Name:_____
Date:_____

6 The Orbit of Mercury

Of the five planets known since ancient times (Mercury, Venus, Mars, Jupiter, and Saturn), Mercury is the most difficult to see. In fact, of the 6 billion people on the planet Earth it is likely that fewer than 1,000,000 (0.0002%) have *knowingly* seen the planet Mercury. The reason for this is that Mercury orbits very close to the Sun, about one third of the Earth's average distance. Therefore it is always located very near the Sun, and can only be seen for short intervals soon after sunset, or just before sunrise. It is a testament to how carefully the ancient peoples watched the sky that Mercury was known at least as far back as 3,000 BC. In Roman mythology Mercury was a son of Jupiter, and was the god of trade and commerce. He was also the messenger of the gods, being "fleet of foot", and commonly dipicted as having winged sandals. Why this god was associated with the planet Mercury is obvious: Mercury moves very quickly in its orbit around the Sun, and is only visible for a very short time during each orbit. In fact, Mercury has the shortest orbital period ("year") of any of the planets. You will determine Mercury's orbital period in this lab. [Note: it is very helpful for this lab exercise to review Lab #1, section 1.5.]

- *Goals:* to learn about planetary orbits
- *Materials:* a protractor, a straight edge, a pencil and calculator

Mercury and Venus are called "inferior" planets because their orbits are interior to that of the Earth. While the planets Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto are called "superior" planets, as their orbits lie outside that of the Earth. Because the orbits of Mercury and Venus are smaller than the Earth's, these planets can never be located very far from the Sun as seen from the Earth. As discovered by Galileo in 1610 (see Fig. 6.1), the planet Venus shows phases that look just like those of the Moon. Mercury also shows these same phases. As can be envisioned from Figure 6.1, when Mercury or Venus are on the far side of the Sun from Earth (a configuration called "superior" conjunction), these two planets are seen as "full". Note, however, that it is almost impossible to see a "full" Mercury or Venus because at this time the planet is very close to, or behind the Sun. When Mercury or Venus are closest to the Earth, a time when they pass between the Earth and the Sun (a configuration termed "inferior" conjunction), we would see a "new" phase. During their new phases, it is also very difficult to see Mercury or Venus because their illuminated hemispheres are pointed away from us, and they are again located *very* close to the Sun in the sky.

The best time to see Mercury is near the time of "greatest elongation". At the time of greatest elongation, the planet Mercury has its largest *angular* separation from the Sun as seen from the Earth. There are six or seven greatest elongations of Mercury each year. At the time of greatest elongation, Mercury can be located up to 28° from the Sun, and sets (or rises) about two hours after (or before) the Sun. In Figure 6.2, we show a diagram for the greatest elongation of Mercury that occurred on August 14, 2003. In this diagram, we

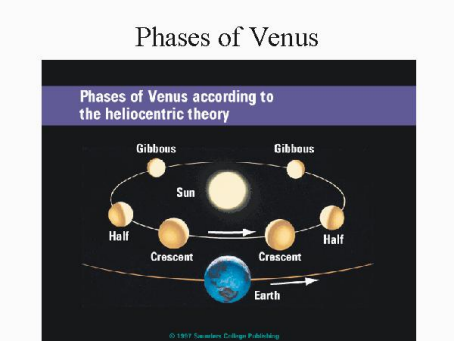


Figure 6.1: A diagram of the phases of Venus as it orbits around the Sun. The planet Mercury exhibits the same set of phases as it too is an "inferior" planet like Venus.

plot the positions of Mercury and the Sun at the time of sunset (actually just a few minutes before sunset!). As this diagram shows, if we started our observations on July 24$^{\text{th}}$, Mercury would be located close to the Sun at sunset. But as the weeks passed, the angle between Mercury and the Sun would increase until it reached its maximum value on August 14$^{\text{th}}$. After this date, the separation between the Sun and Mercury quickly decreases as it heads towards inferior conjunction on September 11$^{\text{th}}$.

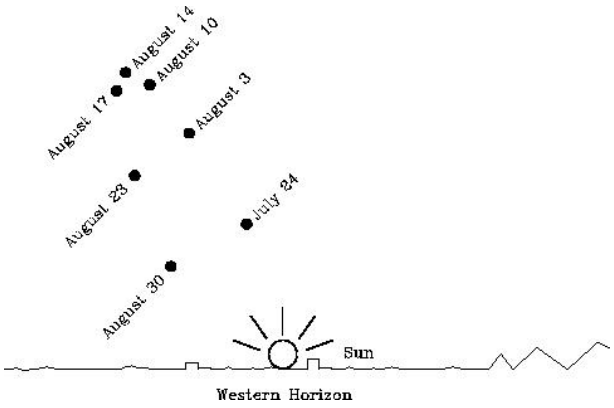


Figure 6.2: The eastern elongation of August, 2003. Mercury was at superior conjunction on July 5$^{\text{th}}$, and quickly moved around its orbit increasing the angular separation between it and the Sun. By July 24$^{\text{th}}$, Mercury could be seen just above the Sun shortly after sunset. As time passed, the angular separation between the Sun and Mercury increased, reaching its maximum value on August 14$^{\text{th}}$, the time of greatest Eastern elongation. As Mercury continued in its orbit it comes closer to the Earth, but the angular separation between it and the Sun shrinks. Eventually, on September 11$^{\text{th}}$, the time of inferior conjunction, Mercury passed directly between the Earth and the Sun.

You can see from Figure 6.2 that Mercury is following an orbit around the Sun: it was "behind" the Sun (superior conjunction) on July 5$^{\text{th}}$, and quickly races around its orbit until the time of greatest elongation, and then passes between the Earth and the Sun on September 11$^{\text{th}}$. If we used a telescope and made careful drawings of Mercury throughout this time, we would see the phases shown in Figure 6.3. On the first date in Figure 6.2 (July

24^{th}), Mercury was still on the far side of the Sun from the Earth, and almost had a full phase (which it only truly has at the time of superior conjunction). The disk of Mercury on July 24^{th} is very small because the planet is far away from the Earth. As time passes, however, the apparent size of the disk of Mercury grows in size, while the illuminated portion of the disk decreases. On August 14^{th}, Mercury reaches greatest elongation, and the disk is half-illuminated. At this time it looks just like the first quarter Moon! As it continues to catch up with the Earth, the distance between the two planets shrinks, so the apparent size of Mercury continues to grow. As the angular separation between Mercury and the Sun shrinks, so does the amount of the illuminated hemisphere that we can see. Eventually Mercury becomes a crescent, and at inferior conjunction it becomes a "new" Mercury.

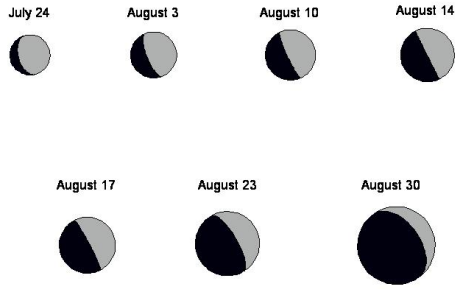


Figure 6.3: A diagram showing the actual appearance of Mercury during the August 2003 apparition. As Mercury comes around its orbit from superior conjunction (where it was "full"), it is far away from the Earth, so it appears small (as on July 24^{th}). As it approaches greatest elongation (August 14^{th}) it gets closer to the Earth, so its apparent size grows, but its phase declines to half (like a first quarter moon). Mercury continues to close its distance with the Earth so it continues to grow in size–but note that the illuminated portion of its disk shrinks, becoming a thin crescent on August 30^{th}. As Mercury passes between the Earth and Sun it is in its "new" phase, and is invisible.

6.1 Eastern and Western Elongations

The greatest elongation that occurred on August 14, 2003 was a "greatest Eastern elongation". Why? As you know, the Sun sets in the West each evening. When Mercury is visible *after* sunset it is located to the East of the Sun. It then sets in the West *after* the Sun has set. As you can imagine, however, the same type of geometry can occur in the morning sky. As Mercury passed through inferior conjunction on September 11^{th}, it moved into the morning sky. Its angular separation from the Sun increased until it reached "greatest Western elongation" on September 27^{th}, 2003. During this time, the phase of Mercury changed from "new" to "last quarter" (half). After September 27^{th} the angular separation between the Sun and Mercury shrinks, as does the apparent size of the disk of Mercury, as it reverses the sequence shown in Figure 6.3. A diagram showing the geometry of eastern and western elongations is shown in Figure 6.4. [Another way of thinking about what each of these means, and an

analogy that might come in useful when you begin plotting the orbit of Mercury, is to think about where Mercury is relative to the Sun at Noon. At Noon, the Sun is due south, and when facing the Sun, East is to the left, and West is to the right. Thus, during an Eastern elongation Mercury is to the left of the Sun, and during a Western elongation Mercury is to the right of the Sun (as seen in the Northern Hemisphere).]

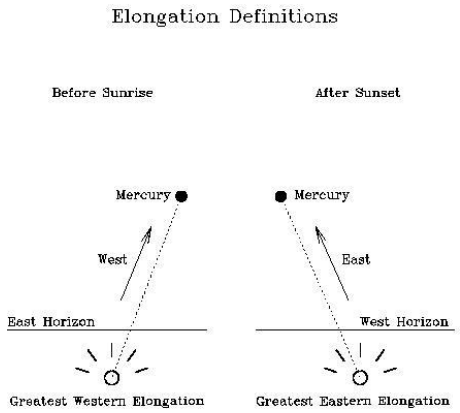


Figure 6.4: A diagram showing the geometry of greatest Western elongations (left side), and greatest Eastern elongations (right side). If you see Mercury–or any other star or planet–above the Western horizon after sunset, that object is located to the East of the Sun. The maximum angular separation between Mercury and the Sun at this time is called the greatest Eastern elongation. A greatest Western elongation occurs when Mercury is seen in the East *before* sunrise.

6.2 Why Greatest Elongations are Special

We have just spent a lot of time describing the greatest elongations of Mercury. We did this because the time of greatest elongation is very special: it is the only time when we know where an inferior planet is in its orbit (except in the rare cases where the planet "transits" across the face of the Sun!). We show why this is true in Fig 6.5. In this figure, we have represented the orbits of Mercury and the Earth as two circles (only about one fourth of the orbits are plotted). We have also plotted the positions of the Earth, the Sun, and Mercury. At the time of greatest elongation, the angle between the Earth, Mercury and the Sun is a right angle. If you were to plot Mercury at some other position in its orbit, the angle between the Earth, Mercury and the Sun would not be a right angle. Therefore, the times of greatest elongation are special, because at this time we know the exact angle between the Earth, Mercury, and the Sun. [You can also figure out from this diagram why Mercury has only one-half of its disk illuminated (a phase of "first quarter").]

Of course, plotting only one elongation is not sufficient to figure out the orbit of Mercury–you need to plot many elongations. In today's lab exercise, you will plot thirteen greatest elongations of Mercury, and trace-out its orbit. There are a lot of angles in this lab, so you need to get comfortable with using a protractor. Your TA will help you figure this out. But the most critical aspect is to not confuse eastern and western elongations. Look at Figure 6.5 again. What kind of elongation is this? Well, as seen from the Earth, Mercury is to the

left of the Sun. As described earlier, if Mercury is to the left of the Sun, it is an *eastern elongation*.

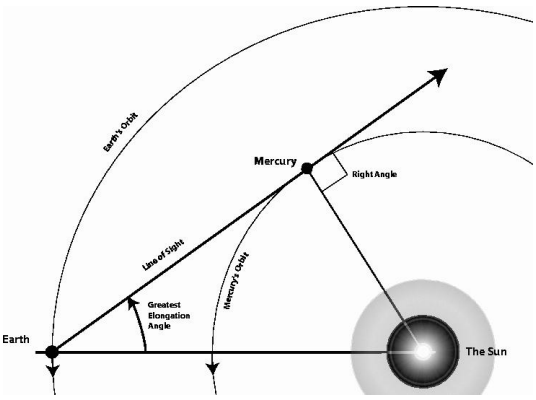


Figure 6.5: A diagram showing the orbital geometry of the Earth and Mercury during a greatest Eastern elongation. The orbits of the Earth and Mercury are the two large circles. The line of sight to Mercury at the time of greatest elongation is indicated. Note that at this time the angle between the Earth, Mercury, and the Sun is a *right* angle. The direction of motion of the two planets is shown by the arrows on the orbits.

6.3 The Orbits of Earth and Mercury

As shown in the previous diagram, both the Earth and Mercury are orbiting the Sun. That means that every single day they are at a different position in their orbits. Before we can begin this lab, we must talk about how we can account for this motion! A year on Earth, the time it takes the Earth to complete one orbit around the Sun, is 365 days. If we assume that the Earth's orbit is a perfect circle, then the Earth moves on that circle by about 1 degree per day. Remember that a circle contains 360 degrees (360°). If it takes 365 days to go 360°, the Earth moves 360°/365 = 0.986 degrees per day (°/day). For this lab, we will assume that the Earth moves exactly one degree per day which, you can see, is very close to the truth. How far does the Earth move in 90 days? 90 degrees! How about 165 days?

6.4 The Data

In Table 6.1, we have listed thirteen dates for greatest elongations of Mercury, as well as the angle of each greatest elongation. **Note that the elongations are either East or West!** In the third column, we have listed something called the Julian date. Over long time intervals, our common calendar is very hard to use to figure out how much time has elapsed. For example, how many days are their between March 13$^{\text{th}}$, 2001 and December 17$^{\text{th}}$ 2004? Remember that 2004 is a leap year! This is difficult to do in your head. To avoid such calculations, astronomers have used a calendar that simply counts the days that have passed since some distant day #1. The system used by astronomers sets Julian date 1 to January 1$^{\text{st}}$, 4713 BC (an arbitrary date chosen in the sixteenth century). Using this calendar, March 13$^{\text{th}}$, 2001 has a Julian date of 2451981. While December 17$^{\text{th}}$ 2004 has a

Table 6.1: Elongation Data For Mercury

#	Date	Elongation Angle	Julian Date	Days	Degrees
#1	Sep. 1, 2002	27.2 degrees east	2452518	–	–
#2	Oct. 13, 2002	18.1 degrees west	2452560	42	42
#3	Dec. 26, 2002	19.9 degrees east	2452634		
#4	Feb. 4, 2003	25.4 degrees west	2452674		
#5	Apr. 16, 2003	19.8 degrees east	2452745		
#6	Jun. 3, 2003	24.4 degrees west	2452793		
#7	Aug. 14, 2003	27.4 degrees east	2452865		
#8	Sep. 27, 2003	17.9 degrees west	2452909		
#9	Dec. 09, 2003	20.9 degrees east	2452982		
#10	Jan. 17, 2004	23.9 degrees west	2453021		
#11	Mar. 29, 2004	18.8 degrees east	2453093		
#12	May 14, 2004	26.0 degrees west	2453139		
#13	Jul. 27, 2004	27.1 degrees east	2453213		

Julian date of 2453356. Taking the difference of these two numbers (2453356 − 2451981) we find that there are 1,375 days between March 13$^{\text{th}}$, 2001 and December 17$^{\text{th}}$ 2004.

Exercise #1: Fill-in the Data Table.

The fourth and fifth columns of the table are blank, and must be filled-in by you. The fourth column is the number of days that have elapsed between elongations in this table (that is, simply subtract the Julian date of the previous elongation from the *following* elongation). We have worked the first one of these for you as an example. The last column lists how far the Earth has moved in degrees. This is simply the number of days times the number 1.0! As the Earth moves one degree per day. (If you wish, instead of using 1.0, you could multiply this number by 0.986 to be more accurate. You will get better results doing it that way.) So, if there are 42 days between elongations, the Earth moves 42 degrees in its orbit (or 41.4 degrees using the correct value of 0.986 °/day). (*10 points*)

Exercise #2: Plotting your data.

Before we describe the plotting process, review Figure 6.5. Unlike that diagram, you do not know what the orbit of Mercury looks like–this is what you are going to figure out during this lab! But we do know two things: the first is that the Earth's orbit is nearly a perfect circle, and two, that the Sun sits at the exact center of this circle. On the next page is a figure containing a large circle with a dot drawn at the center to represent the Sun. At one position on the large circle we have put a little "X" as a reference point. The large circle here is meant to represent the Earth's orbit, and the "X" is simply a good starting point.

To plot the first elongation of Mercury from our data table, using a pencil, draw a line

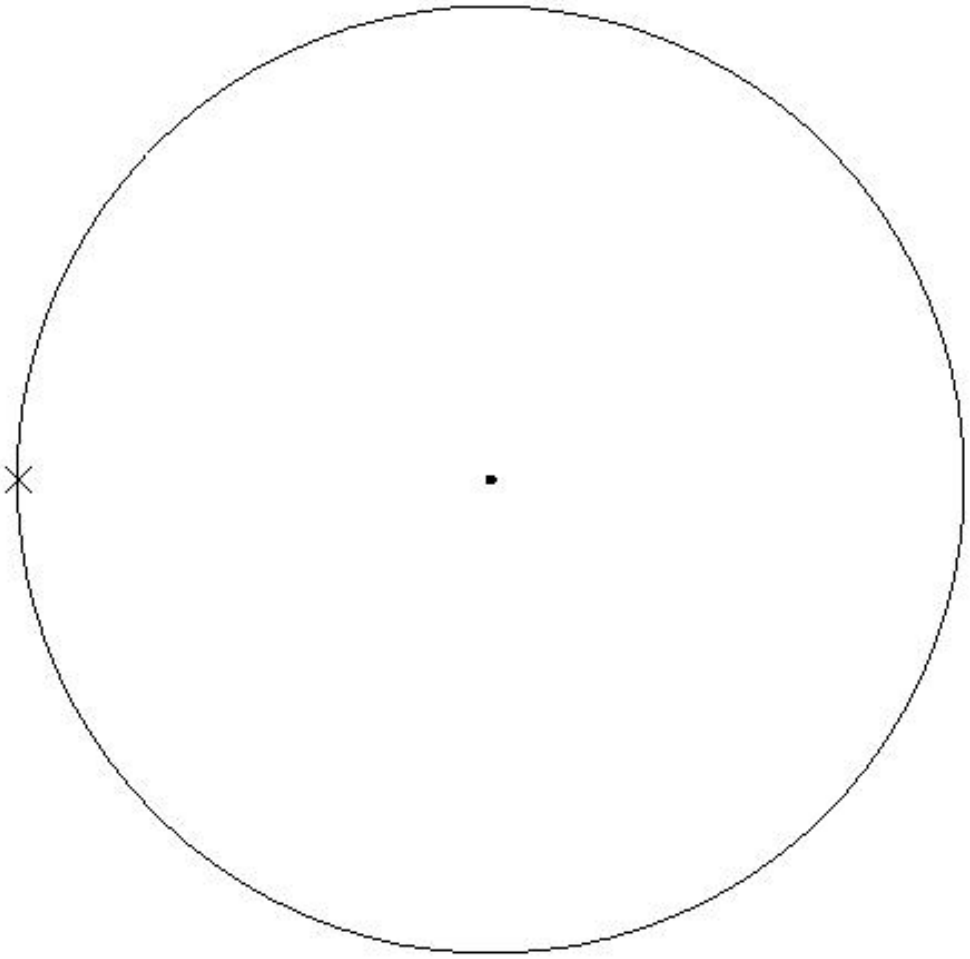
connecting the X, and the Sun using a straight edge (ruler or protractor). The first elongation in the table (September 1, 2002) is 27.2 degrees East. Using your protractor, put the "X" that marks the Earth's location at the center hole/dot on your protractor. Looking back to Figure 6.5, that elongation was also an *eastern* elongation. So, using that diagram as a guide, measure an angle of 27.2 degrees on your protractor and put a small mark on the worksheet. Now, draw a line from the Earth's location through this mark just like the "line of sight" arrow in Figure 6.5. Now, rotate your protractor so that the 90 degree mark is on this line and towards the position of the Earth, while the reference hole/dot is on the same line. Slide the protractor along the line until the 0° (or 180°) reference line intersects the center of the Sun. Mark this spot with a dark circle. This is the position of Mercury!

This is the procedure that you will use for all of the elongations, so if this is confusing to you, have your TA come over and clarify the technique for you so that you don't get lost and waste time doing this incorrectly.

Ok, now things become slightly more difficult—the Earth moves! Looking back to Figure 6.5, note the arrows on the orbits of Earth and Mercury. This is the direction that both planets are moving in their orbits. For the second elongation, the Earth has moved 42 degrees. We have to locate where the Earth is in its orbit before we can plot the next elongation. So, now put the center hole/dot of your protractor on the Sun. Line up the 0/180 degree mark with the first line that connected the Earth and Sun. Measure an angle of 42 degrees (in the correct direction) and put a small mark. Draw a line through this mark that intersects the postion of the Sun, the mark, and the orbit of the Earth. Put an X where this line intersects the Earth's orbit. This is the spot from where you will plot the next elongation of Mercury.

Now, repeat the process for plotting this next elongation angle. Note, however, that this elongation is a western elongation, so that the line of sight arrow this time will be to the right of the Sun. It is extremely important to remember that on eastern elongations the line of sight arrow to Mercury goes to the left of the Sun, while during western elongations it goes to the right of the Sun.

[Hints: It is helpful to label each one of the X's you place on the Earth's orbit with the elongation number from Table 6.1. This will allow you to go back and fix any problems you might find. Note that you will have a large number of lines drawn in this plot by the time you get finished. Use a sharp pencil so that you can erase some/all/pieces of these lines to help clean-up the plot and reduce congestion. You might also find it helpful to simply put a "left" or a "right" each time you encounter East and West in Table 6.1 to insure you plot your data correctly.] **Plot all of your data (*28 points*).**



Exercise #3: Connecting the dots.

Note that planets move on smooth, almost circular paths around the Sun. So try to connect the various positions of Mercury with a smooth arc. Do all of your points fit on this closed curve? If not, identify the bad points and go back and see what you did wrong. Correct any bad elongations.

1) Is Mercury's orbit circular? Describe its shape. (*4 points*)

Exercise #4: Finding the *semi-major axis* of Mercury's orbit.

Using a ruler, find the position on Mercury's orbit that is closest to the Sun ("perihelion") and mark this spot with an "X". Now find the point on the orbit of Mercury that is furthest from the Sun ("aphelion") and mark it with an "X". Draw a line that goes through the Sun that comes closest to connecting these two positions–note that it is likely that these two points will not lie on a line that intercepts the Sun. *Just attempt to draw the best possible line connecting these two points that passes through the Sun.*

2) Measure the length of this line. Astronomers call this line the major axis of the planet's orbit, and abbreviate it as *"a"*. Divide the length you have just measured by two, to get the "semi-major" axis of Mercury's orbit: _____ (mm). Measure the diameter of the Earth's orbit and divide that number by two to get the Earth's semi-major axis: _____ (mm).

Divide the semi-major axis of Mercury by that of the Earth: _____ AU. Since the semi-major axis of the Earth's orbit is defined to be "one astronomical unit", this ratio tells us the size of Mercury's semi-major axis in astronomical units (AU). (*4 points*)

3) As you have probably heard in class, the fact that the orbits of the planet's are ellipses, and not circles, was discovered by Kepler in about 1614. Mercury and Pluto have the most unusual orbits in the solar system in that they are the most non-circular. Going back to the line you drew that went through the Sun and that connected the points of perihelion and aphelion, measure the lengths of the two line segments:

Perihelion (p) = _____ mm. Aphelion (q) = _____ mm.

Astronomers use the term *eccentricity* ("e") to measure how out-of-round a planet's orbit is, and the eccentricity is defined by the equation:

e = (q − p)/(p + q) = _____

Plug your values into this equation and calculate the eccentricity of Mercury's orbit. (*4 points*)

4) The eccentricity for the Earth's orbit is e = 0.017. How does your value of the eccentricity for Mercury compare to that of the Earth? Does the fact that we used a circle for the Earth's orbit now seem justifiable? (5 points)

Exercise #5: The orbital period of Mercury.

Looking at the positions of Mercury at elongation #1, and its position at elongation #2, approximately how far around the orbit did Mercury move in these 42 days? Estimate how long you think it would take Mercury to complete one orbit around the Sun: _____ days. (2 points)

Using Kepler's laws, we can estimate the orbital period of a planet (for a review of Kepler's laws, see lab #4). Kepler's third law says that the orbital period squared (P^2) is proportional to the cube of the semi-major axis (a^3): $P^2 \propto a^3$. This is a type of equation you might not remember how to solve (if you have not done so already, review Lab #1 section 1.5). But let's take it in pieces:

$a^3 = a \times a \times a =$ _____

Plug-in your value of a for Mercury and find its cube.

To find the period of Mercury's orbit, we now need to take the square root of the number you just calculated (see your TA if you do not know whether your calculator can perform this operation). (*4 points*)

$$P = \sqrt{a^3} = \text{\_\_\_\_\_}. \tag{8}$$

Now, the number you just calculated probably means nothing to you. But what you have done is calculate Mercury's orbital period as a fraction of the Earth's orbital period (that is because we have been working in AU, a unit that is defined by the Earth-Sun distance). Since the Earth's orbital period is exactly 365.25 days, find Mercury's orbital period by multiplying the number you just calculated for Mercury by 365.25:

P_{orb}(Mercury) = _____ days.

5) How does the orbital period you just calculated using Kepler's laws compare to the one you estimated from your plot at the beginning of this exercise? (4 points)

Name:_____
Date:_____

6.5 Take Home Questions (*35 points*)

Before you leave lab, your TA will give you the real orbital period of Mercury, as well as its true semi-major axis (in AU) and its orbital eccentricity.

- Compare the precisely known values for Mercury's orbit with the ones you derived. How well did you do?
- What would be required to enable you to do a better job?
- Describe how you might go about making the observations on your own so that you could create a data table like the one in this lab. Do you think this could be done with just the naked eye and some sort of instrument that measured angular separation? What else might be necessary?

6.6 Extra Credit

In this lab you have measured three of the five quantities that completely define a planet's orbit. The other two quantities are the orbital inclination, and the longitude of perihelion. Determining the orbital inclination, the tilt of the plane of Mercury's orbit with respect to the Earth's orbit, is not possible using the data in this lab. But it is possible to determine the longitude of perihelion. Astronomers define the zero point of solar system longitude as the point in the Earth's orbit at the time of the Vernal Equinox (the beginning of Spring in the northern hemisphere). In 2004, the Vernal Equinox occurs on March 20. If you notice, one of the elongations in the table (#11) occurs close to this date. Thus, you can figure out the true location of the Vernal Equinox by moving back from position #11 by the right number of degrees. The longitude of Mercury's perihelion is just the angle measured counterclockwise from the Earth's vernal equinox. You should find that your angle is larger than 180 degrees. Subtract off 180 degrees. How does your value compare with the precise value of 77°? (*3 points*)