Astronomy 505: Homework #9 (Planetary Atmospheres)

Consider a rapidly rotating planet with an atmosphere in monochromatic radiative equilibrium and local thermodynamic equilibrium (LTE). The planet is located at a distance $r_\odot$ of 2 AU from the Sun; its Bond albedo A_b is 0.3 and emissivity ϵ is 0.9 at all wavelengths.

1. Calculate the equilibrium temperature. Why is the effective temperature equivalent to this value? For which planets in the solar system is this not the case, and why? (5 pts)

2. **Infrared Regime**: Assume that the temperature profile of the planet is dominated by incident solar radiation which passed through the atmosphere and is now being re-radiated upward from the surface.

   (a) Use the two-stream approximation to divide the radiation I_ν into an upward component I_ν^+ and a downward component I_ν^-, and define the net flux density F_ν across an atmospheric layer in these terms. Point the flux vector upward, in the direction of increasing optical depth ($\mathrm{F}_\nu \propto \mathrm{I}_\nu^- - \mathrm{I}_\nu^+$). (3 pts)

   (b) Begin with the radiative transfer equation which defines the propagation of energy at an angle θ relative to the vertical through the atmosphere, in terms of the source function S_ν.

   $$\cos\theta \frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

   Use the fact that the flux F_ν does not vary with height to define the blackbody flux B_ν in terms of the mean intensity of the radiation field J_ν; then re-express B_ν in terms of I_ν^+ and I_ν^-. (5 pts)

   (c) Determine I_ν^+ and I_ν^- in terms of B_ν and F_ν, and evaluate them at the upper ($\tau = 0$) and lower ($\tau = \tau_g$) boundary layers of the atmosphere. Define appropriate boundary conditions (I = 0 in one case) to express I in terms of $\mathrm{B}_\nu(\tau_g)$ and $\mathrm{B}_\nu(0)$ in each case. (5 pts)

   (d) Use the radiative transfer equation again, and the Eddington approximation ($\mathrm{K}_\nu = \frac{1}{3}\mathrm{J}_\nu$), to express the net flux density F_ν as a function of the rate of change of B_ν through the atmosphere. Integrate your expression, and use appropriate boundary conditions to define an expression for B_ν as a function of optical depth τ_ν, and $\mathrm{B}_\nu(0)$. (10 pts)

   (e) Show that continuum radiation from the planet's atmosphere is received from a depth where $\tau = 2/3$. *Hint: Integrate B_ν over all frequencies, and show that the temperature $T(\tau)$ is equal to T_{eq} at $\tau = \frac{2}{3}$.* (8 pts)

   (f) Calculate the temperature at the upper boundary of the atmosphere (the skin temperature). (5 pts)

   (g) If the optical depth of the atmosphere $\int_0^\infty \tau(z)\,dz = 10$, estimate the temperature at the bottom layer of the atmosphere (the air temperature just above the ground). (5 pts)

   (h) If the optical depth of the atmosphere $\int_0^\infty \tau(z)\,dz = 10$, estimate the surface temperature of the planet. *Hint: I_ν^+ is equal to the flux from a surface temperature blackbody, at the ground level.* (5 pts)

   (i) How well does this model reproduce the temperature profile of a terrestrial atmosphere? Which of the four physical quantities calculated above – $\mathrm{T}(\tau = \frac{2}{3})$, $\mathrm{T}(\tau = 0)$, $\mathrm{T}(\tau = \tau_g)$, and T(surface) – would be a good match to the temperature profile of the actual planet? (8 pts)

3. **Optical Regime**: Assume that the temperature profile of the atmosphere is determined entirely by incident solar radiation, and that re-radiation from the surface of the planet is negligible.

   (a) Determine I_ν^+ and I_ν^- in terms of B_ν and F_ν, and evaluate them at the upper and lower boundary layers of the atmosphere. Define appropriate boundary conditions. Re-define the flux vector to point downward, in the current direction of increasing optical depth. (5 pts)

   (b) Use appropriate boundary conditions to define an expression for B_ν as a function of optical depth τ_ν, $\mathrm{B}_\nu(0)$, and $\mathrm{B}_\nu(\tau_g)$. (8 pts)

   (c) Determine the optical depth τ_ν at which continuum radiation from the planet's atmosphere is received. (8 pts)

   (d) Calculate the temperature at the upper boundary of the atmosphere, in terms of τ_g. Explain the physics behind the limiting cases of $\tau_g << 1$ and $\tau_g >> 1$. (10 pts)

   (e) If the optical depth of the atmosphere $\int_0^\infty \tau(z)\,dz = 10$, determine the temperature at the bottom layer of the atmosphere. (5 pts)

   (f) Determine the surface temperature of the planet. *Hint: Think about the assumptions of the model.* (3 pts)

   (g) How well does this model reproduce the temperature profile of a terrestrial atmosphere? Which of the the four physical quantities calculated above would be a good match to the temperature profile of the actual planet? (8 pts)

4. (a) Sketch the temperature profile (temperature versus height/optical depth) qualitatively for both of the above models, on a single graph. Add a third line showing the actual profile shape for a terrestrial atmosphere, and discuss its key features. Where does it follow the first model, and why? Where does it follow the second model, and why? Where does radiative transfer dominate, and why? Where is convection most significant, and why? (15 pts)

   (b) Compare the model lines to observed temperature profiles of the Terran and Martian atmospheres. What causes the observed substructure, not reproduced in the models? *Use the attached profiles as a rough guide, but consider the key molecular components and physical processes within each atmosphere.* (10 pts)

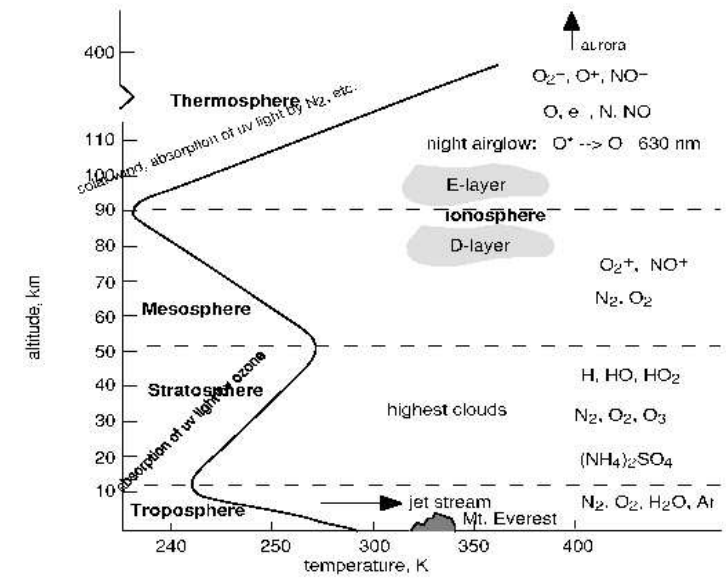

Figure 1: Terran atmospheric profile

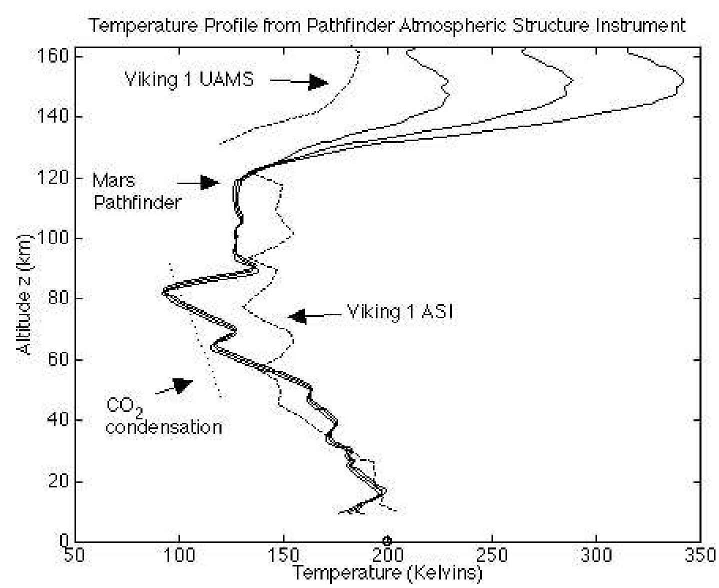

Figure 2: Martian atmospheric profile