Astronomy 505: Homework #1 (Optics)

1. Use the ideas of equal transit times between corresponding points and the orthogonality of rays and wavefronts to derive the following.

   (a) The law of reflection (the angle of incidence is equal to the angle of reflection). (3 pts)

   $$\theta_i = \theta_r$$

   (b) The law of refraction (the ratio of the *sine* of the angle of incidence to that of the angle of transmission is equal to the ratio of the absolute indices of refraction of two media). (3 pts)

   $$\frac{sin(\theta_i)}{sin(\theta_t)} = \frac{n_t}{n_i}$$

2. (a) Use the results of 1(a) and the small angle approximation to show that for a spherical mirror with radius of curvature R, the focal length is $f = \frac{R}{2}$. (5 pts)

   (b) Use the results of 1(a) to show that for a parabolic mirror defined by the equation $y = \left(\frac{1}{4f}\right) x^2$, the focal length is f. (5 pts) *Do not use the small angle approximation!*

3. (a) What is the magnification of a simple two-lens telescope, with an objective lens of focal length f_o and an eyepiece lens of focal length f_e? (2 pts)

   (b) Draw a ray diagram of this telescope, including the eyeball behind it. Include four rays: two from $\pm\theta$ striking the top of the objective, and two from $\pm\theta$ striking the bottom of the objective. (4 pts) *Note: The eyeball is designed to view objects at ∞, so the two rays from $+\theta$ should be parallel to each other when entering the eye (likewise for $-\theta$). We have started the ray diagram below.*

   (c) Why is the exit pupil (the image of the objective by the eyepiece) the best place to put your eye? What happens if the size of the exit pupil is larger than the eye's entrance pupil? (3 pts)

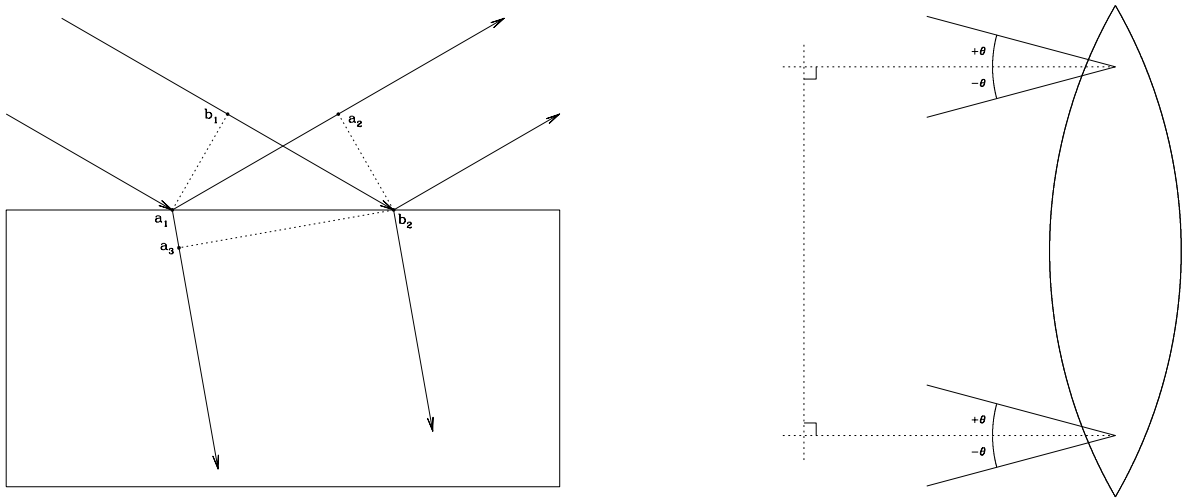

Figure 1: Diagrams for use with problems 1(a,b) and 3(b).

4. Show that a set of N slits (slit width $\ll \lambda$) spaced a distance d apart produces a diffraction pattern with intensity

   $$\mid T^2 \mid \sim \frac{1 - cos(N\delta k)}{1 - cos(\delta k)} \sim \frac{sin^2(N\gamma)}{sin^2(\gamma)}$$

   where

   $$\gamma = \frac{\delta k}{2} = \frac{1}{2}\, kd\, sin(\theta)$$

   which has maxima $\sim N^2$ at $\gamma = 0, \pi, 2\pi, \ldots$ (6 pts) *Hint: Show that the amplitude goes like $1 + x + x^2 + \ldots$, where $x = e^{i\delta k}$, and sum the series; then take the absolute square to get the intensity.*

5. What is the diffraction limit of the Keck telescopes at 5000Å, and at 2.2μ? What is the diffraction limit of HST at 5000Å, and at 2.2μ? In each case, how does the diffraction limit compare to the seeing limit? What is the diffraction limit of the Arecibo radio telescope for HI? Express all values in arcseconds. (5 pts)
6. Identify the primary regimes (by name and wavelength) used for astronomical observations along the electromagnetic spectrum. For each regime, identify a telescope+instrument and experiment combination which has produced a result which you find significant to modern astronomy, and briefly describe the result. (15 pts)