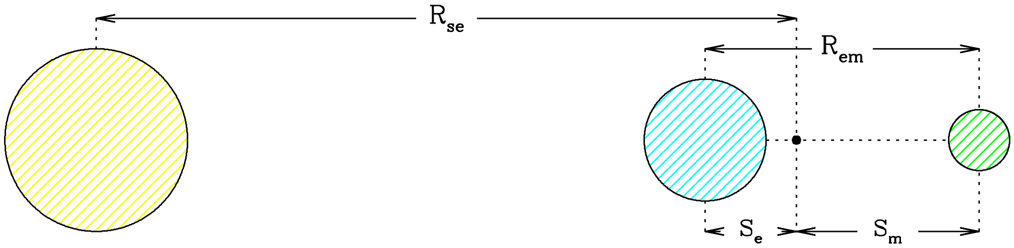
Astronomy 505: Homework #6 (Planets)

4. **Calculate the expected increase in the global average temperature of the Earth at a full Moon compared to a new Moon (neglecting eclipses). Which effect is larger, the change in position of the Earth or the radiation reflected and emitted from the Moon? (10 pts)**

Consider the illumination of the Earth. We will be comparing the effect of two factors: (1) the amount of sunlight reflected off of the Moon onto the Earth, and (2) the change in the direct amount of sunlight due to the Earth's rotation about the center of mass of the Earth-Moon system. During the *new* Moon, the Moon reflects no light onto the Earth, and the Earth lies beyond the center of mass of the Earth-Moon system. During the *full* Moon, the Moon reflects a maximum amount of light onto the Earth, and the Earth lies between the center of mass of the Earth-Moon system and the Sun. The figure below illustrates the full Moon phase.



The first-order solution to the illumination of the Earth is simply the product of the amount of sunlight per unit area on the sky at a distance of one A.U., and the cross-sectional area presented by the Earth. *The Earth presents a disk of area* πr_E^2, **not** *a sphere of area* $4\pi r_E^2$*!*

$$F(Earth) = \frac{L_\odot}{4\pi R_{se}^2}\,\pi r_E^2$$

At the new Moon, the Earth lies beyond the center of mass of the Earth-Moon system and there is no light reflected from the Moon. The flux is thus

$$F_{NM}(Earth) = \frac{L_\odot}{4\pi(R_{se}+S_e)^2}\,\pi r_e^2 = 1.76\times10^{24}\ \mathrm{erg\ sec}^{-1}$$

At the full Moon, the Earth lies between the center of mass of the Earth-Moon system and the Sun, and the Moon reflects light back onto the Earth. We calculate the flux accordingly.

$$F_{FM}(Earth) = \frac{L_\odot}{4\pi(R_{se}-S_e)^2}\,\pi r_e^2 \;+\; \frac{L_\odot}{4\pi(R_{se}+S_m)^2}\,\pi r_m^2 A_m \times \frac{2}{4\pi R_{em}^2}\,\pi r_e^2$$

where the factor of two in the final term is due to the fact that the Moon re-radiates the sunlight into a single hemisphere. (We make use of the fact that the 24 hour period of rotation of the Earth is much less than the 29 day orbital period of the Moon.) We can now calculate the change in the flux incident on the Earth between the new and the full Moon phases.

$$\begin{aligned}\Delta F(Earth) &\equiv F_{FM}(Earth) - F_{NM}(Earth)\\ &= \left(\tfrac{r_e}{2R_{se}}\right)^2 L_\odot\left[\left(1-\tfrac{S_e}{R_{se}}\right)^{-2} - \left(1+\tfrac{S_e}{R_{se}}\right)^{-2} + \left(1+\tfrac{S_m}{R_{se}}\right)^{-2}\times\left(\tfrac{r_m}{R_{em}}\right)^2\tfrac{A_m}{2}\right]\end{aligned}$$

We use a Taylor series expansion to re-express the first two terms, making use of the fact that

$$(1+\delta)^{-2} = 1 - 2\delta - \ldots,\quad \text{for } |\delta| << 1$$

As $S_e \ll R_{se}$,

$$\begin{aligned}\Delta F(Earth) &= \left(\tfrac{r_e}{2R_{se}}\right)^2 L_\odot\left[4\tfrac{S_e}{R_{se}} + \left(1+\tfrac{S_m}{R_{se}}\right)^{-2}\times\left(\tfrac{r_m}{R_{em}}\right)^2\tfrac{A_m}{2}\right]\\ &= \left(\tfrac{r_e}{2R_{se}}\right)^2 L_\odot\left[1.25\times10^{-4} + 7.16\times10^{-7}\right]\end{aligned}$$

where the first term in brackets is due to the change in position of the Earth, and the second term is due to the radiation reflected and emitted from the Moon. We take the ratio of these two terms and observe that

$$\frac{\Delta F_{pos}(Earth)}{\Delta F_{ref}(Earth)} = \frac{1.25\times10^{-4}}{7.16\times10^{-7}} = 175$$

so the effect of the movement of the Earth around the center of mass of the Earth-Moon system is **much** larger than the effect of the reflection of light from the full Moon.