualitative feel for what to expect. Let's revisit the question posed at the beginning of this section: As an object moves farther away from the observer, does its apparent shift increase or decrease?

To check your answer, consider the apparent angular shift for the two objects shown in Figure 4.3. Use the protractor provided at the end of this lab chapter to measure the apparent shift for each object. If you are not sure how to use your protractor, see the section below. (You should measure angles of roughly 12 and 19 degrees.) Is the apparent shift larger for Object 1 or for Object 2? Check your answer to the previous question. (1 point)

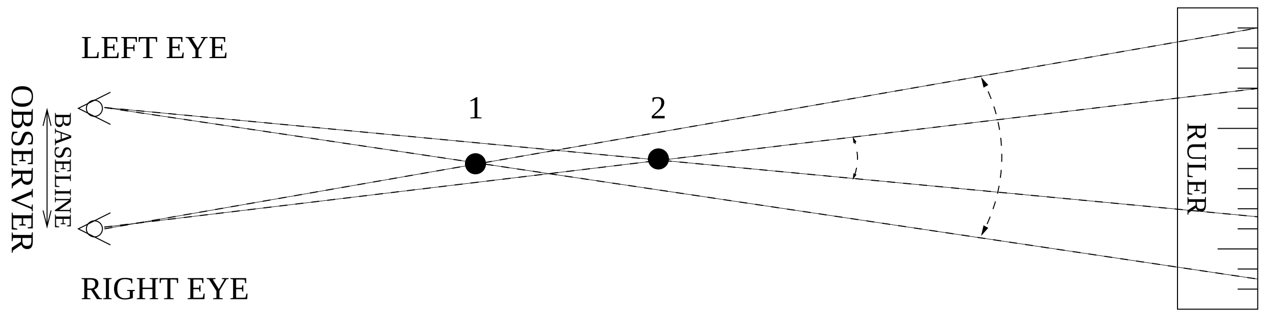


Figure 4.3: Layout of the parallax experiment, comparing the angular shift for objects at two positions.

Using your protractor

The "origin" of your protractor is the center of the circle with the dot in the middle. To measure the apparent angular shifts for the objects in Figure 4.3, center the origin of the protractor on the dot marking the object. Line up the bottom edge of the protractor (the 0–180 degree line) along the downward-sloping dotted line for the object. Read off the angle of the upward-sloping dotted line for the same object. Be careful that you are reading off the angle for the correct line. For Object 1, you should be measuring the angle between the two outer lines; for Object 2, you will be measuring the angle between the two inner lines.

You may find that it will be easier to read off the angles by holding the figure and protractor up to a bright light or against a window.

Determining the apparent angular shift

In order to determine the apparent angular shifts of our object at the three different positions, let us first figure out the angular separation of the inch marks on the parallax ruler as seen from your observation spot. To do this, you will need your protractor and the long piece of string. Hold the end of the string at your coin on the "observation spot." Ask your assistant to take the end of the string that was under the center of the parallax ruler and hold it on the ground directly below one end of your parallax ruler. Pull the string taut. Now carefully place the protractor on the ground underneath the string, and align it so that 0 degrees lies exactly along the string. The origin of the protractor should be centered on your observation spot.

Now, holding the string and protractor stationary at the observation spot, ask your assistant to move his/her end of the string to the other end of the parallax ruler. Carefully read off the angle of the string at its new position, and record it the next line of Table 4.3. Record each value to the nearest thousandth of a degree, writing 0.213 or 0.245 rather than just 0.2, for example.) Repeat this procedure three times, then average your measurements to obtain an estimate of the uncertainty of your measurement technique. Record the average value in the box under "measuring the entire parallax ruler" marked "angular width (degrees)" in Table 4.3.

To determine the angle equivalent to a one-inch length along the ruler, divide (the total angle covered by the entire ruler) by (the total number of inches along the ruler). Record these numbers in Table 4.3.

You can now convert your measurements of apparent *linear* shift at each position (in inches) to apparent *angular* shift (in degrees) by multiplying the number of inches the object shifted at each position by the number of degrees per inch that you just calculated. Do so, and enter the numbers into the next three lines of Table 4.3.

Based on your estimate of the uncertainty in the number of inches each object moved, what is your estimate of the uncertainty in the number of degrees that each object moved? (1 point)

Distance between the vantage points (the "baseline")

You now need to measure the distance between your two different vantage points, the distance between your two eyes. Have your assistant measure this (carefully!) with a ruler. Since you observe the world out of the central pupil of your eyes (the dark region in the center of the eyeball), you want to measure the distance between the centers of your two pupils. As the measurement is made, try to hold your eyes still and focus on a point in the distance. Enter this value into Table 4.3 as the "baseline." Measure it three times, recording each value, and then also record the average value in the box under "eye-to-eye shifts" marked "baseline (inches)" in Table 4.3.

Tables 4.2 and 4.3 should now be complete (all empty spaces should contain numbers). Your assistant has done his/her part, and is free to go!

Using parallax measurements to determine the distance to an object

To determine the distance to an object for which you have a parallax measurement, you can construct an imaginary triangle between the two different vantage points and the object, as shown in Figure 4.4.

The apparent angular shifts that you have just calculated correspond to the angle α on the diagram, and the distance between your pupils (your "baseline") corresponds to the distance b. The distance to the object, which you wish to determine, is d.

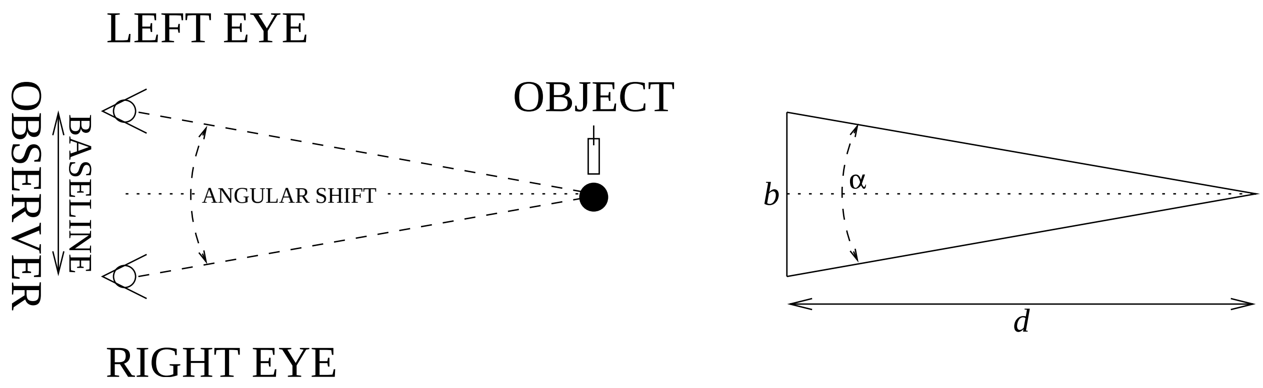


Figure 4.4: The parallax experiment, with real objects (left) and expressed mathematically (right). Can you find the same triangle on both sides?

There are two methods by which you can now calculate d. Method 1 involves a little bit of trigonometry, while Method 2 does not. You will use both methods to calculate the distance of your object in Trial 1 of Position 1. You will compare your results, and then proceed with one method for the rest of your distance calculations.

Method 1: The "Tangent" Way

The three quantities b, d, and α are related by a trigonometric function called the *tangent*. If you can't quite remember what a tangent is, don't worry – we will help you through this step-by-step and then show you how to do this using an easier (but slightly less accurate) way (Method 2).

To find the distance to an object using parallax the "tangent" way, first divide your triangle in half (along the dotted line in Figure 4.4b). This splits both the apparent angular shift, α, and your baseline, b, in half. The tangent of an angle described by a particular triangle is the ratio of the height to the length of the triangle. The tangent of $(\alpha/2)$ is thus equal to the quantity $\frac{(b/2)}{d}$:

$$\tan\left(\frac{\alpha}{2}\right) = \frac{(b/2)}{d}.$$

Since you want to find the distance d, the equation can be rearranged to give:

$$d = \frac{(b/2)}{\tan(\alpha/2)}.$$

To determine the tangent of an angle, use the *tan* button on your calculator. There are several other units for measuring angles besides degrees (such as radians), so **make sure that your calculator is set up to use degrees** for angles before you use the *tangent* function.

Just as a quick check, calculate $tan(10)$. You should get a value of 0.1763.

You are now ready to calculate d via the tangent method, from the angular shift and baseline measurements for Trial 1 of Position 1. Plug the numbers into the equation above, and record your answer here: _______________. (1 point)

Method 2: The Small Angle Approximation Way

Because the angles in astronomical parallax measurements are very small, astronomers do not have to use the tangent method to determine distances from angles – they can use the "small angle approximation" formula. If an angle θ is a small angle (less than 10 degrees), then

$$\tan(\theta) \approx \theta$$

where θ is measured in units of radians. Because there are 57.3 degrees in a radian, we can rewrite this relation as

$$\tan(\theta) \approx \frac{\theta}{57.3}$$

if we wish to express θ in degrees rather than radians. (There are 2π radians, or 360 degrees, in a full circle. The ratio of 360 to 2π is 57.3, so there are 57.3 degrees in a radian.) Replacing $\tan(\alpha/2)$ with $(\alpha/2)/57.3$ in our expression for $\tan(\alpha/2)$, we see that it becomes

$$\frac{(\alpha/2)}{57.3} \approx \frac{(b/2)}{d}.$$

In this equation, α, b, and d are the same angle and distances, respectively, as in the earlier equations (and in Figure 4.4). Rearranging the equation to find d gives:

$$d \approx \frac{57.3 \times (b/2)}{(\alpha/2)} \approx 57.3 \times \frac{b}{\alpha}.$$

Remember that to use this equation your angular shift "α" must be measured in degrees.

Calculate d using the Small Angle Approximation method for Trial 1 of Position 1 by plugging in the appropriate numbers in the equation above. Record your answer here: _______________. (1 point)

To compare the two methods, let's calculate the "percent error" between the values that you got from the two different methods: (2 points)

$$\% \text{ error} = \frac{|\text{Value 1} - \text{Value 2}|}{\text{Value 1}} \times 100.$$

Where Value 1 is measured with the more accurate Method 1, and Value 2 is derived using the Small Angle Approximation. How close were your two values? (1 point)

Now that you have shown that the two methods yield similar answers, choose your favorite method to calculate the rest of your parallax-derived distances.

Use the measurements that you recorded in Table 4.3 and the equation for your method of choice to calculate the distance of the object for all three Trials of each Position of the object. The units of the distances which you determine will be the same as the units you used to measure the distance between your eyes; in this case, inches. Record these values in Table 4.4.

Table 4.4: Parallax-Derived Distances

	Trial 1	Trial 2	Trial 3
Parallax-derived distances from observer to object, in inches (nn.nn).			
Position 1			
Position 2			
Position 3			

(3 points)

Based on your estimate of the uncertainty in the angular measurements and the uncertainty of your measurement of the separation of your eyes, estimate the uncertainty in your measurements of the object distances. (Note that there is no wrong answer to this question. The point is to take a guess, and then to verify or to disprove it.) (1 point)

Now look at the spread in the three values for each position in Table 4.4. Is this spread consistent with your estimated uncertainty? (1 point)

How good are your parallax-derived distances?

At this point, you are ready to average your distance measurements together, and compute an error. Access the plotting tool listed for this lab exercise from the GEAS project lab exercise web page (see the URL on page 10 in §4.1.2). Use the plotting tool to create histograms of your distance measurements, entering the three values measured in Trials 1 through 3 for each quantity in turn. You will not need to save the histogram plots. Instead, simply record the averaged values shown for each plot (labeled "mean value"), and the associated errors. In order to see how good your distance measurements are, you will calculate these averages and errors for each of the quantities recorded three times in Table 4.2 and Table 4.4. Copy these values into Table 4.5.

Just for fun, let's consider how these errors are calculated. Though you will not need to reproduce this calculation yourself, it is helpful to understand how it works.

We can calculate the average value of a set of measurements by adding the numbers together and then dividing by the total number of measurements. Mathematically, we say: *for n data points, we sum the n values and divide by n.* Here is how we write that as an equation:

$$\bar{v} \equiv \frac{1}{n} \sum_{i=1}^{n} v_i .$$

Having found the average value, $\bar{v}$, we would now like to know how accurate it is. We will assume that the scatter, or spread, in the measured values reflects the dominant source of error. To calculate this, we will compare each measurement in turn to the average value, and see how far off each one lies.

Mathematically, we say: *for n data points, we calculate the offset of each point from the average value, we square the offsets, we divide their total by $n-1$, and then for the grand finale we take the square root of the result.* Here is how we write that as an equation:

$$s \equiv \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (v_i - \bar{v})^2} .$$

Let's work through a quick sample case with real numbers. Say that you measured the distance to an object, and recorded values of 11.6, 11.9, and 12.1 inches. We can now calculate the average value, $\bar{v}$:

$$\bar{v} = \frac{1}{3}\,(11.6 + 12.1 + 11.9) = 11.87 \text{ inches},$$

and the standard deviation (the error), s:

$$s = \sqrt{\frac{1}{2}\left[(11.6 - 11.87)^2 + (12.1 - 11.87)^2 + (11.9 - 11.87)^2\right]} = 0.25 \text{ inches}.$$

Your value for the distance is 11.87 ± 0.25 inches! Go ahead and enter "11.6, 12.1, and 11.9" into the plotting tool and plot a histogram of their distribution, to verify these values.

Now compare the distances that you calculated for each position using the parallax method to the distances that you measured directly at the beginning of the experiment (see Table 4.2). How well did the parallax technique work? Are the differences between the direct measurements and your parallax-derived measurements within your errors? (1 point)

If the differences are larger than your errors, can you think of a reason why your measurements might have some additional error in them? We might call this a "systematic" error, if it is connected to a big approximation in our observational setup. *Hint: astronomers can measure parallax angles to stars which are 100 parsecs away, using background stars which are 10,000 parsecs away. Was your background ruler 100 times farther away from you than Position 3 was? If you focused very carefully on a single thin black line on the parallax ruler, would it shift position slightly when viewed through one eye or the other against the distant horizon?* (3 points)

Table 4.5: Comparison of Average Distances

	Direct Distance (from Table 4.2)	Parallax Distance (from Table 4.4)	% error
Measured and parallax-derived distances from observer to object, in inches. Record the average values of the three trials, and the errors calculated with the plotting tool in this form: nn.nn ± n.nn.			
Position 1			
Position 2			
Position 3			

(3 points)

Calculate the percent error between the parallax-derived and directly measured distances using the equation provided earlier, and enter it into Table 4.5. Does this percent error seem reasonable, given your measurement errors and your systematic errors? (1 point)

4.3 Calculating Astronomical Distances With Parallax

Complete the following section, answering the five questions in turn. (Each question is worth either 1 or 2 points.)

4.3.1 Distances on Earth and within the Solar System

We have just demonstrated how parallax works in the classroom, so now let us move to a larger playing field. Use the small angle approximation to determine the angular shift (in degrees) for Organ Summit, the highest peak in the Organ Mountains, if you observed it first with one eye and then the other from NMSU. Organ Summit is located 12 miles (20 kilometers) from Las Cruces. (*If you are working from another location, select a mountain, sky scraper, or other landmark at a similar distance to use in place of the Organ Summit.*) There are 5,280 feet in a mile, and 12 inches in a foot. There are 1,000 meters in a kilometer. (2 points)

You should have gotten a tiny angle!

The smallest angle that the best human eyes can resolve is about 0.02 degrees. Obviously, our eyes provide an inadequate baseline for measuring this large of a distance. How could we create a bigger baseline? Surveyors use a "transit," a small telescope mounted on a (fancy!) protractor, to carefully measure angles to distant objects. By positioning the transit at two different spots separated by exactly 300 feet (and carefully measuring this baseline), they will observe a much larger angular shift. Recall that when you increased the distance between your two vantage points by moving your head from side to side to view your object, the angular shift increased. What this means, then, is that, if an observer has a larger baseline, an object can be farther away from the observer yet still have a measurable shift. With a surveyor's transit's 300-foot baseline, it is thus fairly easy to measure the distances to faraway trees, mountains, buildings or other large objects here on Earth.

What about an object farther out in the solar system? Consider Mars, the planet that comes closest to the Earth. At its closest approach, Mars comes to within 0.4 A.U. of the Earth. (Remember that an A.U. is the average distance between the Earth and the Sun, or

149,600,000 kilometers.) At such a large distance we will need an even larger baseline than a transit could provide, so let us assume we have two telescopes in neighboring states, and calculate the parallax angle for Mars (using the small angle approximation) for a baseline of 1000 kilometers. (2 points)

Wow! Again, a very small angle.

4.3.2 Distances to stars, and the "parsec"

The angular shifts for even our closest neighboring planet are clearly quite small, even with a fairly large baseline. Stars, of course, are *much* farther away. The nearest star is 1.9×10^{13} miles, or 1.2×10^{18} inches, away! At such a tremendous distance, the apparent angular shift is extremely small. When observed through the two vantage points of your two eyes, the angular shift of the nearest star corresponds to the apparent diameter of a human hair seen at the distance of the Sun! This is a truly tiny angle and totally unmeasurable by eye.

Like geological surveyors, we can improve our situation by using two more widely separated vantage points. In order to separate our two observations as far as possible from each other, we will take advantage of the Earth's motion around the Sun. The Earth's orbit forms a large circle around the Sun, and so by observing a star from first one position and then waiting six months for the Earth to revolve around to the other side of the Sun, we will achieve a separation of two A.U. (twice the distance between the Earth and the Sun). This is the distance between our two vantage points, labeled b in Figure 4.5.

An A.U. (astronomical unit) is equal to 1.496×10^8 kilometers, so b is equal to twice that, or 299.2 million kilometers. Even though this sounds like a large distance, we find that the apparent angular shift (α, in Figure 4.5) of even the nearest star is only about 0.00043 degrees. This is unobservable by eye, which is why we cannot directly observe parallax by looking at stars with the naked eye. However, such angles are relatively easy to measure using modern telescopes and instruments.

Let us now introduce the idea of angles that are smaller than a degree. Just as a clock ticks out hours, minutes, and seconds, angles on the sky are measured in degrees, arcminutes, and arcseconds. A single degree can be broken into 60 arcminutes, and each arcminute contains 60 arcseconds. An angular shift of 0.02 degrees is thus equal to 1.2 arcminutes, or to 72 arcseconds. Since the angular shift of even the nearest star (Alpha Centauri) is only 0.00043 degrees (1.56 arcseconds), we can see that arcseconds will be a most convenient unit to use

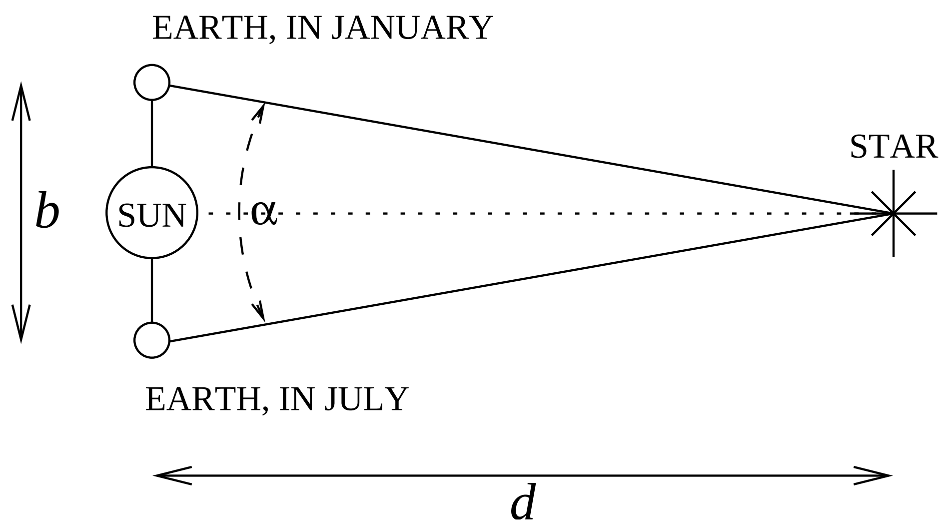


Figure 4.5: The parallax experiment, as done from Earth over a period of six months.

when describing them. Astronomers append a double quotation mark ($''$) at the end of the angle to denote arcseconds, writing $\alpha = 1.56''$ for the nearest star.

Remember that when converting an angle into a distance (using either the tangent or the small angle approximation), we used the angle $\alpha/2$ rather than just α. When astronomers talk about the "parallax" or "parallax angle" of a star, they use this angle: $\alpha/2$. For easier reference, we will give this angle its own symbol, "θ," so the small angle approximation equation now becomes:

$$d = \frac{57.3 \times (b/2)}{\theta}.$$

It is now time to introduce a new distance unit, the "parsec." How far away is a star with parallax angle of $\theta = 1''$? The answer is 3.26 light-years, and this distance is defined to be one parsec. The word parsec comes from the phrase "**par**allax **sec**ond." By definition, an object at 1 parsec has a parallax of $1''$.

An object at 10 parsecs has a parallax angle of $0.1''$. Remember that the farther away an object is from us, the smaller its parallax angle will be. The nearest star has a parallax of $\theta = 0.78''$, and is thus at a distance of $1/\theta = 1/0.78 = 1.3$ parsecs. To convert parsecs into light years, you simply multiply by 3.26 light-years/parsec.

You might read the words parsec, kiloparsec, megaparsec and even gigaparsec in this class. These names are just shorthand methods of talking about distances in astronomy. A kiloparsec is 1,000 parsecs, or 3,260 light-years. A megaparsec is one million parsecs, and a gigaparsec is a whopping one billion parsecs! The parsec may seem like a strange unit, but you have probably already encountered a few other strange units this semester!

Let's go through a couple of examples:

- If a star has a parallax angle of $\theta = 0.25''$, what is its distance in parsecs? (1 point)

- If a star is 5 parsecs away from Earth, what is its parallax angle? (1 point)

- If a star lies 5 parsecs from Earth, how many light-years away is it? (1 point)

4.4 Final (post-lab) Questions

1. How does the parallax angle of an object change as it moves away from us? As we can only measure angles to a certain accuracy, is it easier to measure the distance to a nearby star or to a more distant star? Why? (3 points)

2. Relate the experiment you did in the first part of this lab to the way that parallax is used to measure the distances to nearby stars. Describe the process an astronomer goes through to determine the distance to a star using the parallax method. What did

your two eyes represent in the experiment? (5 points)

3. Imagine that you observe a star field twice, with a six month gap between your observations, and that you see the two sets of stars shown in Figure 4.6:

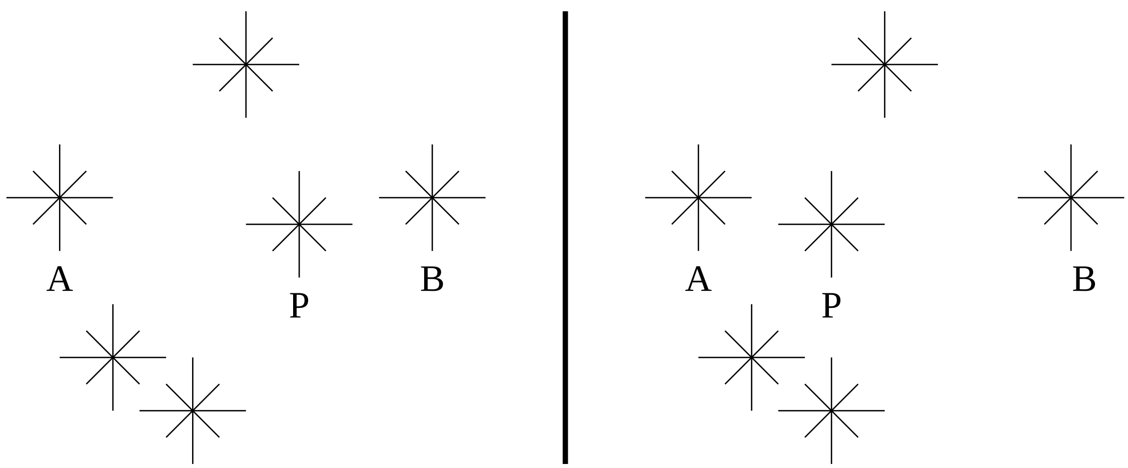


Figure 4.6: A star field, viewed from Earth in January (left) and again in July (right). Which star do you think lies closest to Earth?

The nearby star marked P appears to move between the two images, because of parallax. Consider the two images to be equivalent to the measurements that we made in our experiment, where each image represents the view of an object against the parallax ruler as seen through one of your eyes. All the stars except P do not appear to change position; they correspond to the background ruler in our experiment.

If the angular distance between Stars A and B is 0.5 arcminutes (recall that 60 arcminutes = 1 degree), then how far away would you estimate that Star P lies from Earth?

First, estimate how far Star P has moved between the two images relative to the constant distance between Stars A and B. This tells you the apparent angular shift of P. You also know the distance between the two vantage points (the Earth at two opposite points along its orbit) from the number given in Section 4.3.2. You can then use either the tangent or the small angle approximation parallax equation to estimate the distance to Star P. Remember that you need to convert the angle from arcminutes to degrees (by dividing by 60 arcminutes/degree) before you plug your angle into the equation. (7 points)

4. Imagine that when you performed your experiment, your assistant held an object all the way to the far wall, up against the parallax ruler. How big would the apparent shift of the object be relative to the marks on the ruler? What would you infer about the distance to the object? Why do you think this estimate would be incorrect? Where should the background objects in a parallax experiment be located? (4 points)

4.5 Summary

Summarize the important concepts discussed in this lab. Include a brief description of the basic principles of parallax and how astronomers use parallax to determine the distances to nearby stars. (35 points)

Be sure to think about and answer the following questions:

- Does the parallax method work for all of the stars we can see in our Galaxy? Why, or why not?
- Why is it so important for astronomers to determine the distances to the stars which they study?

Use complete sentences, and be sure to proofread your summary. It should be 300–500 words long.

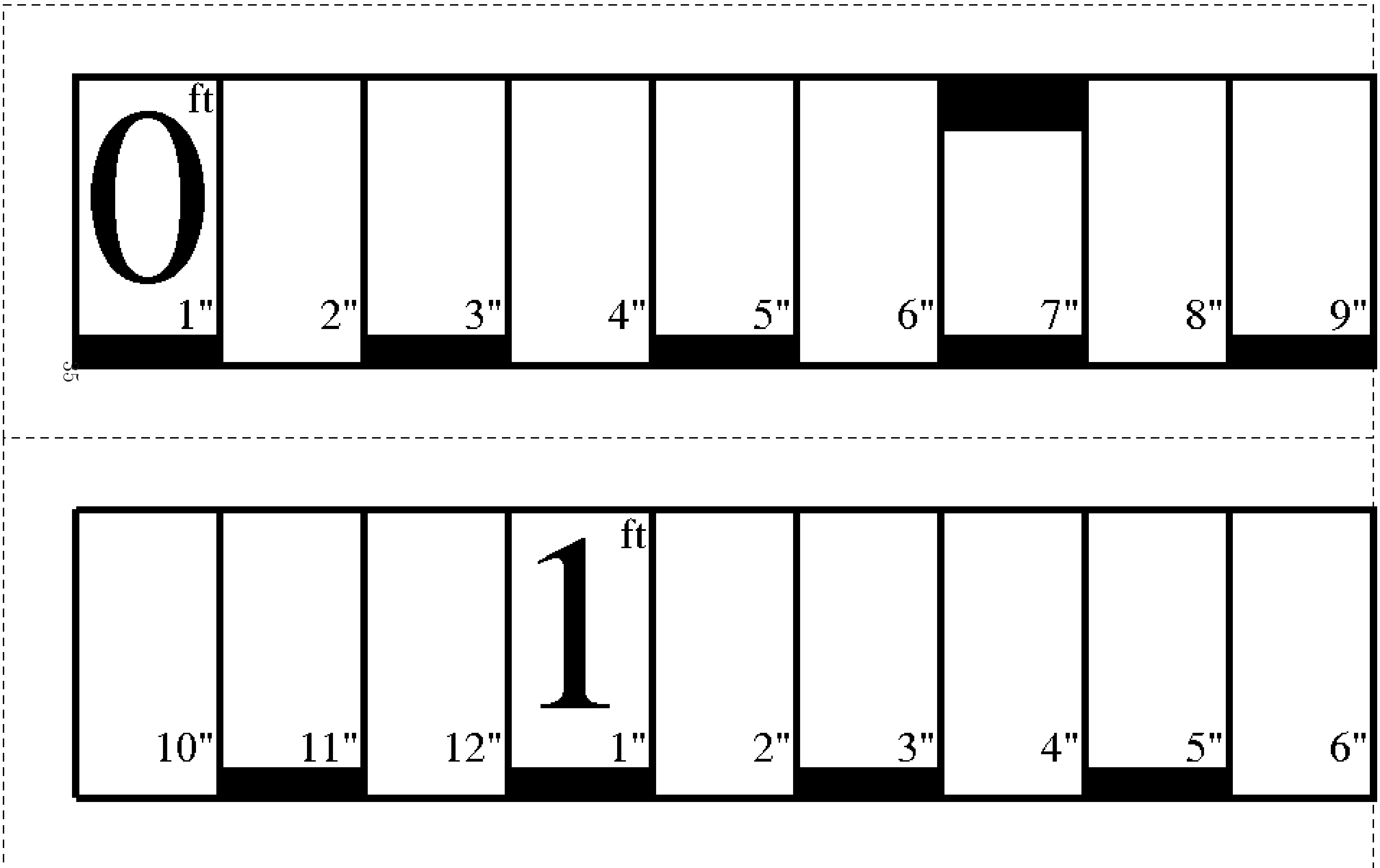
4.6 Extra Credit

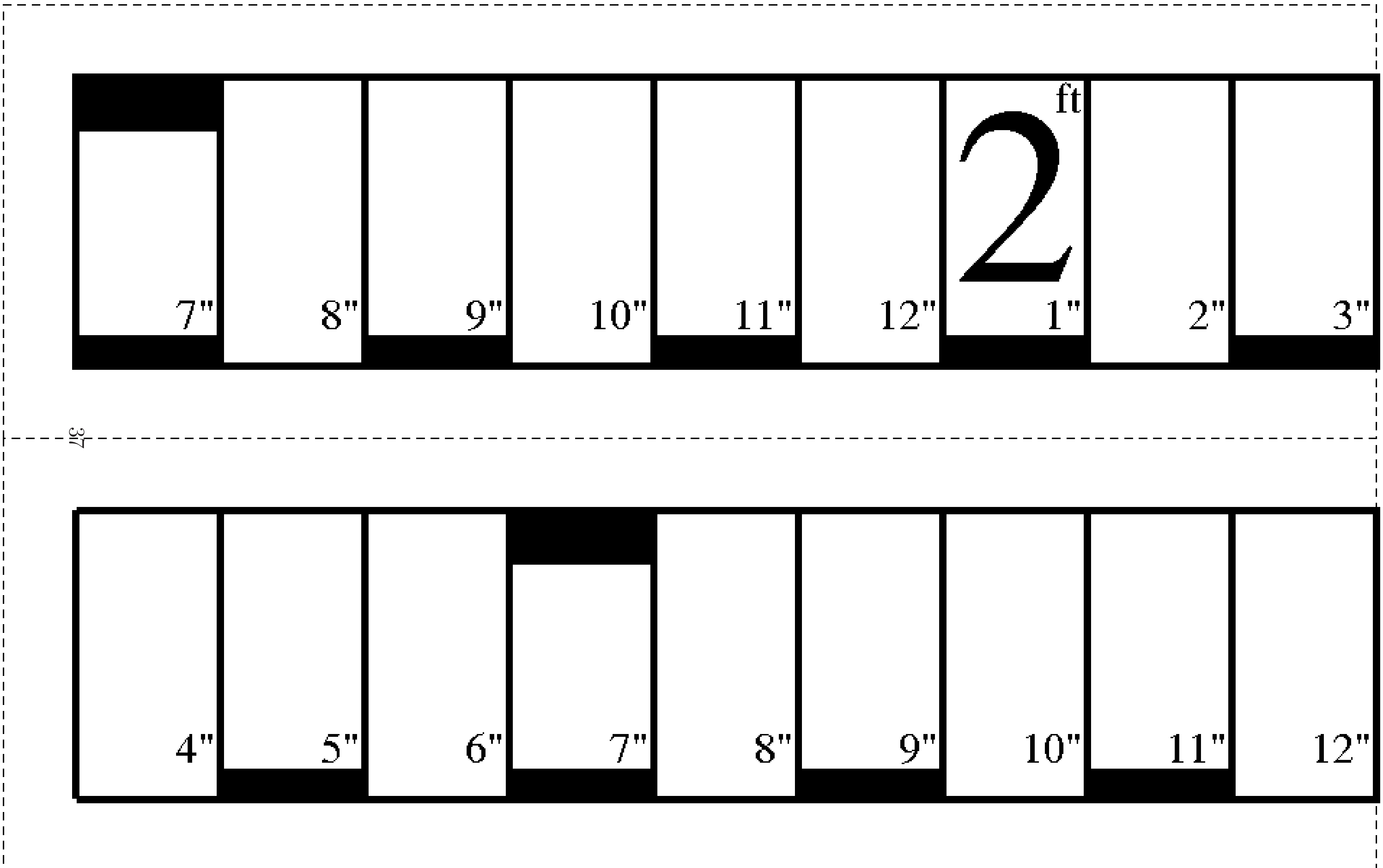
Use the web to learn about the planned Space Interferometry Mission (SIM). What are its goals, and how will it work? How accurately will it be able to measure parallax angles? How much better will SIM be than the best ground-based parallax measurement programs? Be sure that you understand the units of milliarcseconds ("mas") and microarcseconds, and can use them in your discussion. (4 points)

Be sure to cite your references, whether they are texts or URLs.

Parallax Ruler

Cut along dotted lines, and connect pieces with tape to form a six–foot ruler.

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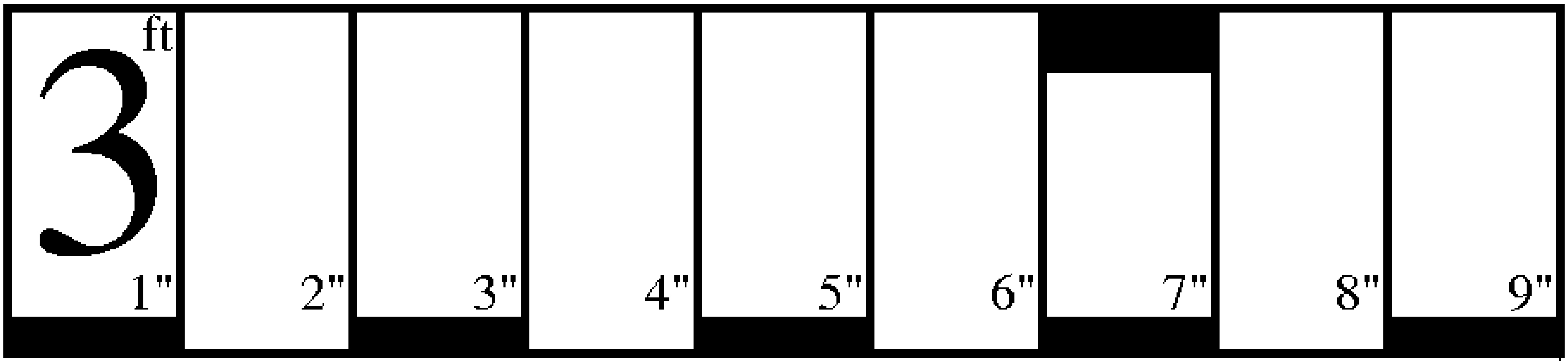
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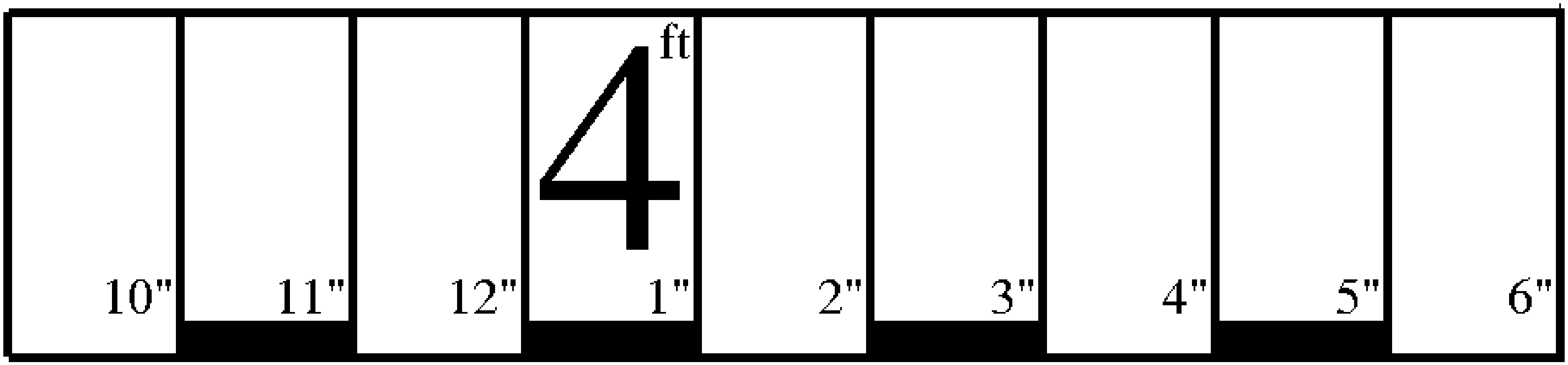
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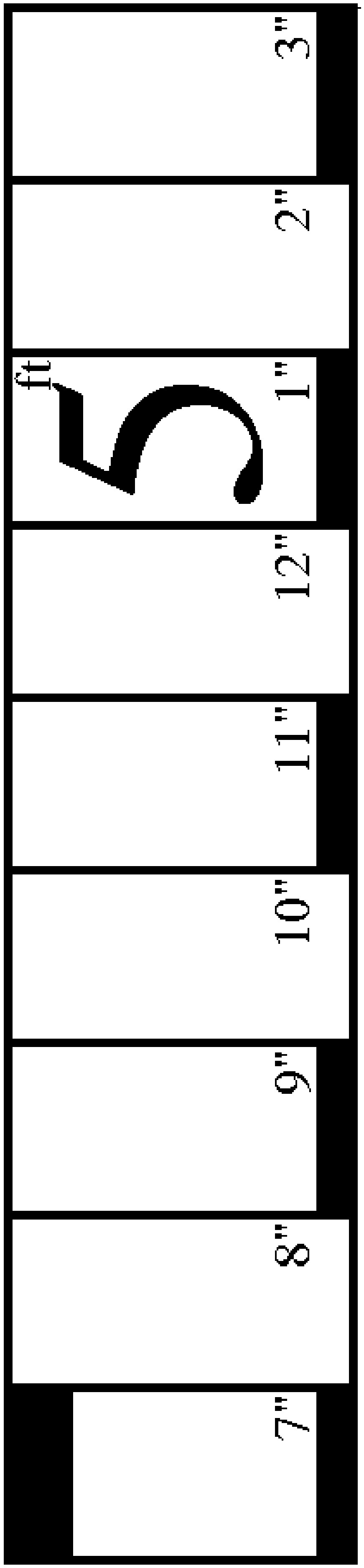
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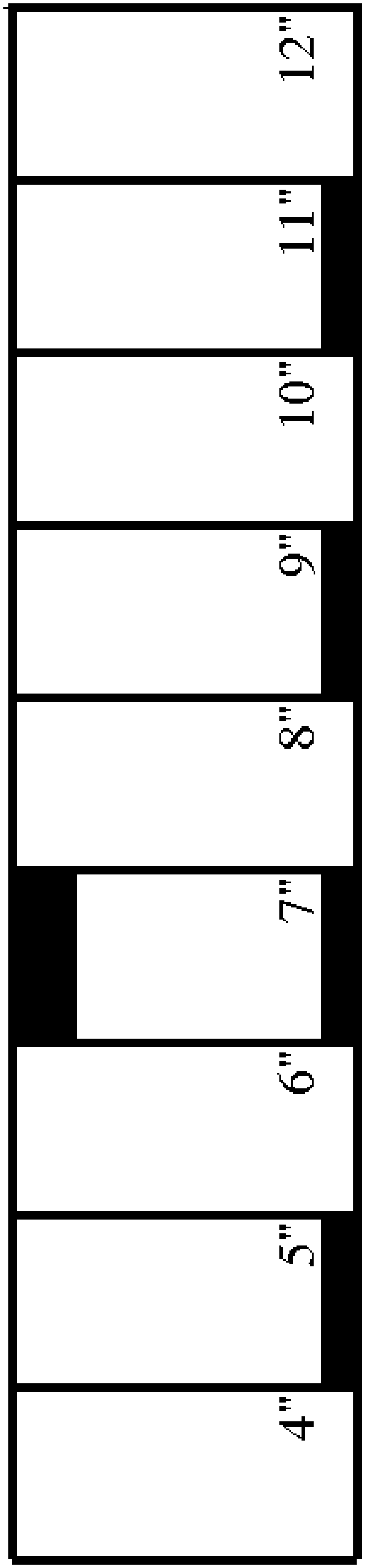
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Protractor

Cut along dotted lines.