Overview: Woes with Waves

Galileo (1632): The Principle of Relativity
The Galilean Transformation

Maxwell's Equations (1873)
Lorentz contraction (1895)?

The Michelson-Morley Experiment (1881, 1887)
Attempts to Preserve the Æther
Lorentz contraction (1895)
... but aberration, etc.

Einstein (1905): The Special Theory of Relativity
The "Lorentz Transformations" for space and time
... also leading to to $E = mc^2$.

The Photon Revival

The Quantization of the Radiation Field (Max Planck 1901)
&
The Photoelectric Effect (Einstein 1905)

$$E = h\nu$$

The Principle of Relativity

Galileo (1632): *Dialogue Concerning the Two Chief World Systems*

All physical laws appear the same to observers in unaccelerated relative motion*

The speed and direction of unaccelerated motion can be defined only with respect to ("relative to") another observer.

* "If, relative to S, S' is a uniformly moving co-ordinate system devoid of rotation, then natural phenomena run their course with respect to S' according to exactly the same general laws as with respect to S."

***i.e.*, Aristotle was wrong!**

Newtonian Reformulation of the Principle of Relativity

Time is absolute, but location and motion are relative.

Accelerations, including those of Newtonian Gravitation, are also invariant.

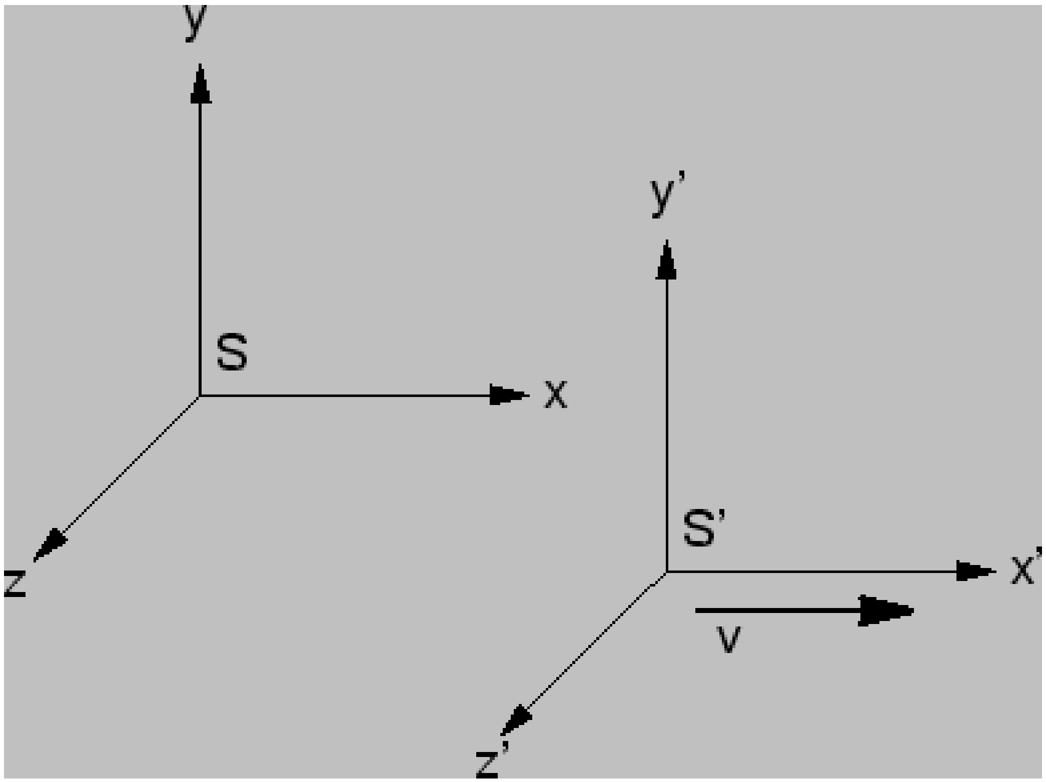
Newton's Laws of Mechanics are invariant under Galilean Transformation.

... so what's a Galilean Transformation?

The Galilean Transformation

Systems S and S' with **event** coordinates (**x**,t) and (**x'**,t')
x = (x,y,z) and **x'** = (x',y',z')

The two systems are in unaccelerated motion with relative velocity **v**



The Galilean Transformation asserts that:

- **Time is absolute**

$$t = t'$$

- **Velocities add linearly**

$$\mathbf{x} = \mathbf{x}' + \mathbf{v}t$$

....where **v** is the (constant) relative velocity.

It then also follows that :

$$\mathbf{u} = \mathbf{u}' + \mathbf{v}$$

....where **u** and **u'** are the velocities observed in the two frames, and

$$\mathbf{a} = \mathbf{a}'$$

....where **a** and **a'** are accelerations observed in the two frames.

....

Again, Newton's Laws of Mechanics (and Newton's Law of Gravity) are invariant under a Galilean Transformation

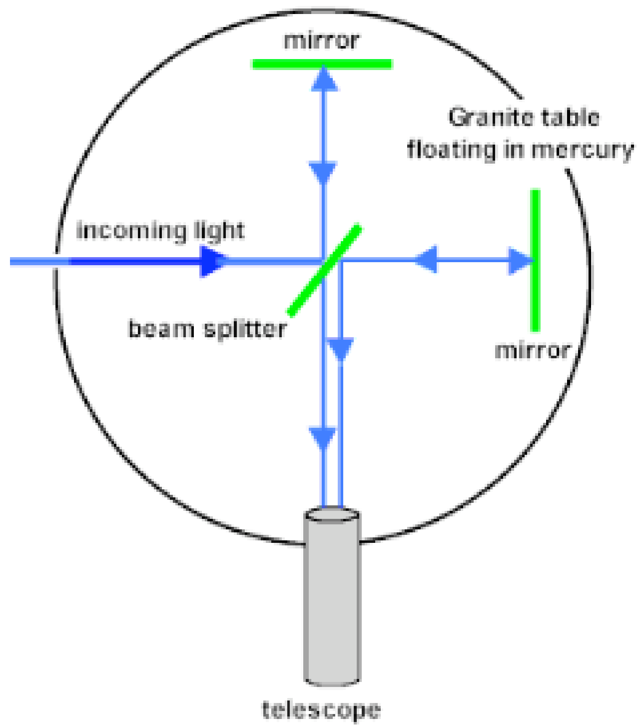
but

Maxwell's Equations are not...............and that was embarrassing!)

The Demise of the Luminiferous Æther

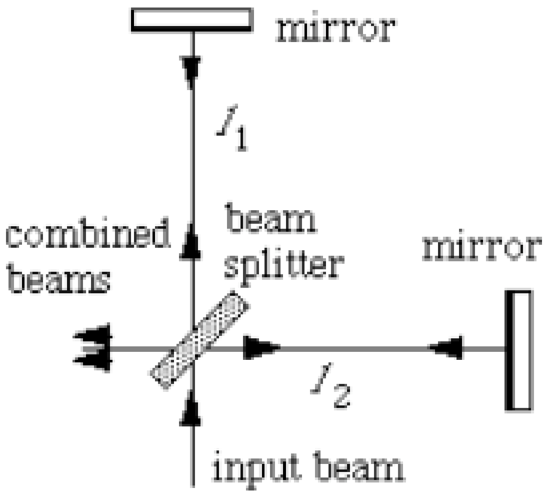
The Michelson-Morley Experiment (1881, 1887)

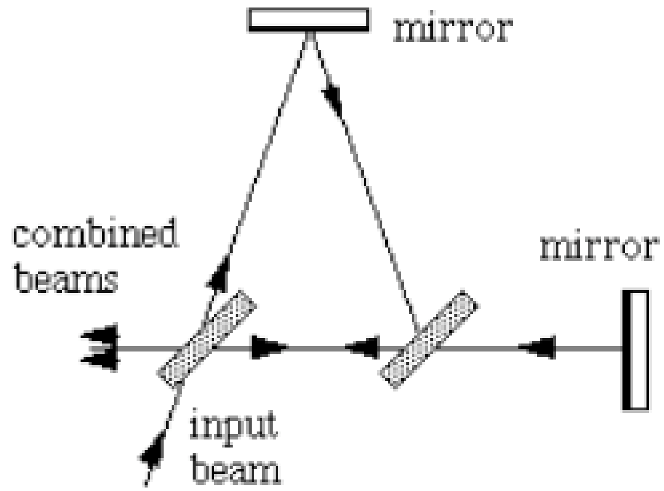
A failed attempt to measure the speed of the Earth through the æther
.... a failure which garnered a Nobel Prize in 1907.

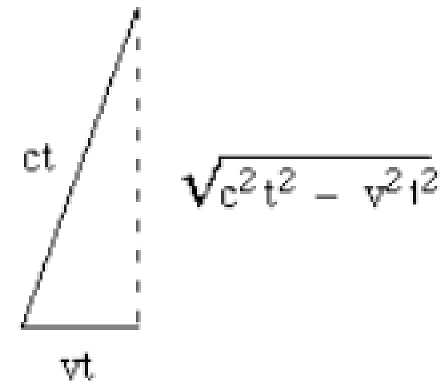


Interference fringes observed
(Recollect Young's Double-Slit Experiment)

Geometry of the Michelson-Morley Experiment







Transit Times for Beams

Fixed		**Moving**
$t_1 = 2l_1/c$	*versus*	$t_1 = (2l_1/c)\gamma$
$t_2 = 2l_2/c$	*versus*	$t_2 = (2l_2/c)\gamma^2$
$\Delta t = 2(l_2 - l_1)/c$	*versus*	$\Delta t = 2(l_2\gamma - l_1)\gamma c.$

.... where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz Factor

Expectations

(Assume equal length arms: $l_1 = l_2 = L$)

$\Delta t = 0$	*versus*	$\Delta t = (2L/c)\ (\gamma - 1)\gamma$

Execution of the Michelson-Morley Experiment

A Numerical Example

Earth's Orbital Speed is $v = 30\ km\ s^{-1} = 1 \times 10^{-4}\ c$

The Lorentz Factor gives $(\gamma - 1) = 5 \times 10^{-9}$

Equal-Arm Interferometer: $l_1 = l_2 = L = 1$ meter

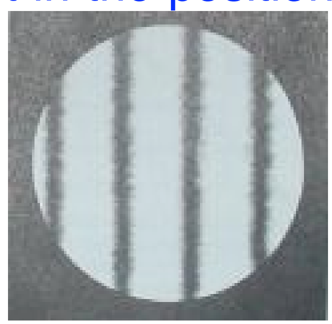
$\Delta t = (2L/c)\ (\gamma - 1)\gamma = 3.3 \times 10^{-17}$ second

but

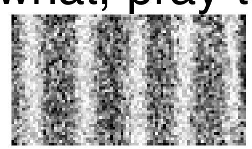
$c\Delta t = 1 \times 10^{-8}\ m = 10\ nm$

... or about 0.02λ for visible light (500 nm)

This 2% shift gives a 4% shift in the positions of the interference fringes.



.........and what, pray tell, is this?



The Return of the Photon

Light is Clearly a Wave!

• **Experiment: Diffraction and Interference (Young 1803)**
(.... and also polarization)
and
• **Theory: Maxwell's Electromagnetism (Maxwell 1862)**
... so light is an electromagnetic wave
but

• **Experiment: There is no Luminiferous Æther (Michelson 1891)**
... so, if it's a wave, what carries the wave?

Photons Redux

Were Aristotle ("photons") and Newton ("corpuscles") right after all?
While waves need a medium to carry them particles do not

• The Quantization of the Radiation Field (Max Planck 1901)
$E = h\nu$?

• The Photoelectric Effect (Einstein 1905)

$E = h\nu$!

Quantizing Radiation

The Thermodynamics of Radiation

Observation and Experiment: $B(\lambda,T)$

- **Wien's Law (1894):** $\lambda_{max}T = const.$

- **Stefan's Law (1879):** $F = \sigma T^4$

but

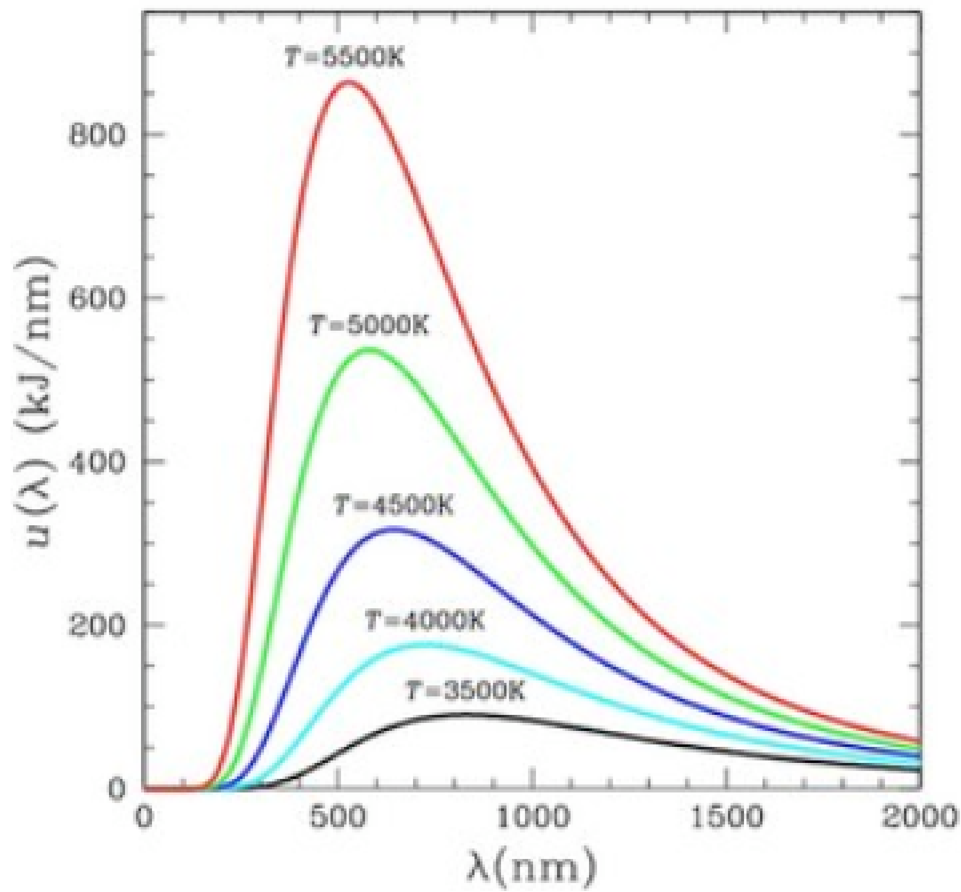
Theoretical Considerations: $B(\lambda,T)$

- **Rayleigh-Jeans Law (1900):** $B(\lambda,T) = 2ckT\,\lambda^{-4}$

a.k.a.

"The Ultraviolet Catastrophe"

The Planck Function



Quantization of Radiation

Max Planck (1901)

Hypothesis: Oscillators can only emit radiation as photons ("quanta")

(Maxwell's equations give continuous emission.)

Wien's Law would then suggest $E = h\nu$ where h is a constant to be determined

Ansatz

At low frequencies the Rayleigh-Jeans form is observed:

$$B(\lambda,T) = 2ckT\,\lambda^{-4}$$

At high frequencies the Wien distribution is observed:

$B(\lambda,T) \sim \lambda^{-5}e^{-a/T\lambda}$ where **a** is a constant

A function that fits both of these is the Planck Function:

$$B(\lambda,T) = (2hc^2/\lambda^5)(e^{hc/\lambda kT} - 1)^{-1}$$

which also happens to fit the "in between" observations as well!

Theoretical Justification & Explanation

Statistical thermodynamics assuming an "oscillator" of frequency ν can only have amplitudes corresponding to energies which are integer multiples of $E = h\nu$ gives the Planck Function.

The Photoelectric Effect

Observations

- **Alexandre Baquerel (1839) - conductivity of solutions**
- **Heinrich Hertz (1887) - enhancement of sparks in gaps**
- **J.J. Thomson (1899) - enhanced discharge in Crookes tubes**

(Note: Radiation must be incident on the cathode!)

Basic Characteristics of the Photoelectric Effect

(1) There is a threshold frequency ν_o for the effect:
Only radiation with $\nu > \nu_o$ generates photoelectrons
This threshold frequency depends upon the material

(2) The current increases linearly with light intensity:
The current is proportional to the number of photoelectrons

(3) Photoelectron energy increases linearly with frequency:

$$E = h(\nu - \nu_o)$$

Suggested by Philipp von Lenard (1902)
Predicted by Albert Einstein (1905)
Confirmed by Robert Millikan (1905)

Explaining the Photoelectric Effect

(Albert Einstein 1905)
"On a Heuristic Viewpoint Concerning the Production and Transformation of Light"

Why waves don't work...and how photons do.
Photoelectric current does show a threshold in frequency
Photoelectric current doesn't show an intensity threshold

Moreover
The photon energy is identical to that suggested by Planck (1901):
$E = h\nu$
where h is identical to Planck's Constant of 1901

Testable prediction (Millikan 1915) : $E = h(\nu - \nu_o)$

... which seems to indicate that light is composed of photons (Aristotle), corpuscles (Newton), or quanta (Planck) after all.

... but what about diffraction, interference, polarization, and Maxwell's electromagnetic waves?

Light in 1905: Prologue to Special Relativity

Observation: Light exhibits all of the properties and phenomena of waves
(frequency, wavelength, speed, amplitude, polarization)
...but there is no medium (æther) to carry the waves
...but some phenomena (see above) were not explained by a wave picture

Observation: Light exhibits all the characteristic of moving particles
(countability, energy, momentum, velocity)
... but the particles all move at the same speed c, independent of energy
... but some phenomena (*e.g., interference)* were not explained by a photon picture

Observation: The speed of light is the same for all observers
... whatever the state of motion of the observer or the source
... which is inconsistent with waves moving in a fixed medium
and
... is inconsistent with the motion of (classical) particles

Fact: Maxwell's Equations are not invariant under Galilean Transformations
... which sounds abstruse, but was one of the oddest results
(because Maxwell's equations survived the most stringent tests)
.

...which was all thought rather strange.